Unit 1, Lesson 6

Describing Transformations

Goals

- Create a drawing on a coordinate grid of the image of a polygon using verbal descriptions of a sequence of transformations.
- Identify what information is needed to perform a sequence of transformations on a polygon. Ask questions to elicit that information.

Learning Targets

- I can apply transformations to a polygon on a grid if I know the coordinates of its vertices.

Lesson Narrative

Prior to this lesson, students have studied and classified different types of transformations (translations, rotations, reflections). They have practiced applying individual transformations and sequences of transformations to figures both on and off of a coordinate grid. In this lesson, they focus on communicating precisely the information needed to apply a sequence of transformations to a polygon on the coordinate grid. They must think carefully about what information they need (MP1) and request this information from their partner in a clear, precise way. They also explain *why* they need each piece of information (MP3). The coordinate plane plays a key role in this work, allowing students to communicate precisely about the locations of polygons and how they are transformed.

Math Community
Today's community building centers on the teacher sharing their draft commitments as part of the mathematical community. At the end of the lesson, students are invited to suggest additions to the teacher sections of the chart.

Standards

Addressing **8.G.A.1, 8.G.A.3**

Instructional Routines

- MLR4: Information Gap Cards

Required Materials

Materials to Gather

- Geometry toolkits: Activity 1
- Math Community Chart: Activity 1, Cool-down

Materials to Copy

- Transformation Information Cards (1 copy for every 2 students): Activity 2

Required Preparation

Activity 1:

In the "Doing Math" teacher section of the Math Community Chart, add 2–5 commitments you have for what your teaching practice "looks like" and "sounds like" this year.

Lesson:

From the geometry toolkits, graph paper and tracing paper are especially helpful.

Student Facing Learning Goals

Let's transform some polygons in the coordinate plane.

6.1 What Do You Want to Know?

Warm-up

10 min

Activity Narrative

The purpose of this *Warm-up* is to prepare students for the *Info Gap* activity that follows. First, students are given a problem with incomplete information. They are prompted to brainstorm what they need to know to solve a problem that involves transformations. Next, they practice asking for information, explaining the rationale for their request, and persevering if their initial questions are unproductive (MP1). Once students have enough information, they solve the problem.

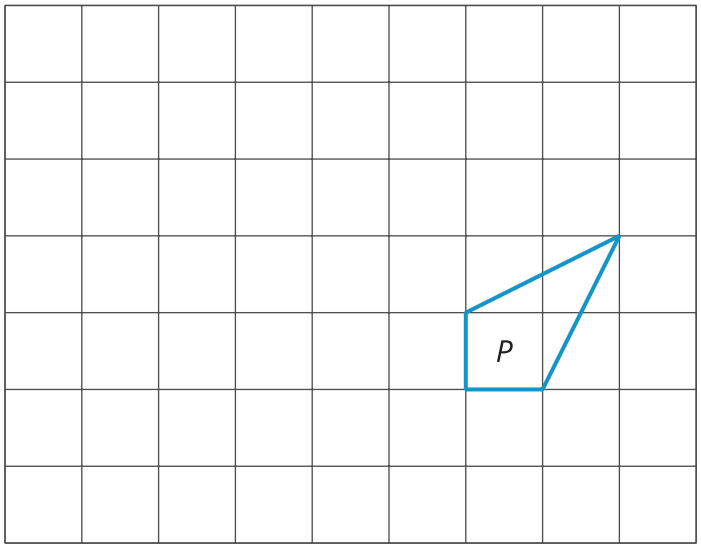
Standards

Addressing **8.G.A.1**

Launch

Display the image and the first sentence of the *Task Statement* for all to see. Ask students what the location of P' is. When they recognize that not enough information is given, display the second sentence and ask students what they need to know to be able to solve the problem. Display the sentence frame "Can you tell me _____." for all to see, and invite students to use it to frame their information requests. Give students 2 minutes of quiet think time.

Student Task Statement



P' is the image of P after some transformations.

What specific information do you need to be able to solve the problem?

Student Response

Sample responses:

- Can you tell me the order of the sequence of transformations?
- Can you tell me how much and which direction P is translated?
- Can you tell me how much and which direction P is rotated?
- Can you tell me the line of reflection?

