



Let's Make a Box

Let's investigate volumes of different boxes.

1.1

Which Three Go Together: Boxes

Which three go together? Why do they go together?

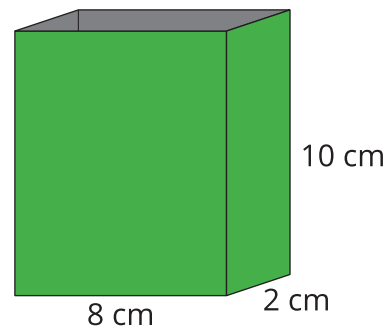
A

length: 4cm

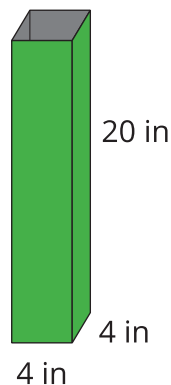
width: 8cm

height: 10cm

B

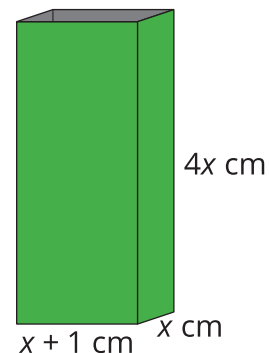


C



D

$$V = 320 \text{ cm}^3$$



1.2

Building Boxes

Your teacher will give you some supplies to construct an open-top box.

- 1. Cut out a square from each corner of a sheet of paper, and then fold up the sides.
- 2. Calculate the volume of your box, and complete the table with your information.

| side length of square cutout (in) | length (in) | width (in) | height (in) | volume of box (in ³) |
|-----------------------------------|-------------|------------|-------------|----------------------------------|
| 1 | | | | |
| | | | | |
| | | | | |

1.3

Building the Biggest Box

- 1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.



Pause here so your teacher can review your plan.

- 2. Write an expression for $V(x)$.
- 3. Use graphing technology to create a graph representing $V(x)$. Approximate the value of x that would allow you to construct an open-top box with the largest volume possible from one piece of paper.



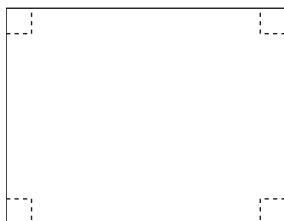
Are you ready for more?

The surface area $A(x)$, in square inches, of the open-top box is also a function of the side length x , in inches, of the square cutouts.

1. Find one expression for $A(x)$ by adding the area of the five faces of our open-top box.
2. Find another expression for $A(x)$ by subtracting the area of the cutouts from the area of the paper.
3. Show algebraically that these two expressions are equivalent.

Lesson 1 Summary

A box can be created by removing squares from each corner of a rectangle of paper.



Let $V(x)$ be the volume of the box in cubic inches, where x is the side length, in inches, of each square removed from the four corners.

To define V using an expression, we can use the fact that the volume of a cube is $(length)(width)(height)$. If the piece of paper we start with is 3 inches by 8 inches, then:

$$V(x) = (3 - 2x)(8 - 2x)(x)$$

What are some reasonable values for x ? Cutting out squares with side lengths less than 0 inches doesn't make sense, and similarly, we can't cut out squares larger than 1.5 inches, since the short side of the paper is only 3 inches (since $3 - 1.5 \cdot 2 = 0$). You may remember that the name for the set of all the input values that make sense to use with a function is the domain. Here, a reasonable domain is somewhere larger than 0 inches but less than 1.5 inches, depending on how well we can cut and fold!

By graphing this function, it is possible to find the maximum value within a specific domain. Here is a graph of $y = V(x)$ for x values between 0 and 1.5. It looks like the largest volume we can get for a box made this way from a 3-inch by 8-inch piece of paper is about 7.4 in^3 .

