

Fractional Lengths

Goals

- Interpret a question (in written language) about multiplicative comparison, such as, “How many times as long?” and write a division equation to represent it.
- Solve a problem about comparing fractional lengths or measuring with non-standard units, and explain (orally and in writing) the solution method.

Learning Targets

- I can use division and multiplication to solve problems involving fractional lengths.

Lesson Narrative

In this lesson, students solve multiplicative comparison problems involving fractional lengths. Familiarity with questions such as “How many groups?” and “What fraction of a group?” helps students answer questions such as “How many objects of this length are in that length?” and “What fraction of this height is that height?”

In the main activity, an *Information Gap* routine, students practice identifying and asking for information that they need in order to solve geometric problems. The context involves the lengths and widths of objects. Two optional activities that follow allow students to practice interpreting other situations involving fractional lengths and using multiplication and division to find unknown values.

Now that students have at their disposal an algorithm for dividing fractions, they encounter a wider range of numbers than in earlier lessons. In working with fractions—including mixed numbers—with various denominators and larger values, they have opportunities to be strategic in how they approach problems based on the numbers involved.

Standards

Addressing 6.NS.A.1

Instructional Routines

- MLR4: Information Gap Cards
- MLR8: Discussion Supports
- Notice and Wonder
- Which Three Go Together?

Required Materials

Materials to Gather

- Geometry toolkits: Activity 2

Materials to Copy

- How Many Would It Take Cards (1 copy for every 4 students): Activity 2

Required Preparation


Activity 2:

For students to verify their answers, consider preparing the objects mentioned in the activity. These objects are $\frac{3}{4}$ -inch



square stickers, $1\frac{1}{4}$ -inch binder clips, and $1\frac{3}{4}$ -inch paper clips.

Student Facing Learning Goals

 Let's solve problems about fractional lengths.

9.1

Which Three Go Together: Working with

 5 min

$\frac{3}{4}$

Warm-up

Activity Narrative

This *Warm-up* prompts students to carefully analyze and compare four situations that involve multiplication or division and a fraction. It gives students a reason to use language precisely (MP6). It gives the teacher an opportunity to hear how students use terminology related to multiplication and division in the context of length and talk about characteristics of the items in comparison to one another.

Standards

Addressing 6.NS.A.1

Instructional Routines

- MLR8: Discussion Supports
- Which Three Go Together?

Launch

Arrange students in groups of 2–4. Display the four items for all to see. Give students 1 minute of quiet think time, and ask them to indicate when they have noticed three items that go together and can explain why. Next, tell students to share their response with their group and then together to find as many sets of three as they can.

Student Task Statement

Which three go together? Why do they go together?

- A. A string that is $\frac{3}{4}$ meter long is cut into 15 equal pieces. How long is each piece?
- B. $? \cdot \frac{3}{4} = 15$
- C. A driver drove $\frac{3}{4}$ km from home to a gas station and then drove 15 times as far to go to work. What is the distance between the gas station and his work?
- D. Mai built a tower that is 21 inches tall by stacking $\frac{3}{4}$ -inch tall cubes. How many cubes did she use?

Student Response

Sample responses:

- A, B, and C go together because they all involve $\frac{3}{4}$, 15, and an unknown number.
- A, B, and D go together because the answer can be found by dividing the given numbers.
- A, C, and D go together because they are all word problems.



- B, C, and D go together because to find the answer, the $\frac{3}{4}$ is being multiplied by another number and not being divided.

Activity Synthesis

Invite each group to share one reason why a particular set of three go together. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which three go together, attend to students' explanations and ensure that the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology that they use to describe the relationship between the known and unknown quantities, such as " $\frac{3}{4}$ split into 15 parts," "15 times as long as $\frac{3}{4}$," and "21 is some number times $\frac{3}{4}$." Ask students to clarify their reasoning as needed. Consider asking:

- "How do you know . . . ?"
- "What do you mean by . . . ?"
- "Can you say that in another way?"



Access for English Language Learners

Speaking: MLR8 Discussion Supports: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.

Design Principle(s): Optimize output (for explanation)

9.2 Info Gap: How Many Would It Take?

🕒 20 min

Activity Narrative

In this activity, students use division to solve problems involving lengths but do not initially have enough information to do so. To bridge the gap, they need to exchange questions and ideas.

The *Information Gap* structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and to ask increasingly more precise questions until they get the information they need (MP6).