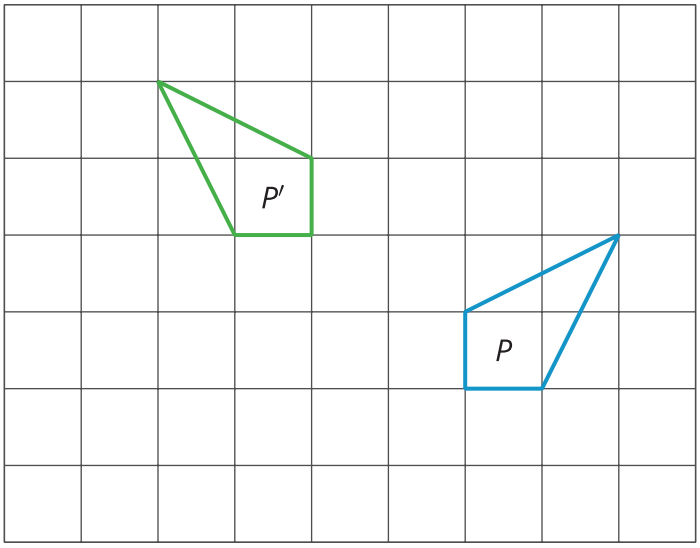
Activity Synthesis

Tell students that the problem is a part of an *Info Gap* routine. In the routine, one person has a problem with incomplete information, and another person has data that can help with solving it. Explain that it is the job of the person with the problem to think about what is needed to answer the question, and then request it from the person with information.

Tell students they will try to solve the problem this way as a class to learn the routine. In this round, the students have the problem, and the teacher has the information needed to solve the problem.

- Ask students, "What specific information do you need to find out where P' is?"
- Select students to ask their questions. Encourage students to use the format of "Can you tell me . . . ?" Respond to each question with, "Why do you need to know _____?"
- Once students justify their question, only answer questions if they can be answered using these data:
  - P is translated and then rotated to get P'.
  - P is not reflected.
  - P is translated up 2 units and left 2 units.
  - P is rotated 90 degrees counterclockwise.
  - The center of rotation is the vertex that is 90 degrees.
- If students ask for information that is not on the data card, respond with, "I don't have that information."

When students think they have enough information, give them 2 minutes to solve the problem.



Tell students they will work in small groups and use the routine to solve problems in the next activity.

Math Community
After the *Warm-up*, display the Math Community Chart with the "doing math" actions added to the teacher section for all to see. Give students 1 minute to review. Then share 2–3 key points from the teacher section and your reasoning for adding them. For example,

- If "questioning vs. telling," a shared reason could focus on your belief that students are capable mathematical thinkers and your desire to understand how students are making meaning of the mathematics.
- If "listening," a shared reason could be that sometimes you want to sit quietly with a group just to listen and hear student thinking and not because you think the group needs help or is off-track.

After sharing, tell students that they will have the opportunity to suggest additions to the teacher section during the *Cool-down*.

6.2 Info Gap: Transformation Information

25 min

Activity Narrative

This is the first *Info Gap* activity in the course. In this activity, students perform transformations in the coordinate plane but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Info Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Students likely need several rounds to determine the information they need.

- They need to know which transformations were applied (i.e., translation, rotation, or reflection).
- They need to determine the order in which the transformations were applied.
- They need to remember what information is needed to describe a translation, rotation, or reflection.

Monitor for students who successfully determine or remember each of these three important pieces of information as

well as students who have partially, but not completely, solved the problem. Students may not realize that the order in which the transformations are applied is important, and this should be addressed in the *Activity Synthesis*.

Access for English Language Learners

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.

