Unit 1, Lesson 17

Rotate and Tessellate

Goals

- Create tessellations and designs with rotational symmetry using rigid transformations.
- Explain (orally and in writing) the rigid transformations needed to move a tessellation or design with rotational symmetry onto itself.

Learning Targets

- I can repeatedly use rigid transformations to make interesting repeating patterns of figures.
- I can use properties of angle sums to reason about how figures will fit together.

Lesson Narrative

In this lesson, students use the language of transformations to produce, describe, and investigate patterns in the plane. This is a direct extension of earlier work with triangles, where three triangles were arranged in the plane to show that the sum of the angles in a triangle is 180 degrees, and where four copies of a triangle were arranged in a large square, cutting out a smaller square in the middle.

Throughout this unit, students have identified different types of rigid transformations, including translations, rotations, reflections, and sequences of these. They have also used rigid transformations to define congruence and investigate the sum of the angles in a triangle. In this lesson, students apply this learning in a creative way, as they examine patterns of shapes.

Both of the activities in this lesson are optional. Depending on time, students can complete one or both activities. In the first optional activity, students create a tessellation using given shapes. In the second optional activity, students create a design of their choosing that highlights rotational symmetry. In both activities, students must use the structure of rigid transformations to create a design (MP7).

Standards

Building On	**4.MD.C, 7.G.B.5**
Addressing	**8.G.A**

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1, Activity 2, Activity 3

Materials to Copy

- Deducing Angle Measures Handout (1 copy for every 2 students): Activity 1

Required Preparation

Activity 3:

Students may benefit from using graph paper and isometric graph paper, but these materials are optional.

Student Facing Learning Goals

Let's make complex patterns using transformations.

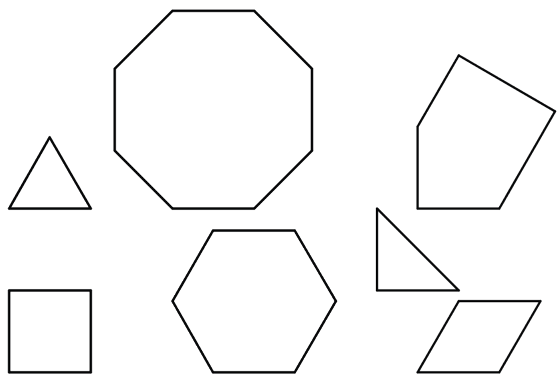
17.1 Deducing Angle Measures

Warm-up

10 min

Activity Narrative

Throughout this lesson, students build different patterns with copies of some polygons. In this activity, they make some copies of each polygon and arrange them in a circle. They calculate some of the angles of the polygons while also gaining an intuition for how the polygons fit together. Here are the figures included in the blackline master:



Students might use a protractor to measure angles, but the measures of all angles can also be deduced. In the first question in the task, students are instructed to fit copies of an equilateral triangle around a single vertex. Six copies fit, leading them to deduce that each angle measures 60 degrees because $360 \div 6 = 60$. For the other shapes, they can reason about angles that sum to 360 degrees, angles that sum to a line, and angles that sum to a known angle.

Standards

Building On **4.MD.C, 7.G.B.5**

Launch

Provide access to geometry toolkits. Distribute one half-sheet (that contains 7 shapes) to each student. Give students 1–2 minutes of individual work time. Pause students after the first question, and invite students to share how they arranged the triangles around a single vertex. Demonstrate using tracing paper if needed. Give students an additional 3–5 minutes of work time before a whole-class discussion.

Student Task Statement

Your teacher will give you some shapes.

1. How many copies of the equilateral triangle can you fit together around a single vertex, so that the triangles' edges have no gaps or overlaps? What is the measure of each angle in these triangles?
2. What are the measures of the angles in the
   a. Square?

b. Hexagon?
c. Parallelogram?
d. Right triangle?
e. Octagon?
f. Pentagon?

Student Response

1. 6, 60°
2. Measures of angles:
    a. Square: 90°
    b. Hexagon: 120°
    c. Parallelogram: 120° and 60°
    d. Right triangle: 45° and 90°
    e. Octagon: 135°
    f. Pentagon: 90°, 120°, and 150°

Building on Student Thinking

When deducing angle measures, it is important to know that angles "all the way around" a vertex sum to 360 degrees. It is also important to know that angles that make a line when adjacent sum to 180 degrees. Monitor for students who need to be reminded of these facts.

Activity Synthesis

For the remainder of the lesson, it is not so important that the degree measures of the angles are known, so don't dwell on the answers. Select a few students who deduced angles' measures by fitting pieces together to present their work. Make sure students see lots of examples of shapes fitting together like puzzle pieces.

