Unit 6, Lesson 5

Squares and Circles

Let's see how the distributive property can relate to equations of circles.

5.1 Math Talk: Distribution

Distribute each expression mentally.

- $5(x + 3)$
- $x(x - 3)$
- $(x + 4)(x + 2)$
- $(x - 5)(x - 5)$

5.2 Perfectly Square

1. Apply the distributive property to each expression.
   a. $(x-7)(x-7)$

   b. $(x+4)^2$

   c. $(x-10)^2$

   d. $(x+1)^2$

2. Look at your results. Each of these expressions is called a *perfect square trinomial*. Why?

3. Which of these expressions are perfect square trinomials? If you get stuck, look for patterns in your earlier work.
   a. x^2-6x+9
   b. $x^2+10x+20$
   c. $x^2+18x+81$
   d. x^2-2x+1
   e. $x^2+4x+16$

4. For each perfect square trinomial that you identified, rewrite it as a squared binomial.

5.3 Back and Forth

1. Here is the equation of a circle: $(x-2)^2 + (y+7)^2 = 10^2$

   a. What are the center and radius of the circle?

   b. Apply the distributive property to the squared binomials and rearrange the equation so that one side is 0. This is the form in which many circle equations are written.

2. This equation looks different, but it also represents a circle:
   $x^2 + 6x + 9 + y^2 - 10y + 25 = 64$

   a. How can you rewrite this equation to find the center and radius of the circle?

   b. What are the center and radius of the circle?

Are you ready for more?

In three-dimensional space, there are 3 coordinate axes, called the x-axis, the y-axis, and the z-axis. Write an equation for a sphere with center (a, b, c) and radius r.

Lesson 5 Summary

Suppose we square several binomials, or expressions that contain 2 terms. We get trinomials, or expressions that contain 3 terms. Does any pattern emerge in the results?

$$(x+6)^2 = x^2 + 12x + 36$$

$$(x-8)^2 = x^2 - 16x + 64$$

$$(x+5)^2 = x^2 + 10x + 25$$

Each of the expressions on the right is called a perfect square trinomial because it is the result of multiplying an expression by itself. There is a pattern in the results: When the coefficient of x^2 in a trinomial is 1, if the constant term is the square of half the coefficient of x, then the expression is a perfect square trinomial.

For example, $x^2 - 14x + 49$ is a perfect square trinomial because the constant term, 49, can be rewritten as $(-7)^2$, and half of -14 is -7. This expression can be rewritten as a squared binomial: $(x-7)^2$.

Two squared binomials show up in the equation for circles: $(x-h)^2 + (y-k)^2 = r^2$. Equations for circles are sometimes written in different forms, but we can rearrange them to help find the center and radius of the circle. For example, suppose the equation of a circle is written like this:

$$x^2 - 22x + 121 + y^2 + 2y + 1 = 225$$

We can't immediately identify the center and radius of the circle. However, if we rewrite the two perfect square trinomials as squared binomials and rewrite the right side in the form r^2, the center and radius will be easier to recognize.

The first 3 terms on the left side, $x^2 - 22x + 121$, can be rewritten as $(x-11)^2$. The remaining terms, $y^2 + 2y + 1$, can be rewritten as $(y+1)^2$. The right side, 225, can be rewritten as 15^2. Let's put it all together.

$$(x-11)^2 + (y+1)^2 = 15^2$$

Now we can see that the center of the circle is $(11, -1)$, and the circle's radius measures 15 units.