Standards

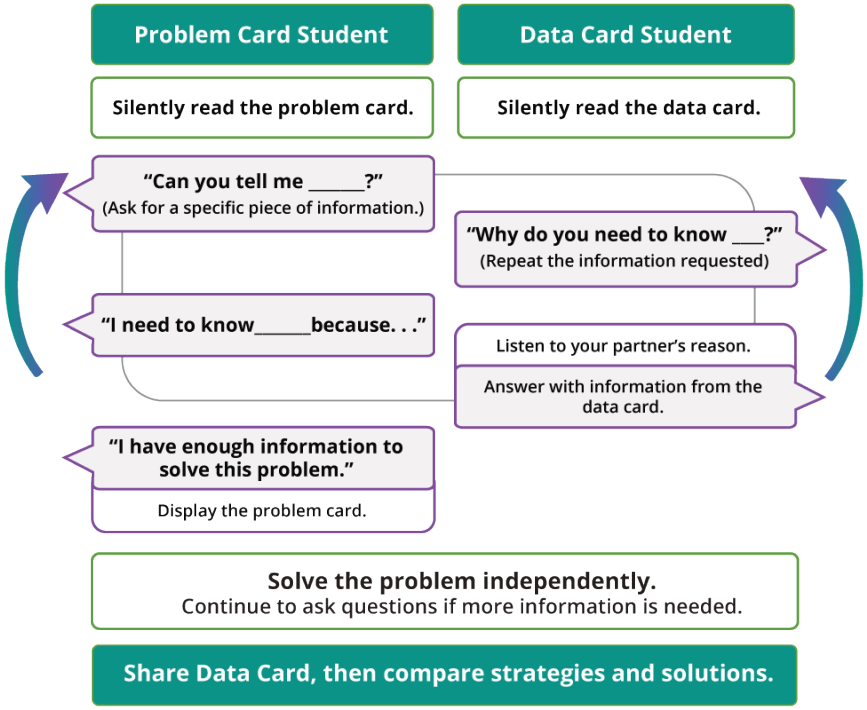
Addressing **8.G.A.1, 8.G.A.3**

Instructional Routines

- MLR4: Information Gap Cards

Launch

Display for all to see the graphic that illustrates a framework for the *Info Gap* routine.



Explain that in an *Info Gap* routine students work with a partner. One partner gets a problem card with a question that doesn't have enough given information, and the other partner gets a data card with information relevant to the problem card.

The person with the problem card asks questions like "Can you tell me _____?" and is expected to explain what they will do with the information. If that person asks for information that is not on the data card (including the answer!) and gives their reason, then the person with the data card must respond with, "I don't have that information." The person with the data card should just be providing information, not making assumptions. Note that it is okay to help a stuck partner by saying something like "I don't have the line of reflection. I only have information about the images of points."

Once the partner with the problem card has enough information, both partners look at the problem card and solve the problem independently.

Arrange students in groups of 2 or 4. If students are new to the *Info Gap* routine, allowing them to work in groups of 2 for each role supports communication and understanding. In each group, distribute a problem card to one student (or group) and a data card to the other student (or group). After reviewing their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Info Gap* graphic visible throughout the activity or provide students with a physical copy.
Supports accessibility for: Memory, Organization

Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . . ."

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card and discuss your reasoning.

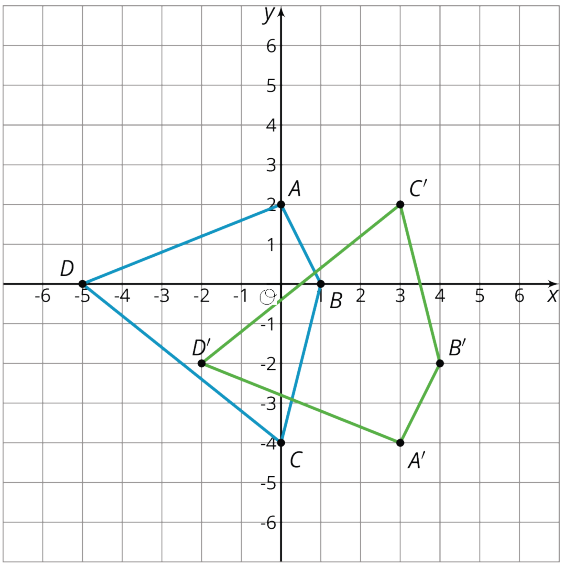
If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

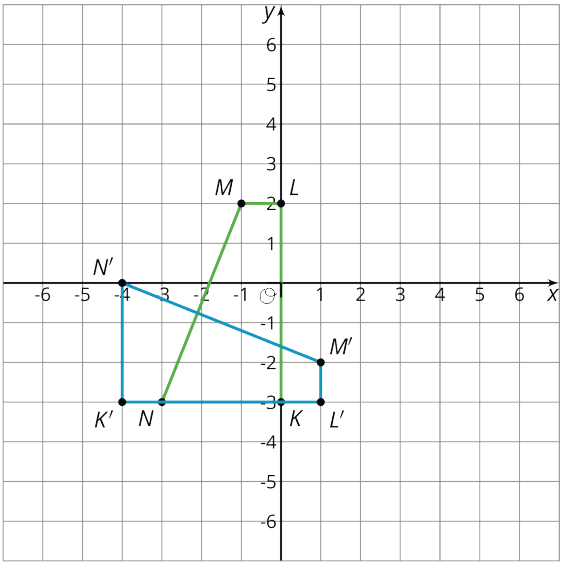
   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Student Response

