Moving Functions

Let's represent vertical and horizontal translations using function notation.

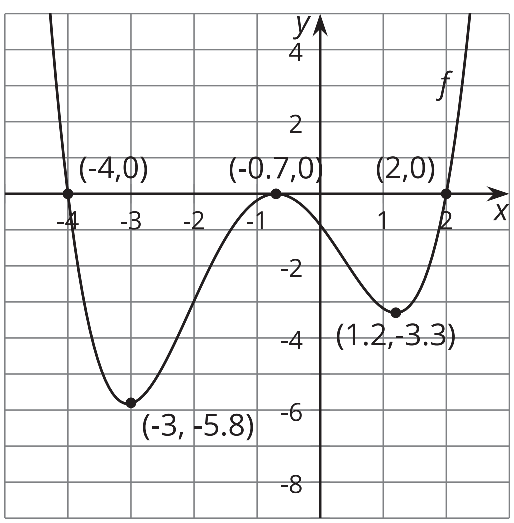
2.1 What Happened to the Equation?

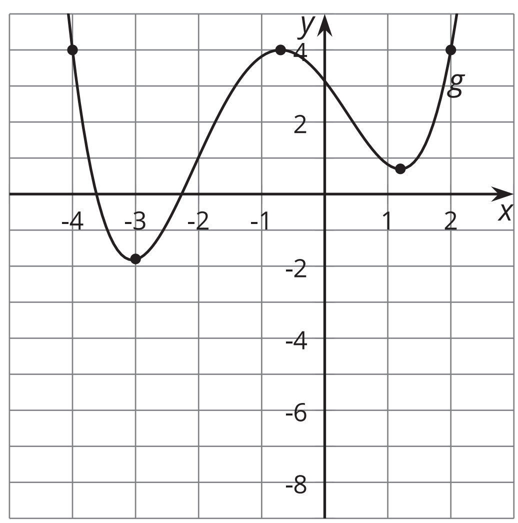
Graph each function using technology. Describe how to transform $f(x) = x^2(x-2)$ to get to the functions shown here, in terms of both the graph and the equation.

1. $h(x) = x^2(x-2) - 5$
2. $g(x) = (x-4)^2(x-6)$

2.2 Writing Equations for Vertical Translations

The graph of function g is a vertical translation of the graph of polynomial f.

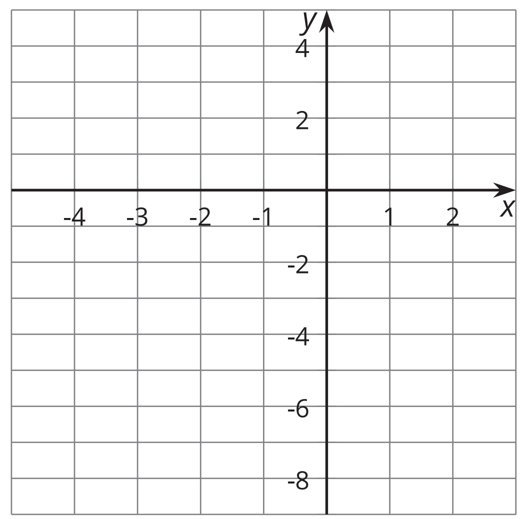




1. Complete the $g(x)$ column of the table.

x	$f(x)$	$g(x)$	$h(x) = f(x) - 2.5$
-4	0		
-3	-5.8		
-0.7	0		
1.2	-3.3		
2	0		

2. If $f(0) = -0.86$, what is $g(0)$? Explain how you know.
3. Write an equation for $g(x)$ in terms of $f(x)$ for any input x.
4. The function h can be written in terms of f as $h(x) = f(x) - 2.5$. Complete the $h(x)$ column of the table.
5. Sketch the graph of function h.

