Comparing Quadratic and Exponential Functions

Let's compare quadratic and exponential changes and see which one grows faster.

4.1 From Least to Greatest

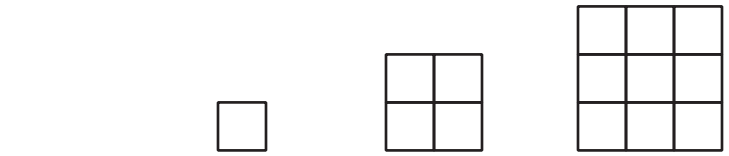
List these quantities in order, from least to greatest, without evaluating each expression. Be prepared to explain your reasoning.

2^{10} 10^2 2^9 9^2

4.2 Which One Grows Faster?

- In Pattern A, the length and width of the rectangle grow by one small square from each step to the next.
- In Pattern B, the number of small squares doubles from each step to the next.
- In each pattern, the number of small squares is a function of the step number, n.

Pattern A



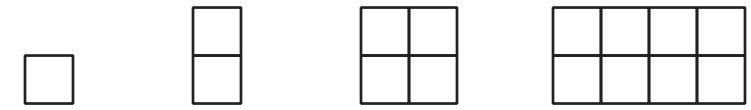
1. Write an equation to represent the number of small squares at Step n in Pattern A.

2. Is the function linear, quadratic, or exponential?

3. Complete the table:

n, step number	$f(n)$, number of small squares
0	
1	
2	
3	
4	
5	
6	
7	
8	

Pattern B



Step 0 Step 1 Step 2 Step 3

1. Write an equation to represent the number of small squares at Step n in Pattern B.

2. Is the function linear, quadratic, or exponential?

3. Complete the table:

n, step number	$g(n)$, number of small squares
0	
1	
2	
3	
4	
5	
6	
7	
8	

How would the two patterns compare if they continue to grow? Make 1–2 observations.

4.3 Comparing Two More Functions

Here are two functions: $p(x) = 6x^2$ and $q(x) = 3^x$.

Investigate the output of p and q for different values of x. For large enough values of x, one function will have a greater value than the other. Which function will have a greater value as x increases?

Support your answer with tables, graphs, or other representations.

Are you ready for more?

1. Jada looks at the function $g(x) = 1.1^x$ and says that this exponential function grows more slowly than the quadratic function $f(x) = x^2$. Do you agree with Jada? Explain your reasoning.

2. Let $f(x) = x^2$. Could you have an exponential function $g(x) = b^x$ so that $g(x) < f(x)$ for all values of x?

Lesson 4 Summary

The graphs of quadratic functions and the graphs of exponential functions with a base that is greater than 1 both curve upward. To compare the two, let's look at the quadratic expression $3n^2$ and the exponential expression 2^n.

A table of values shows that $3n^2$ is initially greater than 2^n, but 2^n eventually becomes greater.

n	$3n^2$	2^n
1	3	2
2	12	4
3	27	8
4	48	16
5	75	32
6	108	64
7	147	128
8	192	256

Here's why exponential growth eventually overtakes quadratic growth.

- When n increases by 1, the exponential expression 2^n always increases by a factor of 2.
- The quadratic expression $3n^2$ increases by different factors, depending on n, but these factors get smaller. For example, when n increases from 2 to 3, the factor is $\frac{27}{12}$ or 2.25. When n increases from 6 to 7, the factor is $\frac{147}{108}$ or about 1.36. As n increases to larger and larger values, $3n^2$ grows by a factor that gets closer and closer to 1.

In general, quadratic functions change with a factor that gets closer and closer to 1 as the input to the function gets larger. Exponential functions always grow with the same factor, so if that growth factor is greater than 1, then the exponential function will eventually grow faster than any quadratic function will.