Problem Card 1:



Problem Card 2:



Building on Student Thinking

Students may struggle to ask their partner for all of the information they need or may ask a question that is not sufficiently precise, such as, "What are the transformations?" Ask these students what kinds of transformations they have worked with. What information is needed to perform a translation? What about a rotation or reflection? Encourage them to find out *which* transformations they need to perform (Is there a translation? Is there a rotation?) and then find out the information they need for each transformation.

Are You Ready for More?

Sometimes two transformations, one performed after the other, have a simpler description as a single transformation. For example, instead of translating 2 units up followed by translating 3 units up, we could simply translate 5 units up. Instead of rotating 20° counterclockwise around the origin followed by rotating 80° clockwise around the origin, we could simply rotate 60° clockwise around the origin.

Can you find a simple description of reflecting across the x-axis followed by reflecting across the y-axis?

Extension Student Response

Reflecting across the x-axis followed by reflecting across the y-axis is the same as rotating 180° (in either direction) around the origin.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- "How did using coordinates help in talking about the problem?"
- "Was the order in which the transformations were applied important? Why?"
- "If this same problem were a picture on a grid without coordinates, how would you talk about the points?"

Highlight for students that one advantage of the coordinate plane is that it allows us to communicate information about transformations precisely. Here is what is needed for each type of transformation (consider showing one example of each while going through the different transformations):

- Translation: the distance of vertical and horizontal components
- Rotation: the center of rotation, the direction of rotation, and the angle of rotation
- Reflection: the line of reflection

Lesson Synthesis

Give students 2–3 minutes to list what information they need to describe a sequence of transformations of a figure. Then, have students share their lists with a partner and then invite them to share with the class.
If not mentioned by students, ask them to consider why the order of transformations is important to know when describing a sequence of transformations. (Sometimes the order doesn't matter, because both orders of the transformations in the sequence result in the same image, but sometimes it will give a different image if there is a different order.)

6.3 Describing a Sequence of Transformations

5 min

Cool-down

Standards

Addressing **8.G.A.1, 8.G.A.3**

Launch

Math Community
Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "What

additions would you make to the teacher 'Doing Math' section of the Math Community Chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the community building question. Use them to add to or revise the teacher "Doing Math" section of the Math Community Chart before Exercise 4.

Student Task Statement

Triangle T' is the image of Triangle T. Han gave this information to Jada to describe the sequence of transformations.

- Triangle T is reflected over line ℓ.
- Triangle T is translated 2 units to the left.
- The order of the sequence of transformations is translation, then reflection.

Which of these figures shows the correct Triangle T'?

Figure 1

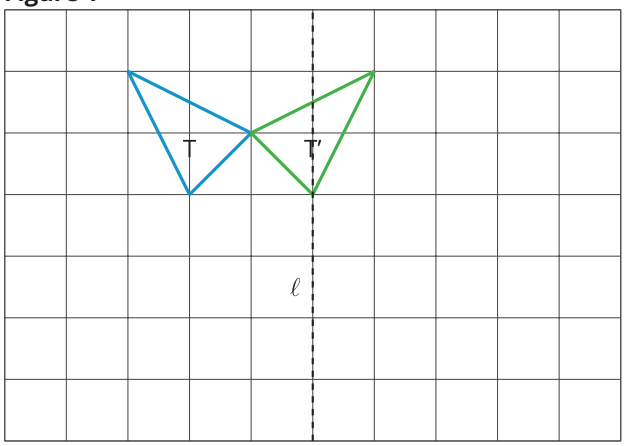
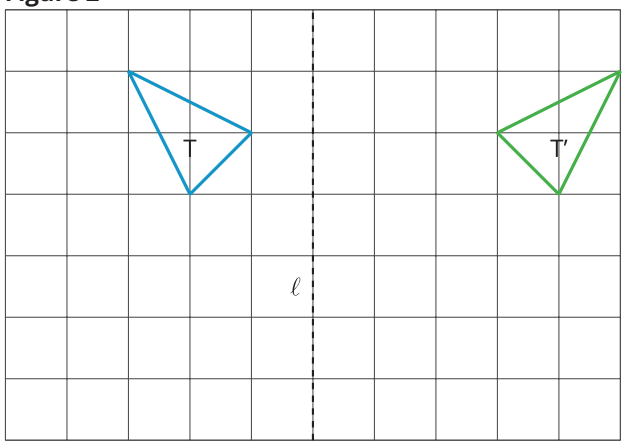


