Transforming Circles

Let's transform some circles.

14.1 Math Talk: Perfect Squares

For each expression, mentally find an equivalent expression that uses perfect squares:

- $x^2 + 4x + 4$
- $2x^2 + 8x + 8$
- $x^2 + 4x + y^2 + 6y + 13$
- $2x^2 + 8x + 3y^2 + 6y + 11$

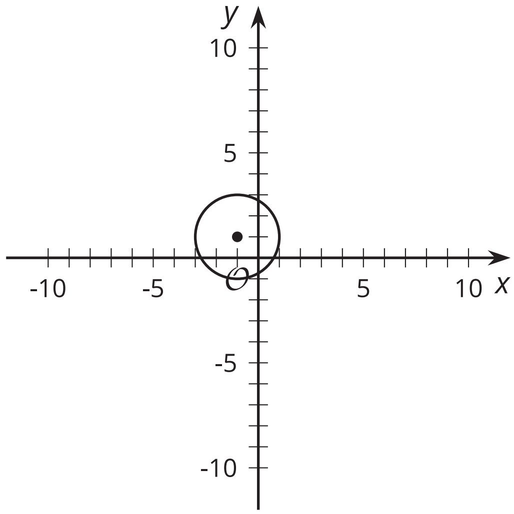
14.2 Circles and Graphs

1. Match these equations of circles with their graphs and centers.

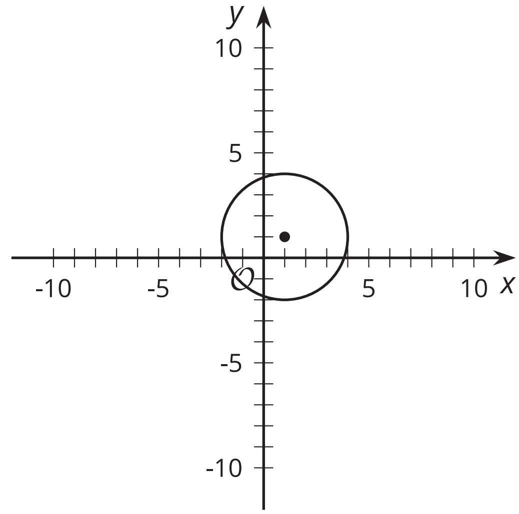
Equations:	Centers:
$(x+1)^2 + (y-1)^2 = 4$	(1, 1)
$(x+3)^2 + (y+2)^2 = 9$	(-1, 1)
$(x-2)^2 + (y-3)^2 = 25$	(2, 3)
$(x-1)^2 + (y-1)^2 = 9$	(-3, -2)

Graphs:

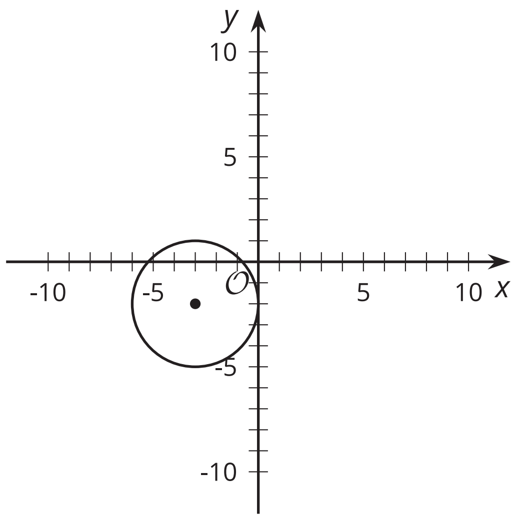
Circle A



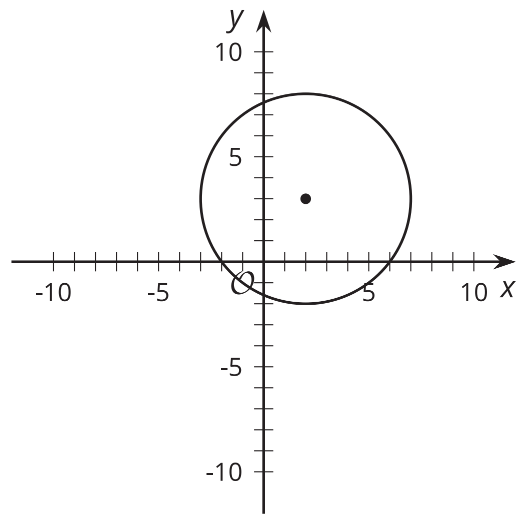
Circle B



Circle C



Circle D



2. What do you notice about the equations and the coordinates of the center of each circle? How does this connect to what you know about transformations of functions?
3. An original equation for a circle could be $x^2 + y^2 = 1$. What would the new equation be for a circle that:
   a. Has a center $(-2, 5)$?
   b. Has a radius of 6?
   c. Has a center $(3, 4)$ with a radius of 5?

Are you ready for more?

When the horizontal and vertical stretch are the same, we have a dilation. Since all circles are similar, a transformation from one circle to another is a dilation. Use graphing technology to experiment with the equation $x^2 + y^2 = 1$.

1. What changes do you need to make to the equation to stretch the graph horizontally but not vertically? What shape is this?
2. Write an equation for an oval that has been stretched horizontally. What is the factor the graph was stretched by?

14.3 Standard Equations

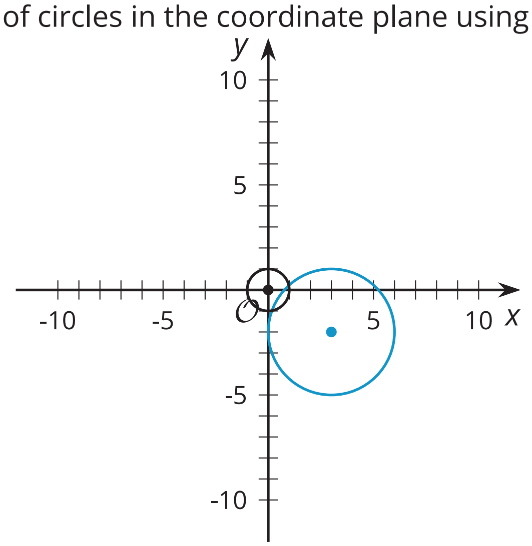
Here are 2 equations:

- $3x^2 - 12x - y = -4$
- $x^2 - 6x + y^2 + 8y = -21$

1. Rewrite the equations so the transformations are visible by completing the square.
2. Which equation represents a circle? Which equation represents a parabola?
3. For the circle:
   a. Identify the center and radius.
   b. Describe the transformations from the original equation $x^2 + y^2 = 1$.
4. For the parabola:
   a. Identify the vertex.
   b. Describe the transformation from the original equation $y = x^2$.

Lesson 14 Summary

We can transform the graphs of circles in the coordinate plane using translations and dilations.



An original circle is centered at $(0, 0)$ with a radius of 1. Its image is centered at $(3, -2)$ with a radius of 3. What transformations were needed to get from the original to the image? In this case, there was a translation right 3 and down 2, with a dilation by a factor of 3. This makes sense because a translation of the circle will translate the center in the same way, and dilating the circle by a factor of 3 also dilates the radius by a factor of 3.

We can also see these transformations in the equations for the circles. We know from studying circles that the equation of any circle with center (h, k) and radius r can be written $(x-h)^2 + (y-k)^2 = r^2$. Therefore, the original circle has an equation $x^2 + y^2 = 1$, and the image or transformed circle has an equation $(x-3)^2 + (y+2)^2 = 9$.

Since the translation of the circle can be determined using the centers, that means that the transformed circle has been translated horizontally by h units and translated vertically by k units. Since the radius of the transformed circle is the same as the scale factor of dilation, we can say that the transformed circle has been dilated by a factor of r.

This means we can tell what transformations have been applied to a circle in the coordinate plane without graphing, given the original circle defined by $x^2 + y^2 = 1$. For example, if a circle has an equation $(x-12)^2 + (y+8)^2 = 81$, we can tell that it was translated right 12 units, translated down 8 units, and dilated by a factor of 9.