Remind students that the 3 congruent angles in an equilateral triangle make a straight angle, so it makes sense that 6 copies of this angle make a full circle.

17.2 Tessellate This

Optional

35 min

Activity Narrative

This optional activity provides students an opportunity to apply their learning about rigid transformations and congruent figures by creating a **tessellation**.

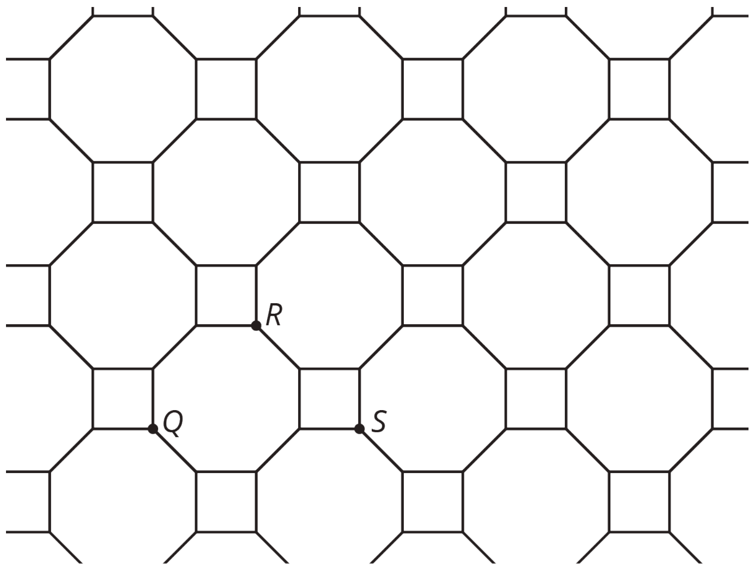
A tessellation of the plane is a regular repeating pattern of one or more shapes that covers the entire plane. Some of the most familiar examples of tessellations are seen in bathroom and kitchen tiles. Tiles (for flooring, ceiling, bathrooms, kitchens) are often composed of copies of the same shapes because they need to fit together and extend in a regular pattern to cover a large surface.

Standards

Addressing **8.G.A**

Launch

Share with students a definition of **tessellation:** "A tessellation is a repeating pattern of one or more shapes. The sides of the shapes fit together perfectly and do not overlap. The pattern goes on forever in all directions." Consider showing several examples of tessellations. A true tessellation covers the entire plane. While this is impossible to show, we can identify a pattern that keeps going forever in all directions. This is important when we think about tessellations and symmetry. One definition of symmetry is, "You can pick it up and put it down a different way and it looks exactly the same." In a tessellation, you can perform a translation and the image looks exactly the same. In the example of this tiling, the translation that takes point Q to point R results in a figure that looks exactly the same as the one you started with. So does the translation that takes S to Q. Describing one of these translations shows that this figure has translational symmetry.



Provide access to geometry toolkits. Suggest to students that if they cut out a shape, it is easy to make many copies of the shape by tracing it. Encourage students to use the shapes from the previous activity (or pattern blocks if available) and experiment putting them together. They do not need to use all of the shapes, so if students are struggling, suggest that they try using copies of a couple of the simpler shapes.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use multiple examples and non-examples to emphasize that a tessellation is a pattern made of shapes that have no gaps or overlaps and has translation symmetry.
Supports accessibility for: Conceptual Processing, Attention

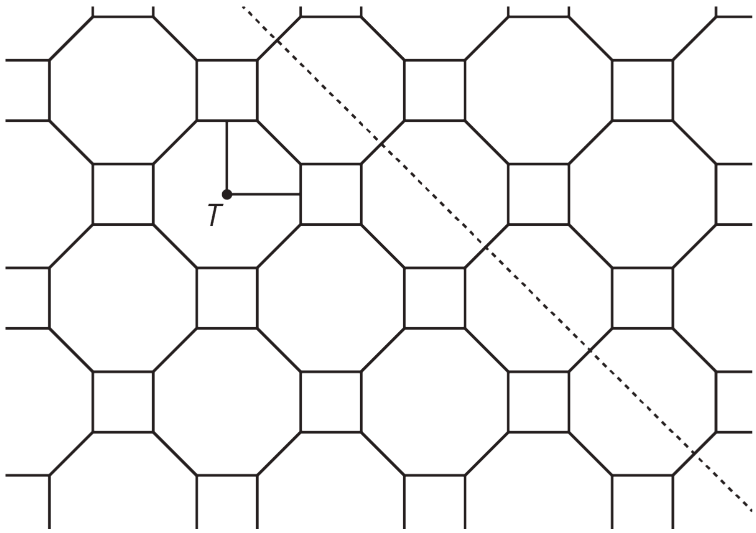
Student Task Statement

1. Design your own **tessellation**. You will need to decide which shapes you want to use and make copies. Remember that a tessellation is a repeating pattern that goes on forever to fill up the entire plane.

