Solving Quadratic Equations with the Zero Product Property

Let's find solutions to equations that contain products that equal zero.

4.1 Math Talk: Solve These Equations

What values of the variables make each equation true?

- $6 + 2a = 0$
- $7b = 0$
- $7(c - 5) = 0$
- $g \cdot h = 0$

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4.2 Take the Zero Product Property Out for a Spin

For each equation, find its solution or solutions. Be prepared to explain your reasoning.

1. $x - 3 = 0$
2. $x + 11 = 0$
3. $2x + 11 = 0$
4. $x(2x + 11) = 0$
5. $(x - 3)(x + 11) = 0$
6. $(x - 3)(2x + 11) = 0$
7. $x(x + 3)(3x - 4) = 0$

Are you ready for more?

1. Use factors of 48 to find as many solutions as you can to the equation $(x - 3)(x + 5) = 48$.

2. Once you think you have all the solutions, explain why these must be the only solutions.

4.3 Revisiting a Projectile

We have seen quadratic functions modeling the height of a projectile as a function of time.

Here are two ways to define the same function that approximates the height of a projectile in meters, t seconds after launch:

$$h(t) = -5t^2 + 27t + 18 \qquad h(t) = (-5t - 3)(t - 6)$$

1. Which way of defining the function allows us to use the zero product property to find out when the height of the object is 0 meters?

2. Without graphing, determine at what time the height of the object is 0 meters. Show your reasoning.

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Lesson 4 Summary

The **zero product property** says that if the product of two numbers is 0, then one of the numbers must be 0. In other words, if $a \cdot b = 0$, then either $a = 0$ or $b = 0$. This property is handy when an equation we want to solve states that the product of two factors is 0.

Suppose we want to solve $m(m + 9) = 0$. This equation says that the product of m and $(m + 9)$ is 0. For this to be true, either $m = 0$ or $m + 9 = 0$, so both 0 and -9 are solutions.

Here is another equation: $(u - 2.345)(14u + 2) = 0$. The equation says the product of $(u - 2.345)$ and $(14u + 2)$ is 0, so we can use the zero product property to help us find the values of u. For the equation to be true, one of the factors must be 0.

- For $u - 2.345 = 0$ to be true, u would have to be 2.345.
- For $14u + 2 = 0$ or ($14u =$ -2) to be true, u would have to be -$\frac{2}{14}$, or -$\frac{1}{7}$.

The solutions are 2.345 and -$\frac{1}{7}$.

In general, when a quadratic expression in factored form is on one side of an equation and 0 is on the other side, we can use the zero product property to find its solutions.

This property is unique to 0. Given an equation like $a \cdot b = 6$, the factors could be 2 and 3, 1 and 6, -12 and -$\frac{1}{2}$, π and $\frac{6}{\pi}$, or any other of the infinite number of combinations. This type of equation does not give insight into the value of a or b.

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