Figure 2



Student Response

Figure 2

Responding to Student Thinking

Press Pause
By this point in the unit, there should be some student mastery of identifying sequences of transformations. If most students struggle, make time to revisit the work in the activity referred to here. See the Course Guide for ideas to help students re-engage with earlier work.

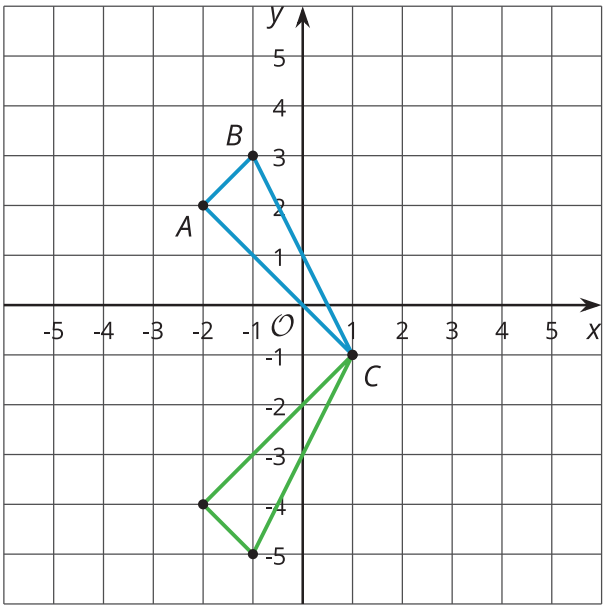
Grade 8, Unit 1, Lesson 4, Activity 2 Make That Move

Lesson 6 Summary

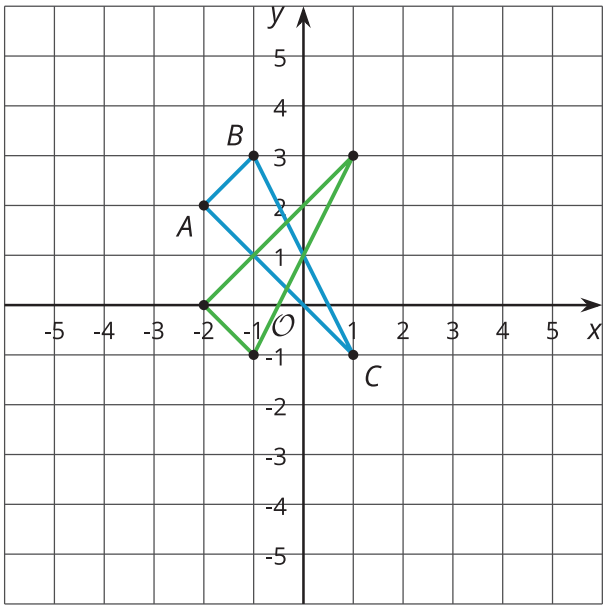
When describing a sequence of transformations, there are several pieces of information that are important to know. For a translation, we need to know distance and direction. For a rotation, we need the center of rotation, direction, and amount of rotation. For a reflection, we need a line of reflection. There is one more piece of information that is helpful though.

When we perform a sequence of transformations, the order of the transformations can be important.

Here is triangle ABC translated up two units and then reflected over the x-axis.



Here is triangle ABC reflected over the x-axis and then translated up 2 units.

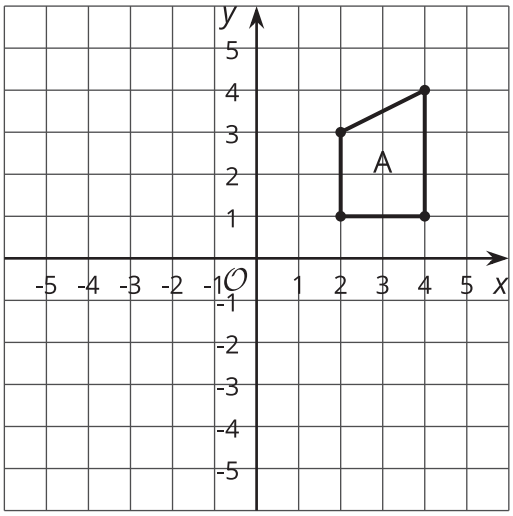


Triangle ABC ends up in different places when the transformations are applied in the opposite order!

