



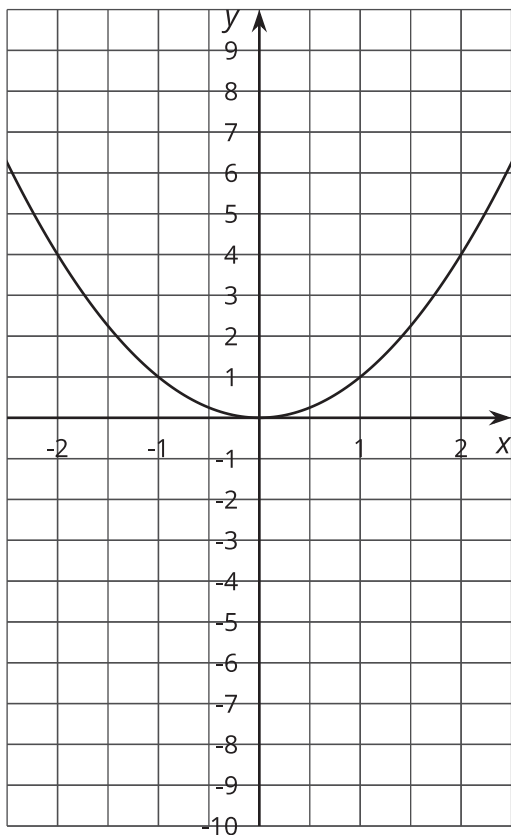
# Cubes and Cube Roots

Let's compare equations with cubes and cube roots.

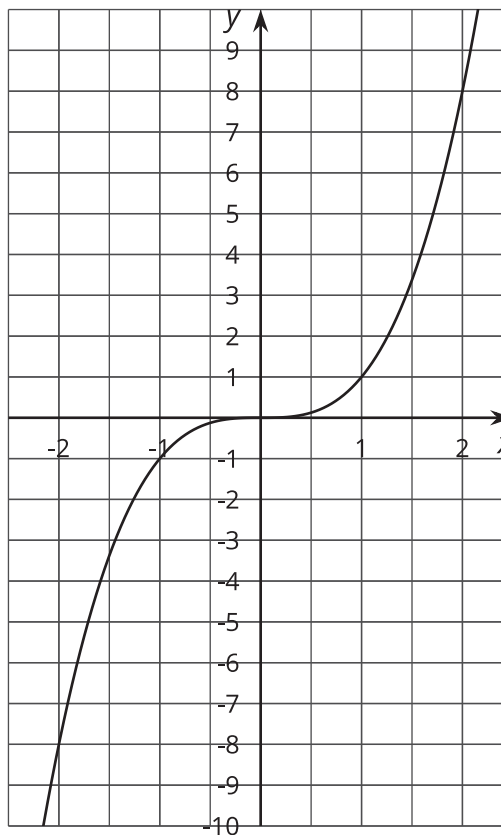
## 9.1 Comparing Two Graphs

How are these graphs the same? How are they different?

