Unit 9, Lesson 9

# Interpreting Exponential Functions

Let's find some meaningful ways to represent exponential functions.

## 9.1 Equivalent or Not?

Lin and Diego are discussing two expressions: $x^2$ and $2^x$.

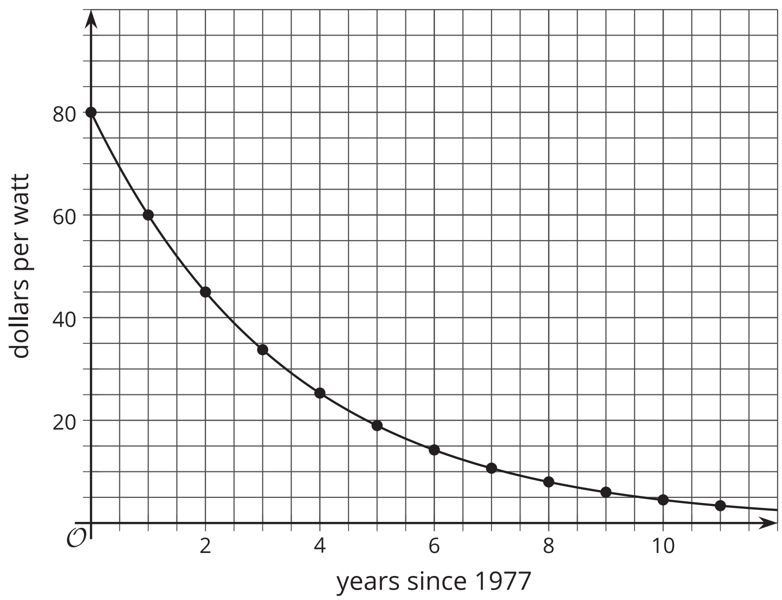
- Lin says, "I think the two expressions are equivalent."
- Diego says, "I think the two expressions are equal only for *some* values of $x$."

Do you agree with either of them? Explain or show your reasoning.

## 9.2 Cost of Solar Cells

The cost, in dollars, to produce 1 watt of solar power is a function of the number of years since 1977, $t$.

From 1977 to 1987, the cost could be modeled by an exponential function, $f$. Here is the graph of the function.




1. What is the statement $f(9) \approx 6$ saying about this situation?

2. What is $f(4)$? What about $f(3.5)$? What do these values represent in this context?

3. When $f(t) = 45$, what is $t$? What does that value of $t$ represent in this context?

4. By what factor did the cost of solar cells change each year? (If you get stuck, consider creating a table.)

## 9.3 Paper Folding

1. The thickness, $t$, in millimeters of a folded sheet of paper after it is folded $n$ times is given by the equation $t = (0.05) \cdot 2^n$.
   a. What does the number $0.05$ represent in the equation?
   b. Use graphing technology to graph the equation $t = (0.05) \cdot 2^n$.
   c. How many folds does it take before the folded sheet of paper is more than 1 mm thick? How many folds before it is more than 1 cm thick? Explain how you know.

2. The area of a sheet of paper is 93.5 square inches.
   a. Find the remaining, visible area of the sheet of paper after it is folded in half once, twice, and three times.
   b. Write an equation expressing the visible area, $a$ of the sheet of paper in terms of the number of times it has been folded, $n$.
   c. Use graphing technology to graph the equation.
   d. In this context, can $n$ take negative values? Explain your reasoning.
   e. Can $a$ take negative values? Explain your reasoning.

## Are you ready for more?

1. Using the model in this task, how many folds would be needed to get 1 meter in thickness? 1 kilometer in thickness?

2. Do some research: what is the current world record for the number of times humans were able to fold a sheet of paper?

## 9.4 Information Gap: Smartphone Sales

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me ________?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know ________ because . . . .” Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know ________?”
3. Listen to your partner’s reasoning, and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner!
   These steps may be repeated.
4. When your partner has enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

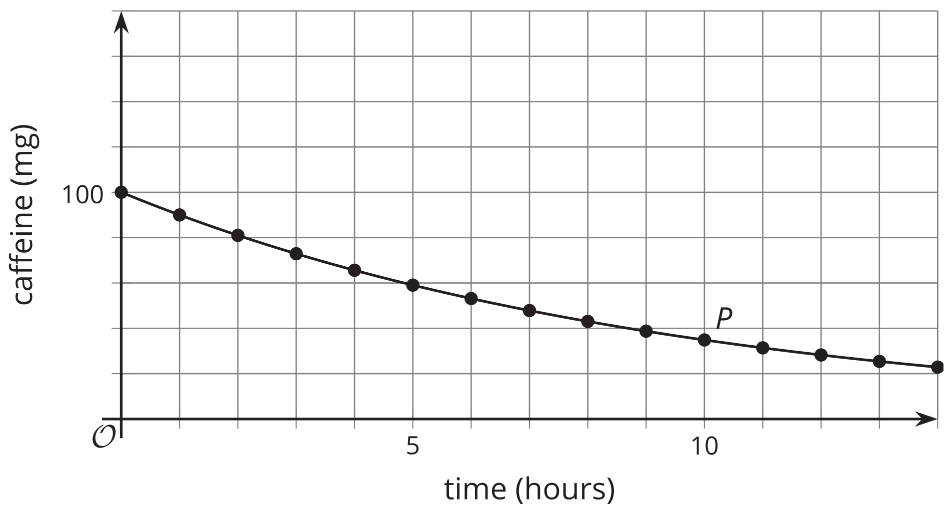
## Lesson 9 Summary

Earlier, we used equations to represent situations characterized by exponential change. For example, to describe the amount of caffeine, $c$, in a person's body $t$ hours after an initial measurement of 100 mg, we used the equation $c = 100 \cdot \left(\frac{9}{10}\right)^t$.

Notice that the amount of caffeine is a *function* of time, so another way to express this relationship is $c = f(t)$, where $f$ is the function given by $f(t) = 100 \cdot \left(\frac{9}{10}\right)^t$.

We can use this function to analyze the amount of caffeine. For example, when $t$ is 3, the amount of caffeine in the body is $100 \cdot \left(\frac{9}{10}\right)^3$ or $100 \cdot \frac{729}{1,000}$, which is 72.9. The statement $f(3) = 72.9$ means that 72.9 mg of caffeine are present 3 hours after the initial measurement.

We can also graph the function $f$ to better understand what is happening. The point labeled $P$, for example, has the approximate coordinates $(10, 35)$ so it takes about 10 hours after the initial measurement for the caffeine level to decrease to 35 mg.




A graph can also help us think about the values in the domain and range of a function. Because the body breaks down caffeine continuously over time, the domain of the function—the time in hours—can include non-whole numbers (for example, we can find the caffeine level when $t$ is 3.5). In this situation, negative values for the domain would represent the time *before* the initial measurement. For example $f(-1)$ would represent the amount of caffeine in the person's body 1 hour before the initial measurement. The range of this function would not include negative values, as a negative amount of caffeine does not make sense in this situation.