Lesson 13: Reasoning about Exponential Graphs (Part 2)

Let's investigate what we can learn from graphs that represent exponential functions.

13.1: Which One Doesn't Belong: Four Functions

Which one doesn't belong?

$f(n) = 8 \cdot 2^n$

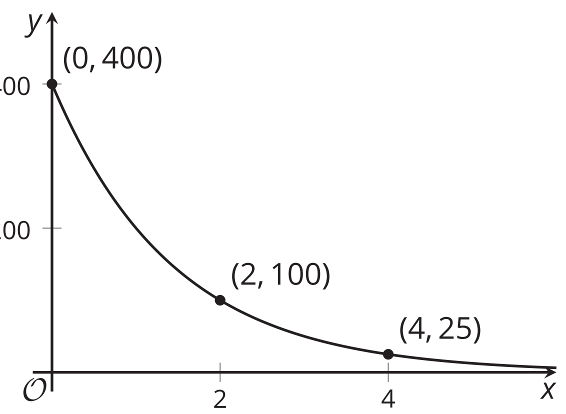
$g(n) = 2 \cdot 8^n$

$h(n) = 8 + 2n$

$j(n) = 8 \cdot \left(\frac{1}{2}\right)^n$

13.2: Value of A Computer

1. Here is a graph representing an exponential function f. The function f gives the value of a computer, in dollars, as a function of time, x, measured in years since the time of purchase.



Based on the graph, what can you say about the following?

a. The purchase price of the computer

b. The value of f when x is 1

c. The meaning of $f(1)$

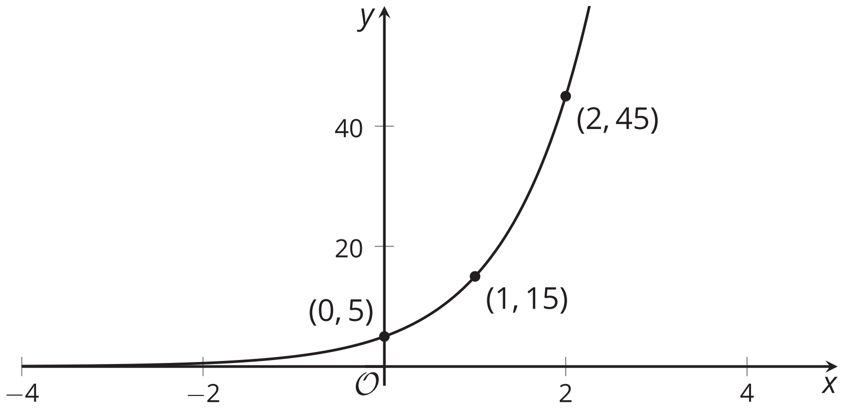
d. How the value of the computer is changing each year

e. An equation that defines f

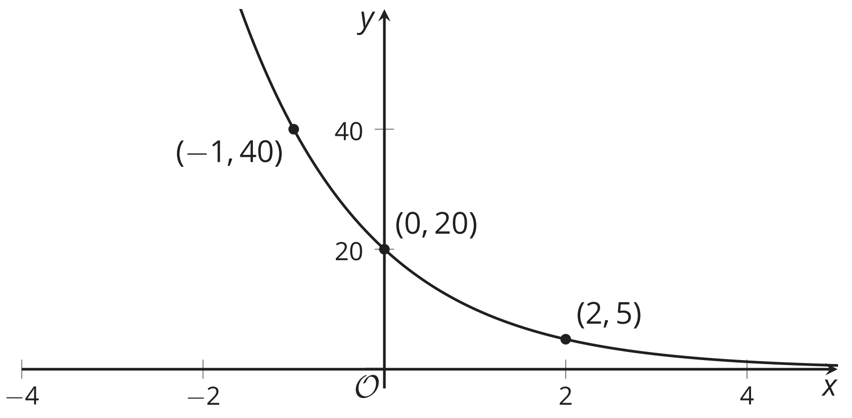
f. Whether the value of f will reach 0 after 10 years

2. Here are graphs of two exponential functions. For each, write an equation that defines the function and find the value of the function when x is 5.

a.



b.



