Exponential Rules

Let's explore how exponents combine.

8.1 Combining Exponents

Rewrite these expressions so that there is a single variable with only one exponent.

1. $a \cdot a \cdot a \cdot a$
2. $\frac{b \cdot b \cdot b \cdot b}{b \cdot b}$
3. $(c \cdot c) \cdot (c \cdot c) \cdot (c \cdot c)$
4. $(d \cdot d) \cdot (d \cdot d \cdot d \cdot d)$

8.2 Exponent Rules

1. Rewrite these expressions so that there is a single exponent.
    a. $a^3 \cdot a^4$
    b. $\frac{b^8}{b^3}$
    c. $\left(c^3\right)^2$
    d. $\frac{x^3 \cdot x^5}{x^6}$

2. Write a rule for each expression that shows how the exponents can be combined.
    a. $a^b \cdot a^c$
    b. $\frac{e^d}{e^f}$
    c. $\left(g^h\right)^j$

8.3 Revisiting Roots

1. A square has a side length of $\sqrt{10}$ centimeters. What is the area of the square?

2. A cube has an edge of length $\sqrt[3]{5}$ centimeters. What is the volume of the cube?

3. If $x = \sqrt[4]{13}$, then what must be true about x^4?

4. Find the value of $\left(\sqrt{5}\right)^4$. Explain or show your reasoning.

Are you ready for more?

What about negative values like $\sqrt{(-6)^2}$ or $\left(\sqrt[3]{-8}\right)^3$?

1. Use technology to find the value of each expression.

2. Explore other roots (like $\sqrt[4]{-5}$ or $\sqrt[5]{-5}$) of negative values raised to an exponent that matches the root to find a pattern. Describe the pattern you find.

Lesson 8 Summary

There is a connection between exponents and roots that will be explored in later lessons. To understand the connection, it is helpful to know some exponent rules and to understand roots in terms of powers that "undo" them.

We can understand $b^m \cdot b^n$ as a lot of bs multiplied together. In particular, there are m of them multiplied together and then another n of them multiplied together. This means that there are a total of $m+n$ of the bs multiplied together, which can be written as b^{m+n}.

Similarly, we can understand $(b^m)^n$ as n groups of b^m, all multiplied together. That means there are a total of $m \cdot n$ of the bs multiplied together, which can be written as $b^{m \cdot n}$.

When dividing expressions involving exponents, like $\frac{b^m}{b^n}$, let's consider two situations.

First, let's look at the situation in which there are at least as many bs in the numerator as in the denominator ($m \geq n$). We can separate out n of the bs from the numerator and still have $m-n$ of them left. This can be written as $\frac{b^n \cdot b^{m-n}}{b^n}$. By rewriting the fraction, we can get $\frac{b^n}{b^n} \cdot \frac{b^{m-n}}{1}$ which is the same as b^{m-n} because the first fraction has a value of 1.

Second, let's look at the situation in which there are more bs in the denominator than in the numerator ($m < n$). Again, we can separate the fraction into $\frac{b^m}{b^n} = \frac{b^n}{b^n} \cdot \frac{1}{b^{n-m}}$, or $\frac{1}{b^{n-m}}$. Then if we recall the meaning of a negative exponent, we can see that $\frac{1}{b^{n-m}} = b^{-(n-m)}$, which is the same as b^{m-n} again.

Here are all of the exponent rules we looked at in this lesson:

$$b^m \cdot b^n = b^{m+n}$$
$$(b^m)^n = b^{m \cdot n}$$
$$\frac{b^m}{b^n} = b^{m-n}$$

It is also helpful to recall the meaning of $\sqrt[n]{a}$. We can say that $x = \sqrt[n]{a}$ is a solution to the equation $x^n = a$ (when $a \geq 0$).