Lesson 6 Practice Problems

1 Student Task Statement

Here is Polygon A in the coordinate plane:

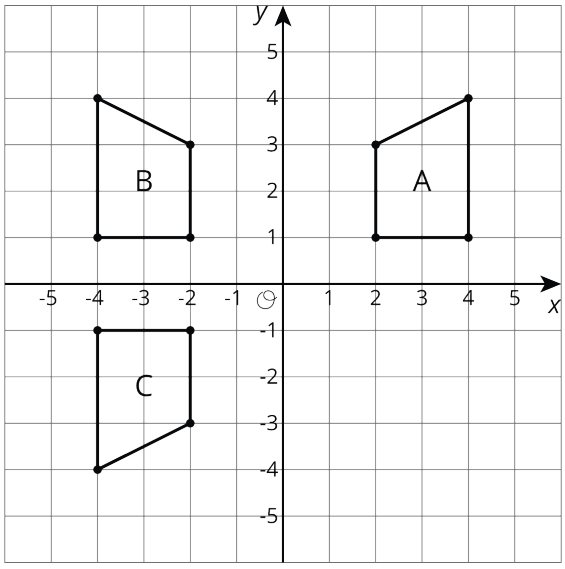


a. Draw Polygon B, the image of A, using the y-axis as the line of reflection.

b. Draw Polygon C, the image of B, using the x-axis as the line of reflection.

c. Draw Polygon D, the image of C, using the x-axis as the line of reflection.

Solution



Polygon D is the same as B: reflecting a polygon twice over the x-axis returns it to its original position.

2 Student Task Statement

The point $(-4, 1)$ is rotated $180°$ counterclockwise using center $(-3, 0)$. What are the coordinates of the image?

A. $(-5, 2)$

B. $(-4, -1)$

C. $(-2, -1)$

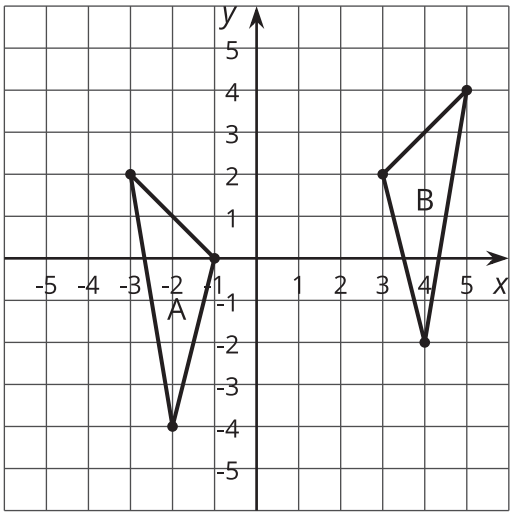
D. $(4, -1)$

Solution

C

3 Student Task Statement

Describe a sequence of transformations for which Triangle B is the image of Triangle A.



Solution

Sample response: B is the image of A under a reflection over the y-axis, then a translation 2 units to the right and 2 units up.

4 from Unit 1, Lesson 2

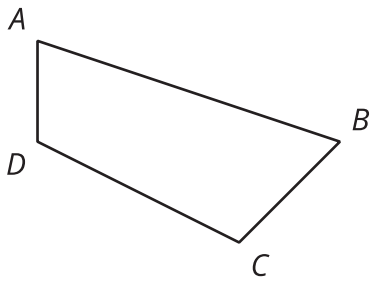
Student Task Statement

Here is quadrilateral $ABCD$. Draw the image of quadrilateral $ABCD$ after each transformation.

a. The translation that takes B to D.

b. The reflection over segment BC.

c. The rotation about point A by angle DAB, counterclockwise.



Solution

a. Image of trapezoid moved to the left so that B lines up with D

b. Image of trapezoid sharing segment BC with $ABCD$

c. Image of trapezoid rotated so that the side corresponding to AD is now part of segment AB