2. Find a partner and trade pictures. Describe a transformation of your partner's picture that takes the pattern to itself. How many different transformations can you find that take the pattern to itself? Consider translations, reflections, and rotations.
3. If there's time, color and decorate your tessellation.

Student Response

1. Answers vary
2. Sample response: In the tessellation given previously, we could reflect across the dashed line, or rotate 90 degrees clockwise around the point marked T.



Building on Student Thinking

Watch out for students who choose shapes that almost-but-don't-quite fit together. Reiterate that the pattern has to keep going forever. Often small gaps or overlaps become more obvious when the pattern continues.

Activity Synthesis

Invite students to share their designs and also describe a transformation that takes the design to itself. Consider displaying their tessellations around the room.

17.3 Rotate That

Optional

35 min

Activity Narrative

This optional activity provides students an opportunity to apply their learning about rigid transformations and congruent figures by creating designs with rotational symmetry.

They then share designs and find the different rotations (and possibly reflections) that make the shape match up with itself.

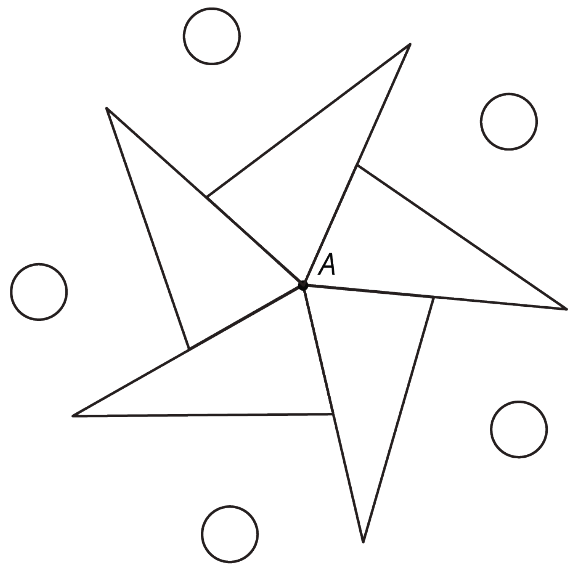
Standards

Addressing **8.G.A**

Instructional Routines

- MLR8: Discussion Supports

Launch



Ask students what transformation they could perform on the figure so that it matches up with its original position. There are a number of rotations using A as the center that would work: 72 degrees or any multiple of 72 degrees. Make sure students understand that the 5 triangles in this pattern are congruent and that $5 \cdot 72 = 360$: This is why multiples of 72 degrees with center A match this figure up with itself. They need to be careful in selecting angles for the shapes in their pattern. If they struggle, consider asking them to use pattern tiles or copies of the shapes from the previous activity to help build a pattern.

If possible, show students several examples of figures that have rotational symmetry.

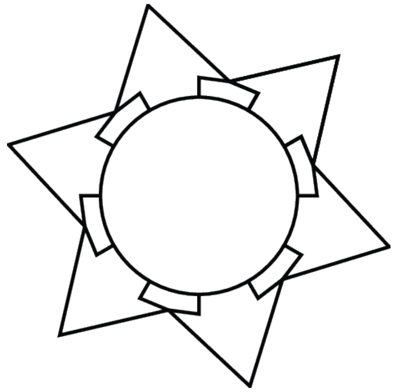
Provide access to geometry toolkits. If possible, provide access to square graph paper or isometric graph paper.

Student Task Statement

1. Make a design with rotational symmetry.
2. Find a partner who has also made a design. Exchange designs and find a transformation of your partner's design that takes it to itself. Consider rotations, reflections, and translations.
3. If there's time, color and decorate your design.

Student Response

Sample response:



Building on Student Thinking

Before now, students may think that reflection symmetry is the only kind of symmetry. Because of this, they may create a design that has reflection symmetry but not rotational symmetry. Steer students in the right direction by asking them to perform a rotation that takes the figure to itself. Acknowledge that reflection symmetry is a type of symmetry, but the task here is to create a design with rotational symmetry.

Activity Synthesis

Invite students to share their designs and also describe a transformation that takes the design to itself. Consider displaying their designs around the room.

Access for English Language Learners

MLR8 Discussion Supports. For each design and transformation that is shared, invite students to turn to a partner and restate what they heard using precise mathematical language.
Advances: Listening, Speaking

Glossary

- tessellation