Unit 4, Lesson 3

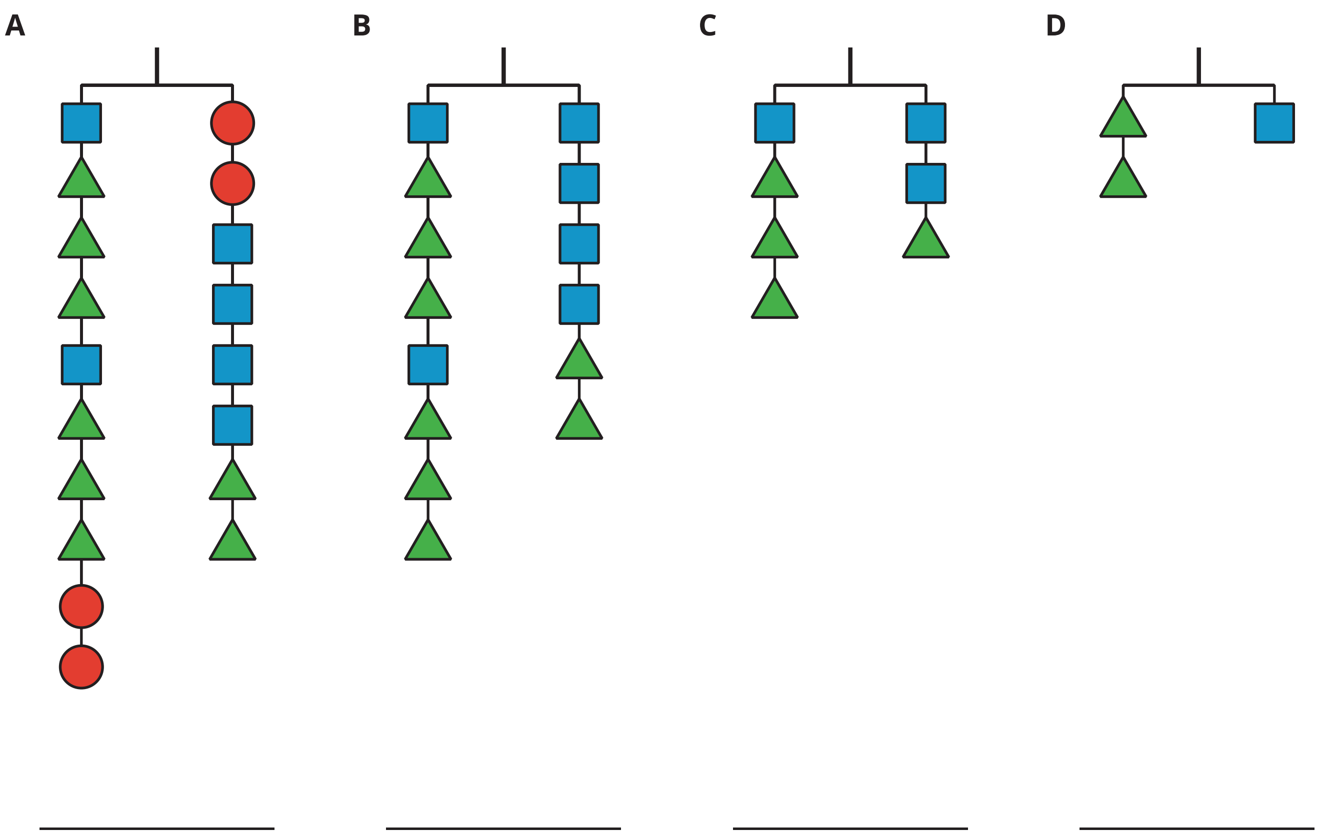
# Balanced Moves

Let's rewrite equations while keeping the same solutions.

## 3.1 Matching Hangers

Figures B, C, and D show the result of simplifying the hanger in Figure A by removing equal weights from each side.

Let $x$ be the weight of the blue square, $y$ be the weight of a green triangle, and $z$ be the weight of a red circle.




1. Draw an arrow from each hanger to the next and describe the move that keeps the hanger in balance near each arrow.

2. Here are some equations. Each equation represents one of the hanger diagrams. Write the matching equation on the line below each hanger diagram.

$$2(x+3y)=4x+2y$$
$$2y=x$$
$$2(x+3y)+2z=2z+4x+2y$$
$$x+3y=2x+y$$

3. Describe the move that keeps the equation for diagram A equivalent to the equation for diagram B.

## 3.2 Card Sort: Matching Equation Moves

1. Your teacher will give you a set of cards. Take turns with your partner to match a pair of equations with a description of the valid move that is used to rewrite the first equation as the second.
   a. For each match that you find, explain to your partner how you know it’s a match.
   b. For each match that your partner finds, listen carefully to your partner's explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. One of the letter cards does not have a match. For this card, write two equations showing the described move.

## 3.3 Keeping Equality

1. Noah and Lin both solve the equation $14a = 2(a - 3)$.

   Do you agree with either of them? Explain your reasoning.

   Noah's solution:

   $$\begin{aligned}14a &= 2(a-3)\\ 14a &= 2a - 6\\ 12a &= \text{-}6\\ a &= \text{-}\frac{1}{2}\end{aligned}$$

   Lin's solution:

   $$\begin{aligned}14a &= 2(a-3)\\ 7a &= a - 3\\ 6a &= \text{-}3\\ a &= \text{-}\frac{1}{2}\end{aligned}$$

2. Elena is asked to solve $15 - 10x = 5(x + 9)$. What do you recommend she does to each side first?

3. Diego is asked to solve $3x - 8 = 4(x + 5)$. What do you recommend he does to each side first?

### Are you ready for more?

In a cryptarithmetic puzzle, the digits 0–9 are represented with letters of the alphabet. Use your understanding of addition to find which digits go with the letters A, B, E, G, H, L, N, and R.

HANGER + HANGER + HANGER = ALGEBRA

## Lesson 3 Summary

An equation tells us that two expressions have equal value. For example, if $4x+9$ and $-2x-3$ have equal value, we can write the equation

$$4x+9=-2x-3$$

Earlier, we used hangers to understand that if we add the same positive number to each side of the equation, the sides will still have equal value. It also works if we add negative numbers! For example, we can add -9 to each side of the equation.

$$\begin{aligned} 4x+9 &= -2x-3 \\ &\text{add -9 (each side)} \\ 4x+9+-9 &= -2x-3+-9 \\ &\text{combine like terms (each side)} \\ 4x &= -2x-12 \end{aligned}$$

Because expressions represent numbers, we can also add expressions to each side of an equation. For example, we can add $2x$ to each side and still maintain equality.

$$\begin{aligned} 4x &= -2x-12 \\ &\text{add } 2x \text{ (each side)} \\ 4x+2x &= -2x-12+2x \\ &\text{combine like terms (each side)} \\ 6x &= -12 \end{aligned}$$

If we multiply or divide the expressions on each side of an equation by the same number, we will also maintain the equality (as long as we do not divide by zero).

$$\begin{aligned} 6x &= -12 \\ &\text{multiply by } \tfrac{1}{6} \text{ (each side)} \\ 6x \cdot \tfrac{1}{6} &= -12 \cdot \tfrac{1}{6} \end{aligned}$$

or

$$\begin{aligned} 6x &= -12 \\ &\text{divide by 6 (each side)} \\ 6x \div 6 &= -12 \div 6 \end{aligned}$$

Now we can see that $x=-2$ is the solution to our equation.