Access for English Language Learners

This activity uses the *Information Gap* math language routine, which facilitates meaningful interactions by positioning some students as holders of information that is needed by other students, creating a need to communicate.



Standards

Addressing 6.NS.A.1



Instructional Routines

- MLR4: Information Gap Cards



Launch

Tell students they will solve problems that involve multiplying or dividing fractions. Display, for all to see, the *Information Gap* graphic that illustrates a framework for the routine.

Remind students of the structure of the *Information Gap* routine, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. In each group, give a problem card to one student and a data card to the other student. After reviewing their work on the first problem, give students the cards for a second problem and instruct them to switch roles.

To orient students to the meaning of “spine” as used in the activity, consider holding up a book and pointing out where its spine is.



Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Check for understanding by inviting students to rephrase directions in their own words. Keep a display of the *Information Gap* graphic visible throughout the activity, or provide students with a physical copy.

Supports accessibility for: Memory, Organization



Student Task Statement

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me _____?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know _____ because”
Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know _____?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!
These steps may be repeated.
4. Once your partner says there is enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Student Response

Problem Card 1: It takes 14 stickers to make a line that is $10\frac{1}{2}$ inches long. $10\frac{1}{2} \div \frac{3}{4} = \frac{21}{2} \cdot \frac{4}{3} = 14$.

Problem Card 2: It takes 32 binder clips to make two lines that are each 20 inches long. $20 \div \frac{5}{4} = 20 \cdot \frac{4}{5} = 16$ and $16 \cdot 2 = 32$.



Building on Student Thinking

If students aren't sure how to represent the situations mathematically, suggest that they first draw sketches of the objects to make sense of the given information. When they have a better understanding of the situation, then they can reason in other more-abstract or more-efficient ways, such as by creating tape diagrams or by writing equations.

Are You Ready for More?

Lin has a work of art that is 14 inches by 20 inches. She wants to frame it with large paper clips laid end to end.

1. If each paper clip is $1\frac{3}{4}$ inch long, how many paper clips would she need? Show your reasoning and be sure to think about potential gaps and overlaps. Consider making a sketch that shows how the paper clips could be arranged.
2. How many paper clips are needed if the paper clips are spaced $\frac{1}{4}$ inch apart? Describe the arrangement of the paper clips at the corners of the frame.

Extension Student Response

Sample response:

1. 38 paper clips.
 - On the side that is 20 inches long, Lin can fit 11 paper clips along the side with a gap of $\frac{3}{4}$ inch because $11 \cdot 1\frac{3}{4} = 19\frac{1}{4}$. If the paper clips are centered along the 20-inch length, there will be $\frac{3}{8}$ inch of gap on either side.
 - On the side that is 14 inches long, Lin can fit 8 paper clips along the side with no gap at all because $14 \div 1\frac{3}{4} = 8$.
 - At each corner, two paper clips will meet. If the paper clip is about $\frac{3}{8}$ inch in width (to fit in the $\frac{3}{8}$ -inch gap left by the 11 paper clips along the longer side), then there will be no gap or overlap.
 - In total, Lin will need 38 paper clips. $11 + 11 + 8 + 8 = 38$
2. 34 paper clips. If a gap of $\frac{1}{4}$ inch is between the paper clips, then each paper clip could have $\frac{1}{8}$ inch of space on either end so that the paper clip and its space takes up 2 inches. Then there are 7 paper clips along the 14-inch side of the frame, and there are 10 paper clips along the 20-inch side. There is a gap of $\frac{1}{8}$ inch between the end of the paper clip and the end of the frame.

Activity Synthesis

After students have completed their work, share the correct answers, and ask students to discuss the process of solving the problems. Here are some questions for discussion:

- “How are the situations about stickers and binder clips alike?” (They are both about finding how many of a fraction are in a larger number. They both involve lengths. The answers are both whole numbers.)
- “How are the problems different?” (One dividend is not a whole number and the other one is.)
- “Was your strategy for finding $20 \div \frac{5}{4}$ different from the one you used to find $10\frac{1}{2} \div \frac{3}{4}$? Why or why not?”

Highlight for students that we can reason about equal-size groups in the context of lengths using the same



representations and strategies as used in other contexts. In some cases, it might be easier to reason in terms of multiplication, such as by thinking, “What number times $\frac{5}{4}$ is 20, or $\frac{80}{4}$?” Other times, it might be quicker to draw a tape diagram or use an algorithm.