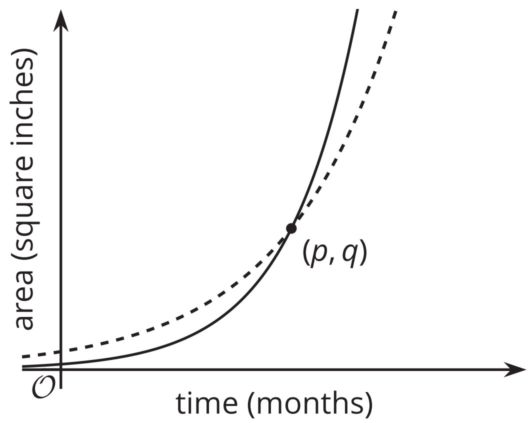
Are you ready for more?

Consider a function f defined by $f(x) = a \cdot b^x$.

- If the graph of f goes through the points $(2, 10)$ and $(8, 30)$, would you expect $f(5)$ to be less than, equal to, or greater than 20?
- If the graph of f goes through the points $(2, 30)$ and $(8, 10)$, would you expect $f(5)$ to be less than, equal to, or greater than 20?

13.3: Moldy Wall

Here are graphs representing two functions, and descriptions of two functions.



- Function f: The area of a wall that is covered by Mold A, in square inches, doubling every month.
- Function g: The area of a wall that is covered by Mold B, in square inches, tripling every month.

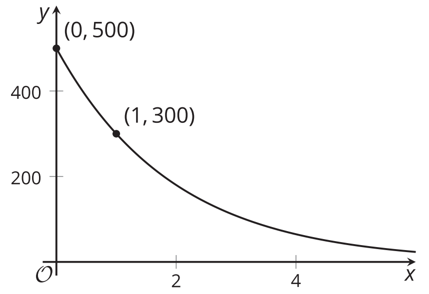
1. Which graph represents each function? Label the graphs accordingly and explain your reasoning.

2. When the mold was first spotted and measured, was there more of Mold A or Mold B? Explain how you know.

3. What does the point (p, q) tell us in this situation?

Lesson 13 Summary

If we have enough information about a graph representing an exponential function f, we can write a corresponding equation. Here is a graph of $y = f(x)$.

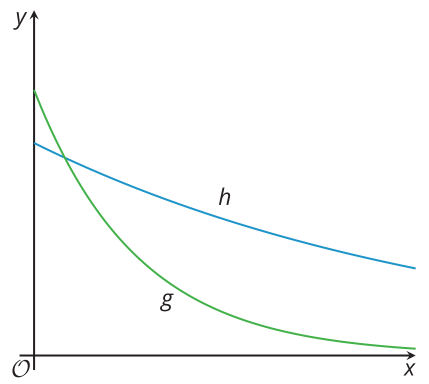


An equation defining an exponential function has the form $f(x) = a \cdot b^x$. The value of a is the starting value or $f(0)$, so it is the y-intercept of the graph. We can see that $f(0)$ is 500 and that the function is decreasing.

The value of b is the growth factor. It is the number by which we multiply the function's output at x to get the output at $x + 1$. To find this growth factor for f, we can calculate $\frac{f(1)}{f(0)}$, which is $\frac{300}{500}$ or $\frac{3}{5}$. So an equation that defines f is:

$$f(x) = 500 \cdot \left(\frac{3}{5}\right)^x$$

We can also use graphs to compare functions. Here are graphs representing two different exponential functions, labeled g and h. Each one represents the area of algae (in square meters) in a pond, x days after certain fish were introduced.



- Pond A had 40 square meters of algae. Its area shrinks to $\frac{8}{10}$ of the area on the previous day.
- Pond B had 50 square meters of algae. Its area shrinks to $\frac{2}{5}$ of the area on the previous day.

Can you tell which graph corresponds to which algae population?

We can see that the y-intercept of g's graph is greater than the y-intercept of h's graph. We can also see that g has a smaller growth factor than h because as x increases by the same amount, g is retaining a smaller fraction of its value compared to h. This suggests that g corresponds to Pond B and h corresponds to Pond A.