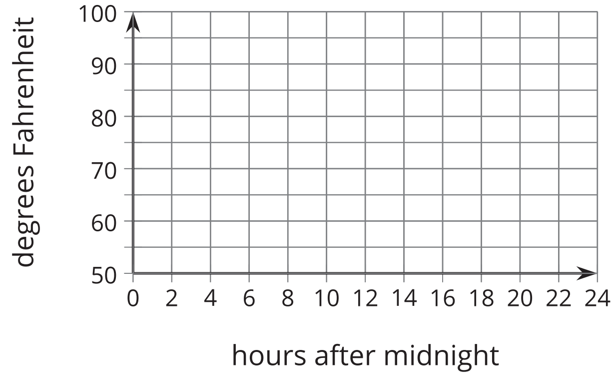


6. Write an equation for $g(x)$ in terms of $h(x)$ for any input x.

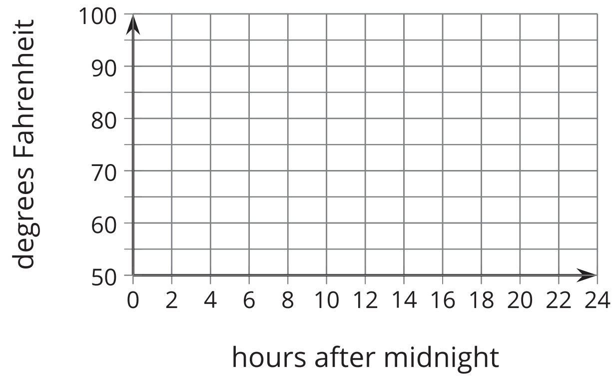
2.3 Heating the Kitchen

A bakery kitchen has a thermostat set to 65°F. Starting at 5:00 a.m., the temperature in the kitchen rises to 85°F when the ovens and other kitchen equipment are turned on to bake the daily breads and pastries. The ovens are turned off at 10:00 a.m. when the baking finishes.

1. Sketch a graph of the function H that gives the temperature in the kitchen $H(x)$, in degrees Fahrenheit, x hours after midnight.



2. The bakery owner decides to change the shop hours to start and end 2 hours earlier. This means the daily baking schedule will also start and end two hours earlier. Sketch a graph of the new function G, which gives the temperature in the kitchen as a function of time.



3. Explain what $H(10.25) = 80$ means in this situation. Why is this reasonable?
4. If $H(10.25) = 80$, then what would the corresponding point on the graph of G be? Use function notation to describe the point on the graph of G.
5. Write an equation for G in terms of H. Explain why your equation makes sense.

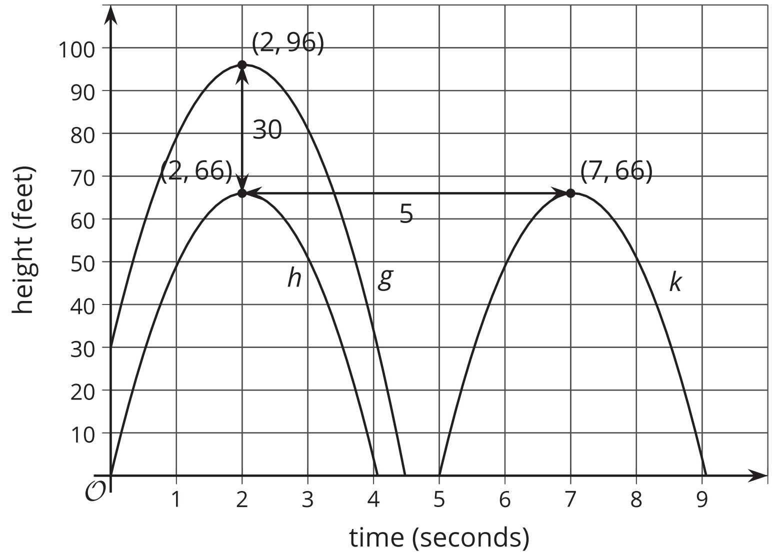
Are you ready for more?

Write an equation that defines your piecewise function, H, algebraically.

Lesson 2 Summary

A pumpkin catapult is used to launch a pumpkin vertically into the air. The function h gives the height $h(t)$, in feet, of this pumpkin above the ground t seconds after launch.

Now consider what happens if the pumpkin had been launched at the same time, but from a platform 30 feet above the ground. Let function g represent the height $g(t)$, in feet, of this pumpkin. How would the graphs of h and g compare?



Since the height of the second pumpkin is 30 feet greater than the first pumpkin at all times t, the graph of function g is translated up 30 feet from the graph of function h. For example, the point $(2, 66)$ on the graph of h tells us that $h(2) = 66$, so the original pumpkin was 66 feet high after 2 seconds. The new pumpkin would be 30 feet higher than that, so $g(2) = 96$. Since all the outputs of g are 30 more than the corresponding outputs of h, we can express $g(t)$ in terms of $h(t)$, using function notation as $g(t) = h(t) + 30$.

Now suppose instead the pumpkin launched 5 seconds later. Let function k represent the height $k(t)$, in feet of this pumpkin. The graph of k is translated right 5 seconds from the graph of h. We can also say that the output values of k are the same as the output values of h 5 seconds earlier. For example, $k(7) = 66$ and $h(7-5) = h(2) = 66$. This means we can express $k(t)$ in terms of $h(t)$ as $k(t) = h(t-5)$.