9.3

How Many Times as Tall or as Far?

🕒 15 min

Optional

Activity Narrative

In this optional activity, students practice dividing fractions as they solve multiplicative comparison problems. The reasoning here builds on the work in an optional activity earlier in the unit, “Fractions of Ropes.” There, students used diagrams to reason about how many times as long one rope is compared to another. Here, students can decide what representations or strategies would be fruitful. If needed for scaffolding, consider facilitating that activity before this one.

As students create expressions, equations, or diagrams to make sense of quantities and answer questions in context, they practice reasoning abstractly and quantitatively (MP2).

Standards

Addressing 6.NS.A.1

Launch

Keep students in groups of 2. Provide access to graph paper.

Display and read aloud the first set of questions. Give students a minute to estimate whether the answer to each question will be less than 1 or greater than 1. Invite a few students to share their estimates and reasoning.

Give students 2–3 minutes of quiet time to calculate the answers to these questions and time to share their responses with their partner. Ask them to come to an agreement before moving on. Remind students that they can check their quotients using multiplication.

Give students 4–5 minutes to complete the last set of questions, either independently or collaboratively with their group. Encourage students to estimate the answer before calculating and to check it afterward.

Student Task Statement

Write a division expression that can help answer each question. Then find the answer and show your reasoning. You can draw a tape diagram if you find it helpful.

1. A young giraffe is 4 meters tall. An adult giraffe is $5\frac{2}{3}$ meters tall.
 - a. How many times as tall as the young giraffe is the adult giraffe?
 - b. What fraction of the adult giraffe’s height is the young giraffe’s height?
2.
 - a. A runner ran $1\frac{4}{5}$ miles on Monday and $6\frac{3}{10}$ miles on Tuesday. How many times her Monday’s distance was her Tuesday’s distance?
 - b. A cyclist planned to ride $9\frac{1}{2}$ miles but only managed to travel $3\frac{7}{8}$ miles. What fraction of his planned trip





did he travel?

Student Response

- $5\frac{2}{3} \div 4$. The adult giraffe is [...] as tall as the young giraffe. Sample reasoning: $5\frac{2}{3} \div 4 = \frac{17}{3} \cdot \frac{1}{4} = \frac{17}{12}$
 - $4 \div 5\frac{2}{3}$. The young giraffe is [...] as tall as the adult giraffe. Sample reasoning: $4 \div 5\frac{2}{3} = 4 \cdot \frac{3}{17} = \frac{12}{17}$
- $6\frac{3}{10} \div 1\frac{4}{5}$. On Tuesday, she ran $3\frac{1}{2}$ times Monday's distance. Sample reasoning: $6\frac{3}{10} \div 1\frac{4}{5} = \frac{63}{10} \cdot \frac{5}{9} = \frac{7}{2} = 3\frac{1}{2}$
 - $3\frac{7}{8} \div 9\frac{1}{2}$. He traveled $\frac{31}{76}$ of his planned trip. Sample reasoning: $3\frac{7}{8} \div 9\frac{1}{2} = \frac{31}{8} \cdot \frac{2}{19} = \frac{31}{76}$

Building on Student Thinking

Students may be unsure how to use the algorithm to divide two mixed numbers. Ask them to recall or revisit how they divided a mixed number by a fraction or by a whole number in earlier activities. Remind them that each mixed number could be written as a fraction with only a numerator and denominator.

Activity Synthesis

Consider giving students access to the answers so they can check their work. If time permits, reconvene as a class to discuss the last set of questions and the different ways in which they were represented and solved.

9.4 Comparing Paper Rolls

Optional

 10 min

Activity Narrative

This activity gives students another opportunity to solve a contextual problem using what they know about fractions, the relationship between multiplication and division, and diagrams. Students observe a photograph of two paper rolls of differing lengths and estimate the relationship between the lengths. The photograph shows that the longer roll is about $2\frac{1}{2}$ or $(\frac{5}{2})$ times as long as the shorter roll. Students use this observation to find out the length of the shorter roll.

The two paper rolls are from paper towels and toilet paper. If possible, consider providing one of each roll to each group of students so they can physically compare their lengths in addition to observing the picture.

As students work, monitor for the different starting equations or diagrams that they use to begin solving the last problem. Ask students who use different entry points to share later.

