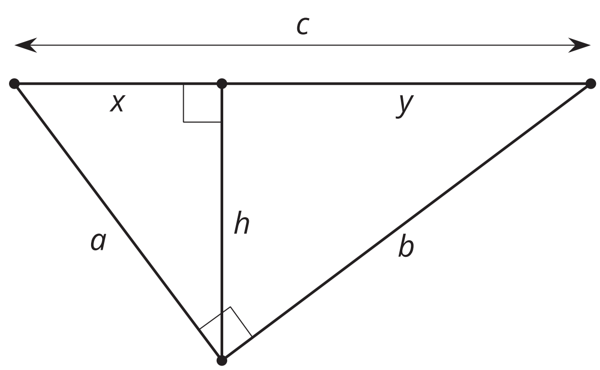
Proving the Pythagorean Theorem

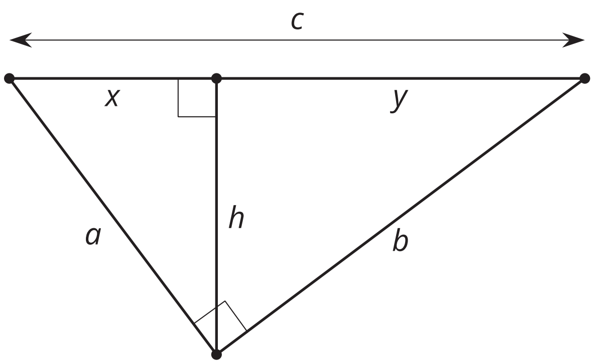
Let’s prove the Pythagorean Theorem.

14.1 Notice and Wonder: Variable Version

What do you notice? What do you wonder?



14.2 Prove Pythagoras Right



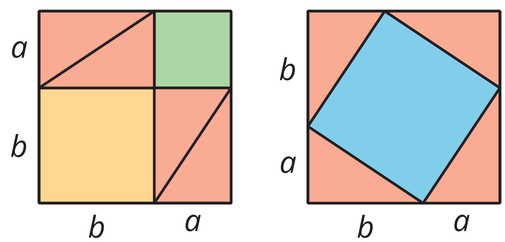
Elena is playing with the equivalent ratios she wrote using this diagram. She rewrites $\frac{a}{x} = \frac{c}{a}$ as $a^2 = xc$. Diego notices and comments, “I got $b^2 = yc$. The a^2 and b^2 remind me of the Pythagorean Theorem.” Elena says, “The Pythagorean Theorem says that $a^2 + b^2 = c^2$. I bet we could figure out how to show that.”

1. How did Elena get from $\frac{a}{x} = \frac{c}{a}$ to $a^2 = xc$?

2. What equivalent ratios of side lengths did Diego use to get $b^2 = yc$?

3. Prove $a^2 + b^2 = c^2$ in a right triangle with legs length a and b and hypotenuse length c.

14.3 An Alternate Approach



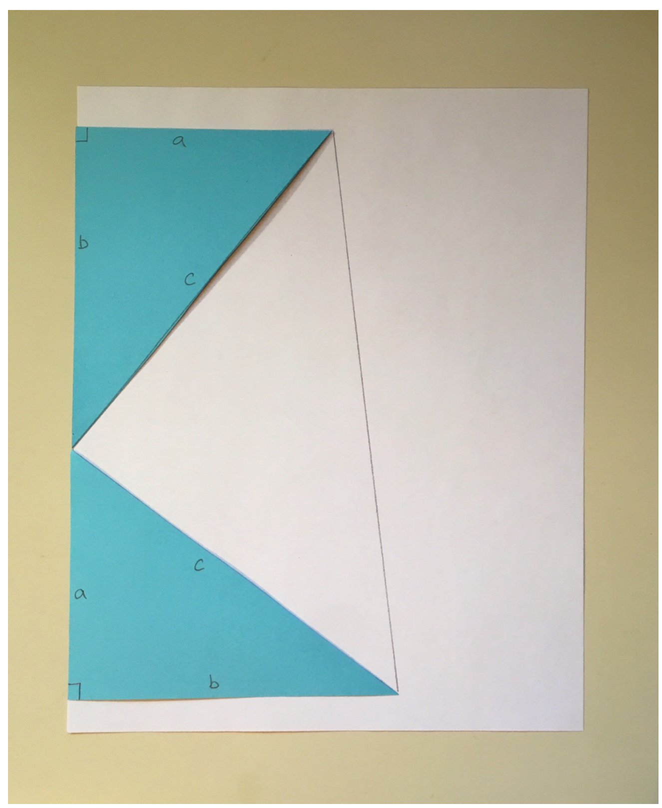
When Pythagoras proved his theorem, he used the two images shown here. Can you figure out how he used these diagrams to prove that $a^2 + b^2 = c^2$ in a right triangle with a hypotenuse of length c?

Are you ready for more?

James Garfield, the 20th president, proved the Pythagorean Theorem in a different way.

- Cut out 2 congruent right triangles
- Label the long sides b, the short sides a and the hypotenuses c.
- Align the triangles on a piece of paper, with one long side and one short side in a line. Draw the line connecting the other acute angles.

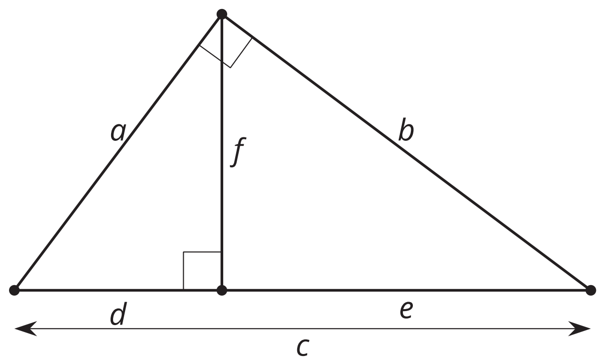
How does this diagram prove the Pythagorean Theorem?



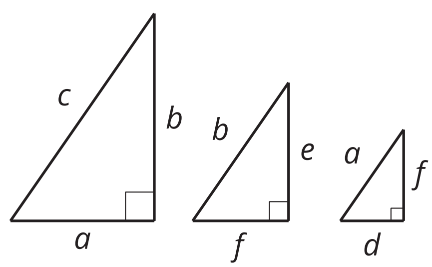
Lesson 14 Summary

In any right triangle with legs a and b and hypotenuse c, we know that $a^2 + b^2 = c^2$. We call this the Pythagorean Theorem. But why does it work?

We can use an altitude drawn to the hypotenuse of a right triangle to prove the Pythagorean Theorem.



We can use the Angle-Angle Triangle Similarity Theorem to show that all 3 triangles are similar. Because the triangles are similar, the corresponding side lengths are in the same proportion.



Because the largest triangle is similar to the smaller triangle, $\frac{c}{a} = \frac{a}{d}$. Because the largest triangle is similar to the middle triangle, $\frac{c}{b} = \frac{b}{e}$. We can rewrite these equations as $a^2 = cd$ and $b^2 = ce$.

We can add the 2 equations to get that $a^2 + b^2 = cd + ce$, or $a^2 + b^2 = c(d + e)$. From the original diagram we can see that $d + e = c$, so $a^2 + b^2 = c(c)$, or $a^2 + b^2 = c^2$.