$$y = x^2$$



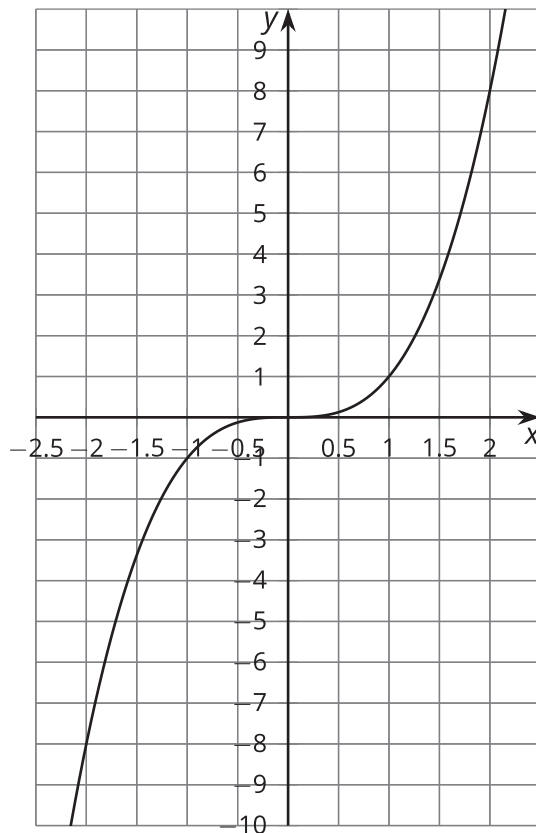
$$y = x^3$$



## 9.2 Finding Cube Roots with a Graph

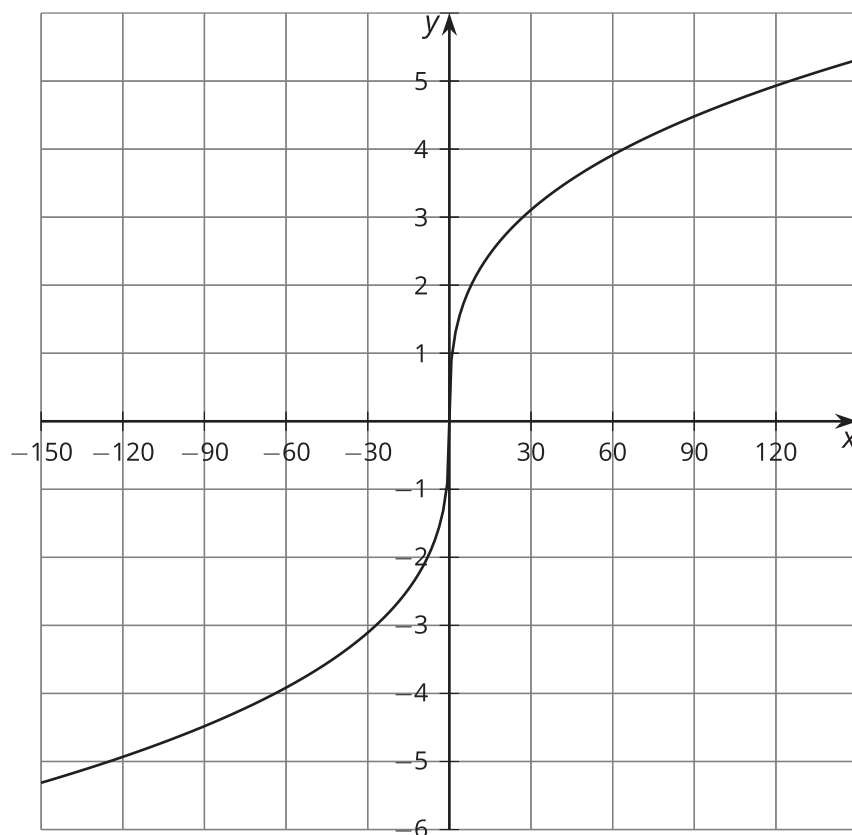
How many real solutions are there to each of the following equations? Estimate the solution(s) from the graph of  $y = x^3$ . Check your estimate by substituting it back into the equation.

1.  $x^3 = 8$
2.  $x^3 = 2$
3.  $x^3 = 0$
4.  $x^3 = -8$
5.  $x^3 = -2$



## 9.3

## Cube Root Equations



1. Use the graph of  $y = \sqrt[3]{x}$  to estimate the real solution(s) to  $\sqrt[3]{x} = -4$ .
2. Use the meaning of cube roots to find an exact real solution to the equation  $\sqrt[3]{x} = -4$ . How close was your estimate?
3. Find the real solution of the equation  $\sqrt[3]{x} = 3.5$  using the meaning of cube roots. Use the graph to check that your solution is reasonable.

## 9.4

## Solve These Equations with Cube Roots in Them

Here are a lot of equations:

$$\bullet \sqrt[3]{t+4} = 3$$

$$\bullet -10 = -\sqrt[3]{a}$$

$$\bullet \sqrt[3]{3-w} - 4 = 0$$

$$\bullet \sqrt[3]{z} + 9 = 0$$

$$\bullet \sqrt[3]{r^3 - 19} = 2$$

$$\bullet 5 - \sqrt[3]{k+1} = -1$$

$$\bullet \sqrt[3]{p+4} - 2 = 1$$

$$\bullet 6 - \sqrt[3]{b} = 0$$

$$\bullet \sqrt[3]{2n} + 3 = -5$$

$$\bullet 4 + \sqrt[3]{-m} + 4 = 6$$

$$\bullet -\sqrt[3]{c} = 5$$

$$\bullet \sqrt[3]{s-7} + 3 = 0$$

1. Without solving, identify 3 equations that you think would be the least difficult to solve and 3 equations that you think would be the most difficult to solve. Be prepared to explain your reasoning.
2. Choose 4 equations and solve them. At least one should be from your “least difficult” list and at least one should be from your “most difficult” list.

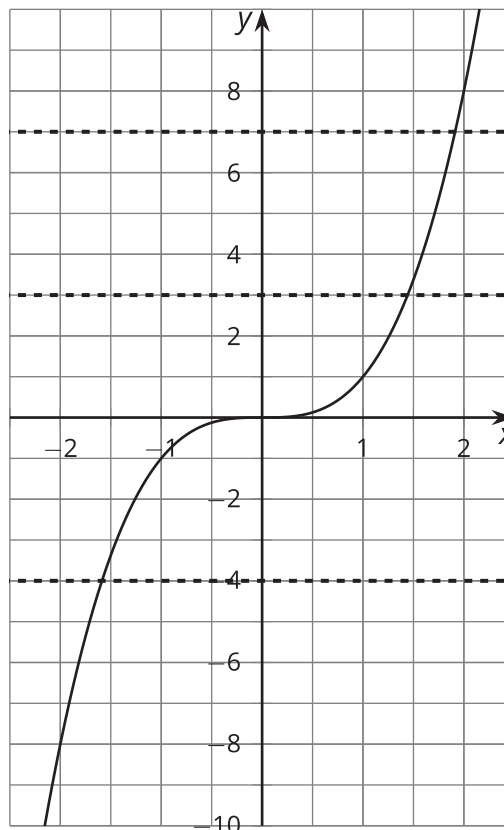
## Are you ready for more?

All of these equations are equivalent to an equation in the form  $\sqrt[3]{ax + b} + c = 0$  for some constants  $a$ ,  $b$ , and  $c$ . Find a formula that would solve any such equation for  $x$  in terms of  $a$ ,  $b$ , and  $c$ .

## Lesson 9 Summary

Every real number has exactly one real cube root. You can see this by looking at the graph of  $y = x^3$  on the real coordinate plane.

If  $y$  is any number, for example,  $-4$ , then we can see that  $y = -4$  crosses the graph in one and only one place, so the equation  $x^3 = -4$  will have the real solution  $-\sqrt[3]{4}$ . This is true for any real number  $a$ :  $y = a$  will cross the graph in exactly one place, and  $x^3 = a$  will have one real solution,  $\sqrt[3]{a}$ .



In an equation like  $\sqrt[3]{t} + 6 = 0$ , we can isolate the cube root, and then cube each side:

$$\begin{aligned}\sqrt[3]{t} + 6 &= 0 \\ \sqrt[3]{t} &= -6 \\ t &= (-6)^3 \\ t &= -216\end{aligned}$$

While cubing each side of an equation won't create an equation with solutions that are different from the original equation, it is still a good idea to always check solutions in the original equation because little mistakes can creep in along the way.