Standards

Addressing 6.NS.A.1

Instructional Routines

- Notice and Wonder

Launch

Tell students to close their books or devices (or to keep them closed). Display the image of the paper rolls for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder. Record and display responses without editing or commentary. If possible, record the relevant



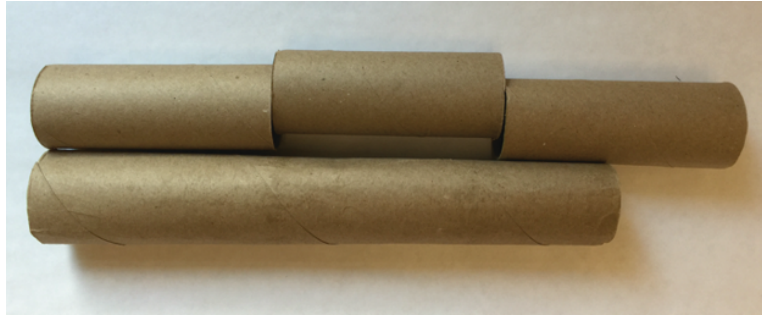
reasoning on or near the image.

If the relationship between the lengths of the rolls does not come up during the conversation, ask students to discuss this idea.

Tell students to open their books or devices. Keep students in groups of 2. Give students 3–4 minutes of quiet time to complete the questions and time to share their response and reasoning with their partner.

Student Task Statement

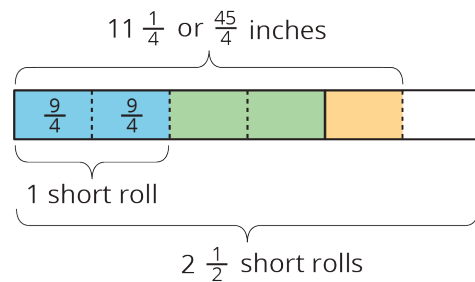
The photo shows a situation that involves fractions.



1. Complete the sentences. Be prepared to explain your reasoning.
 - a. The length of the long tube is about _____ times the length of a short tube.
 - b. The length of a short tube is about _____ times the length of the long tube.
2. If the length of the long paper roll is $11\frac{1}{4}$ inches, what is the length of each short paper roll?

Student Response

1. a. About $\frac{5}{2}$ (or $2\frac{1}{2}$ or 2.5) times.
b. About $\frac{2}{5}$ (or 0.4) times.
2. $4\frac{1}{2}$ (or equivalent) inches. Sample reasoning:
 - $11\frac{1}{4} \div 2\frac{1}{2} = \frac{45}{4} \div \frac{5}{2} = \frac{45}{4} \cdot \frac{2}{5} = 4\frac{1}{2}$
 - $\frac{2}{5} \cdot 11\frac{1}{4} = \frac{2}{5} \cdot \frac{45}{4} = \frac{9}{2} = 4\frac{1}{2}$



Building on Student Thinking

Students might round too much when estimating the relationship between the lengths of the rolls. For example, they might say that the length of the shorter roll is $\frac{1}{3}$ that of the longer roll, or that the longer roll is twice as long as the shorter roll. If this happens, ask students to make a more precise estimate. Suggest that they divide the larger roll into smaller segments, each of which matches the length of the shorter rolls.

Activity Synthesis

Select previously identified students to share their strategies for finding the length of the short roll. Display their diagram, and record their reasoning for all to see.



To find the length of the short roll, some students may use $11\frac{1}{4} \div \frac{5}{2} = ?$ and others $\frac{2}{5} \cdot \frac{45}{4} = ?$, depending on how they view the relationship between the rolls. Highlight the idea that to find the length of the short roll, one way is to partition the length of the large roll into 5 equal pieces, find that length, and multiply it by 2, because the length of the shorter roll is about $\frac{2}{5}$ of that of the longer roll. This is an opportunity to reinforce the structure behind the division algorithm.

Lesson Synthesis

The goal of this discussion is to help students make connections between the geometric problems students solved in this lesson and the problems they saw earlier in the unit. Select and display 2–3 questions students answered in the lesson, for instance:

- “How many $\frac{5}{8}$ -inch paper clips, laid end to end, are in a length of $12\frac{1}{2}$ inches?”
- “A young giraffe is 4 meters tall. An adult giraffe is $5\frac{2}{3}$ meters tall. What fraction of the adult giraffe’s height is the young giraffe’s height?”

Ask questions such as:

- “How are these questions like the ones in earlier lessons?” (We can think of them in terms of finding the number of groups or fraction of a group. We can write multiplication equations to represent them and answer by dividing.)
- “How are they different?” (These are about lengths or comparisons of two lengths. The latter isn’t about equal-size groups.)
- “What are some ways to answer comparison questions such as the ones about the giraffes?” (Draw a diagram to represent the two values, and see what fraction one is of the other. Write an equation like $? \cdot 5\frac{2}{3} = 4$, and find $4 \div 5\frac{2}{3}$ to find the answer.)

If time allows, or if the question arose during the activity, discuss whether or when to write fractions greater than 1 in the form of mixed numbers versus $\frac{a}{b}$. Mixed numbers, such as $12\frac{1}{2}$ and $5\frac{2}{3}$, are easier to visualize, but $\frac{25}{2}$ and $\frac{17}{3}$ are easier to work with in calculations. In fact, we have to use the latter to easily multiply. Explain that the two forms serve different purposes and that we can write one as the other depending on what we aim to do. When writing them as solutions, both forms are mathematically correct.

9.5 Building A Fence

Cool-down

🕒 5 min

Standards

Addressing 6.NS.A.1

Launch

Give students continued access to graph paper, if needed.



Student Task Statement

A builder was building a fence. In the morning, he worked for $\frac{2}{5}$ of an hour. In the afternoon, he worked for $\frac{9}{10}$ of an hour. How many times as long as in the morning did he work in the afternoon?

Write a division equation to represent this situation, then answer the question. Show your reasoning. If you get stuck, consider drawing a diagram.

Student Response

Division equation: $\frac{9}{10} \div \frac{2}{5} = ?$ (or $\frac{9}{10} \div ? = \frac{2}{5}$). In the afternoon, he worked $2\frac{1}{4}$ times as long as he did in the morning.

Sample reasoning: $\frac{9}{10} \div \frac{2}{5} = \frac{9}{10} \cdot \frac{5}{2} = \frac{45}{20} = \frac{9}{4}$.

Responding to Student Thinking

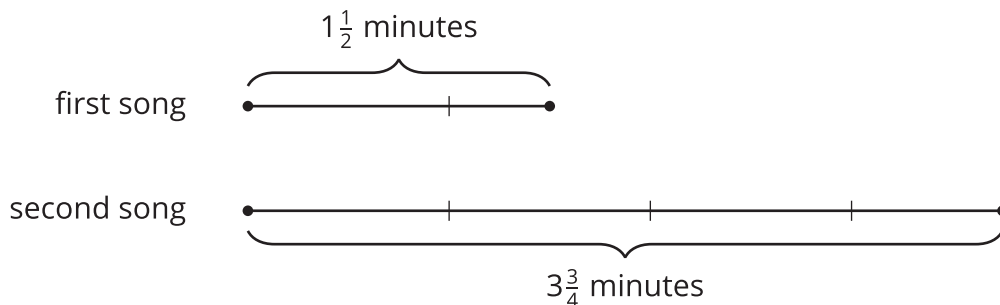
Points to Emphasize

If students struggle with writing a division equation to represent a situation, focus on starting with a diagram and a multiplication equation that describe the relationship between the quantities. For example, when doing the referenced practice problem, create a tape diagram that represents the relationship between the two heights and ask students to write a multiplication equation before writing a division equation.

Accelerated 6, Unit 3, Lesson 9, Practice Problem 2

Lesson 9 Summary

Division can help us solve comparison problems in which we find out how many times as large or as small one number is compared to another. For example, a student is playing two songs for a music recital. The first song is $1\frac{1}{2}$ minutes long. The second song is $3\frac{3}{4}$ minutes long.



We can ask two different comparison questions and write different multiplication and division equations to represent each question.

- How many times as long as the first song is the second song?

$$? \cdot 1\frac{1}{2} = 3\frac{3}{4}$$
$$3\frac{3}{4} \div 1\frac{1}{2} = ?$$

- What fraction of the second song is the first song?

$$? \cdot 3\frac{3}{4} = 1\frac{1}{2}$$
$$1\frac{1}{2} \div 3\frac{3}{4} = ?$$



We can use the algorithm we learned to calculate the quotients.

$$= \frac{15}{4} \div \frac{3}{2}$$

$$= \frac{15}{4} \cdot \frac{2}{3}$$

$$= \frac{30}{12}$$

$$= \frac{5}{2}$$

$$= \frac{3}{2} \div \frac{15}{4}$$

$$= \frac{3}{2} \cdot \frac{4}{15}$$

$$= \frac{12}{30}$$

$$= \frac{2}{5}$$

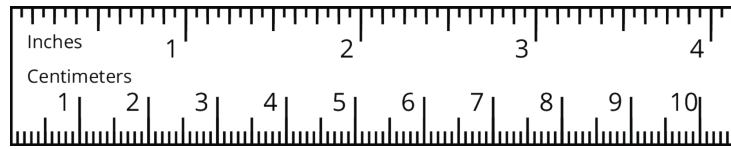
This means the second song is $2\frac{1}{2}$ times as long as the first song.

This means the first song is $\frac{2}{5}$ as long as the second song.

Lesson 9 Practice Problems

1 Student Task Statement

One inch is around $2\frac{11}{20}$ centimeters.



- How many centimeters long is 3 inches? Show your reasoning.
- What fraction of an inch is 1 centimeter? Show your reasoning.
- What question can be answered by finding $10 \div 2\frac{11}{20}$ in this situation?

Solution

- $7\frac{13}{20}$ centimeters. $3 \cdot 2\frac{11}{20} = \frac{3}{1} \cdot \frac{51}{20} = \frac{153}{20}$, which is $7\frac{13}{20}$.
- $\frac{20}{51}$, because $1 \div 2\frac{11}{20} = 1 \cdot \frac{20}{51}$, which is $\frac{20}{51}$.
- How many inches are in 10 centimeters?

2 Student Task Statement

A zookeeper is $6\frac{1}{4}$ feet tall. A young giraffe in his care is $9\frac{3}{8}$ feet tall.

- How many times as tall as the zookeeper is the giraffe?
- What fraction of the giraffe's height is the zookeeper's height?

Solution

- $9\frac{3}{8} \div 6\frac{1}{4} = \frac{75}{8} \div \frac{25}{4}$, and $\frac{75}{8} \div \frac{25}{4} = \frac{75}{8} \cdot \frac{4}{25}$, which equals $\frac{3}{2}$. The giraffe is $\frac{3}{2}$ or $1\frac{1}{2}$ times as tall as the zookeeper.
- $6\frac{1}{4} \div 9\frac{3}{8} = \frac{25}{4} \div \frac{75}{8}$, and $\frac{25}{4} \div \frac{75}{8} = \frac{25}{4} \cdot \frac{8}{75}$, which equals $\frac{2}{3}$. The zookeeper's height is $\frac{2}{3}$ of the giraffe's height.

3 Student Task Statement

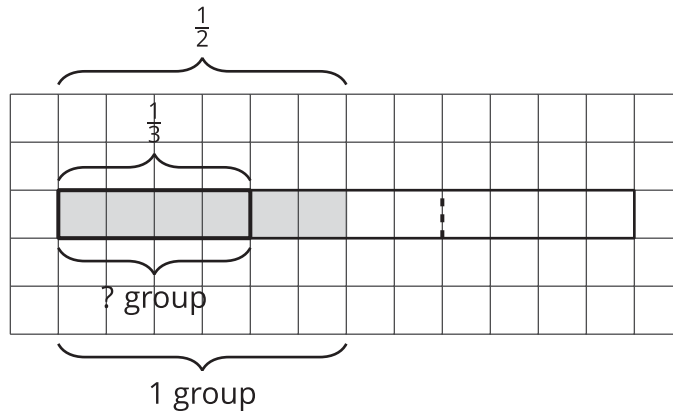
A rectangular bathroom floor is covered with square tiles that are $1\frac{1}{2}$ feet by $1\frac{1}{2}$ feet. The length of the bathroom floor is $10\frac{1}{2}$ feet and the width is $6\frac{1}{2}$ feet.

- How many tiles does it take to cover the length of the floor?
- How many tiles does it take to cover the width of the floor?



Solution


$\frac{2}{3}$. Sample reasoning:



6

from Unit 3, Lesson 3

Student Task Statement

 Noah says, “There are $2\frac{1}{2}$ groups of $\frac{4}{5}$ in 2.” Do you agree with him? Draw a tape diagram to show your reasoning. Use graph paper, if needed.

Solution

Agree. Sample diagram:

