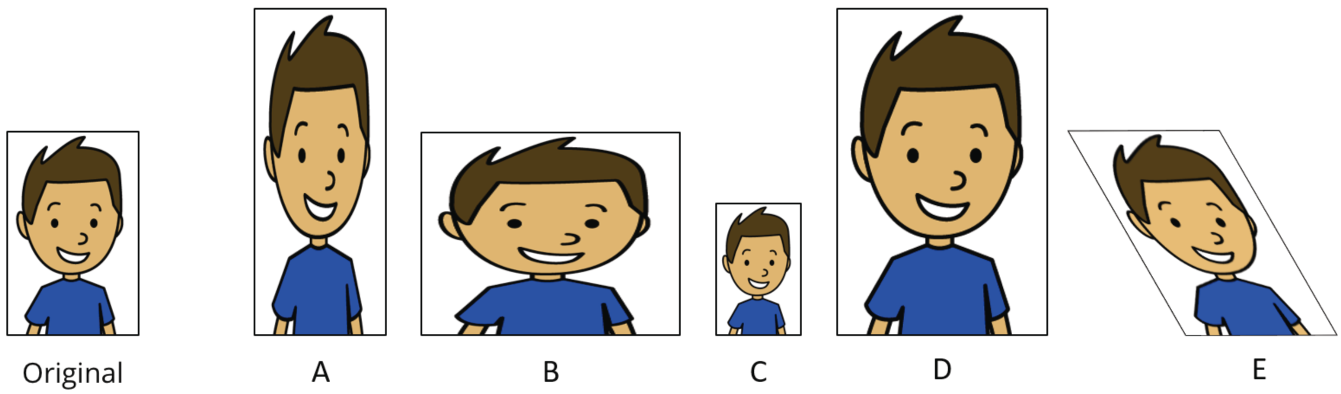
Unit 2 Family Support Materials

Scale Drawings, Similarity, and Slope

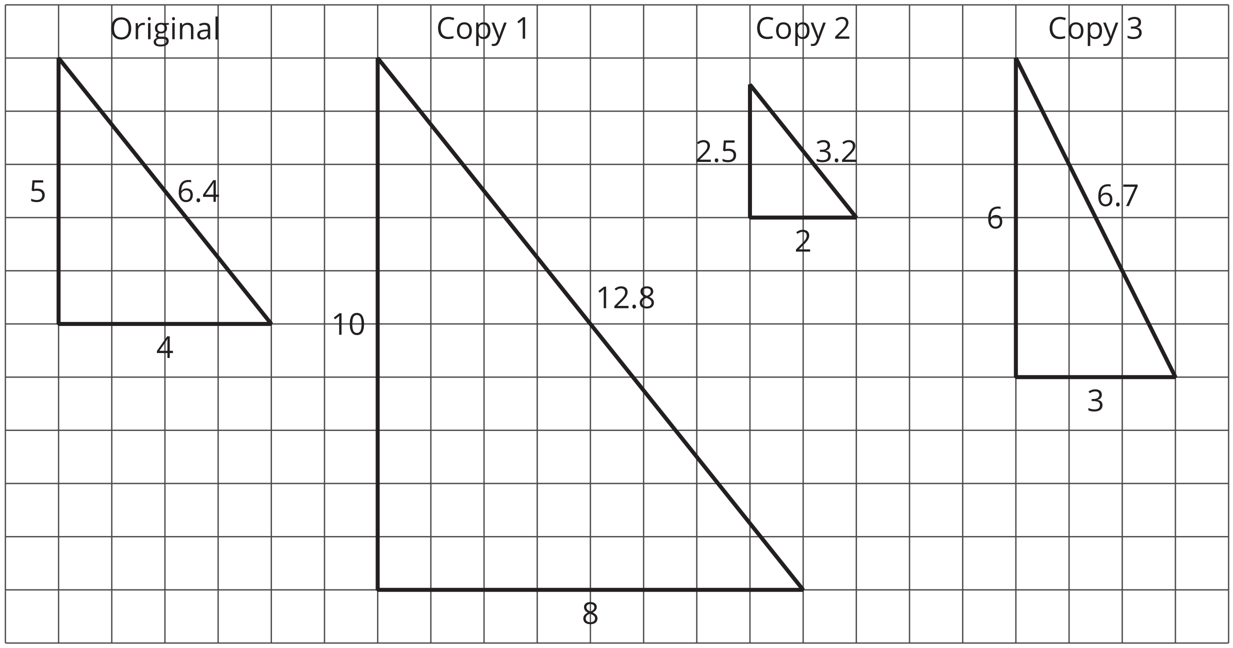
Section A: Scaled Copies

This week your student will learn about scaling shapes. An image is a **scaled copy** of the original if the shape is stretched in a way that does not distort it. For example, here is an original picture and five copies. Pictures C and D are scaled copies of the original, but pictures A, B, and E are not.



In each scaled copy, the sides are a certain number of times as long as the corresponding sides in the original. We call this number the **scale factor**. The size of the scale factor affects the size of the copy. A scale factor greater than 1 makes a copy that is larger than the original. A scale factor less than 1 makes a copy that is smaller.

Here is a task to try with your student:



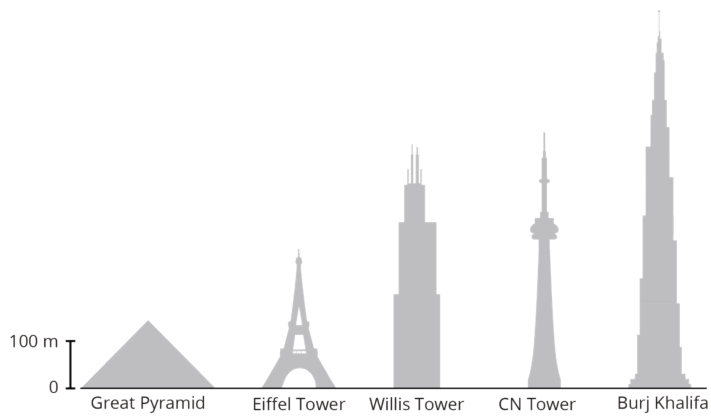
1. For each copy, tell whether it is a scaled copy of the original triangle. If so, what is the scale factor?
2. Draw another scaled copy of the original triangle using a different scale factor.

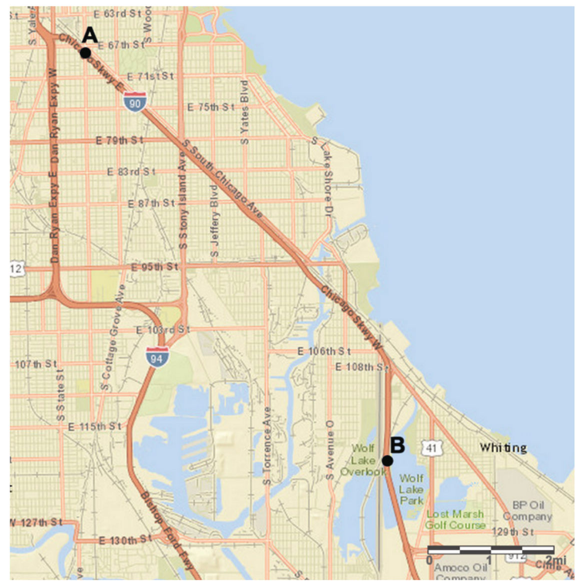
Solution:

1. a. Copy 1 is a scaled copy of the original triangle. The scale factor is 2, because each side in Copy 1 is twice as long as the corresponding side in the original triangle. $5 \cdot 2 = 10$, $4 \cdot 2 = 8$, $(6.4) \cdot 2 = 12.8$
   b. Copy 2 is a scaled copy of the original triangle. The scale factor is $\frac{1}{2}$ or 0.5, because each side in Copy 2 is half as long as the corresponding side in the original triangle. $5 \cdot (0.5) = 2.5$, $4 \cdot (0.5) = 2$, $(6.4) \cdot (0.5) = 3.2$
   c. Copy 3 is not a scaled copy of the original triangle. The shape has been distorted. The angles are different sizes and there is not one number that we can multiply by each side length of the original triangle to get the corresponding side length in Copy 3.
2. Answers vary. Sample response: A right triangle with side lengths of 12, 15, and 19.2 units would be a scaled copy of the original triangle using a scale factor of 3.

Section B: Scale Drawings

This week your student will be learning about scale drawings. A **scale drawing** is a two-dimensional representation of an actual object or place. Maps and floor plans are some examples of scale drawings.





The **scale** tells us what some length on the scale drawing represents in actual length. For example, a scale of "1 inch to 5 miles" means that 1 inch on the drawing represents 5 actual miles. If the drawing shows a road that is 2 inches long, we know that the road is actually $2 \cdot 5$, or 10, miles long.

Scales can be written with units (for example, 1 inch to 5 miles), or without units (for example, 1 to 50, or 1 to 400). When a scale does not have units, the same unit is used for distances on the scale drawing and actual distances. For example, a scale of "1 to 50" means 1 centimeter on the drawing represents 50 actual centimeters, 1 inch represents 50 actual inches, etc.

Here is a task to try with your student:

Kiran drew a floor plan of his classroom using the scale 1 inch to 6 feet.

1. Kiran's drawing is 4 inches wide and $5\frac{1}{2}$ inches long. What are the dimensions of the actual classroom?
2. A table in the classroom is 3 feet wide and 6 feet long. What size should it be on the scale drawing?
3. Kiran wants to make a larger scale drawing of the same classroom. Which of these scales could he use?

A. 1 to 50

B. 1 to 72

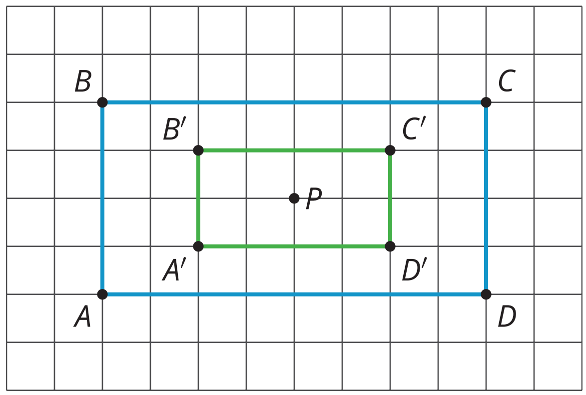
C. 1 to 100

Solution:

1. 24 feet wide and 33 feet long. Because each inch on the drawing represents 6 feet, we can multiply by 6 to find the actual measurements. The actual classroom is 24 feet wide because $4 \cdot 6 = 24$. The classroom is 33 feet long because $5\frac{1}{2} \cdot 6 = (5 \cdot 6) + (\frac{1}{2} \cdot 6) = 30 + 3 = 33$
2. $\frac{1}{2}$ inch wide and 1 inch long. We can divide by 6 to find the measurements on the drawing. $3 \div 6 = \frac{1}{2}$ and $6 \div 6 = 1$.
3. Choice A. 1 to 50. The scale “1 inch to 6 feet” is equivalent to the scale “1 to 72” because there are 72 inches in 6 feet. The scale “1 to 100” would make a scale drawing that is smaller than the scale “1 to 72” because each inch on the new drawing would represent more actual length. The scale “1 to 50” would make a scale drawing that is larger than the scale “1 to 72” because Kiran would need more inches on the drawing to represent the same actual length.

Section C: Dilations

This week your student will expand their understanding of transformations to include non-rigid transformations. Specifically, they will learn to make and describe dilations of figures. A **dilation** is a process to make a scaled copy of a figure, and it is described using a center point and a number (the scale factor). The scale factor can be any positive number, including fractions and decimals. If the scale factor is less than 1, the dilated figure is smaller than the original. If it is greater than 1, the dilated figure is larger than the original. In this dilation, the center is point P, and the scale factor is $\frac{1}{2}$.



When dilating figures, the distance from the center of dilation to a point on the figure is multiplied by the scale factor to get the location of the corresponding point. In this example, the distance between center P and B multiplied by $\frac{1}{2}$ results in the distance between P and B'. Notice also how the side lengths of the dilated figure, $A'B'C'D'$, are all exactly $\frac{1}{2}$ the side lengths of the original figure, $ABCD$, while the angle measures remain the same.

Here is a task to try with your student:

Rectangle A measures 10 cm by 24 cm. Rectangle B is a scaled copy of Rectangle A.

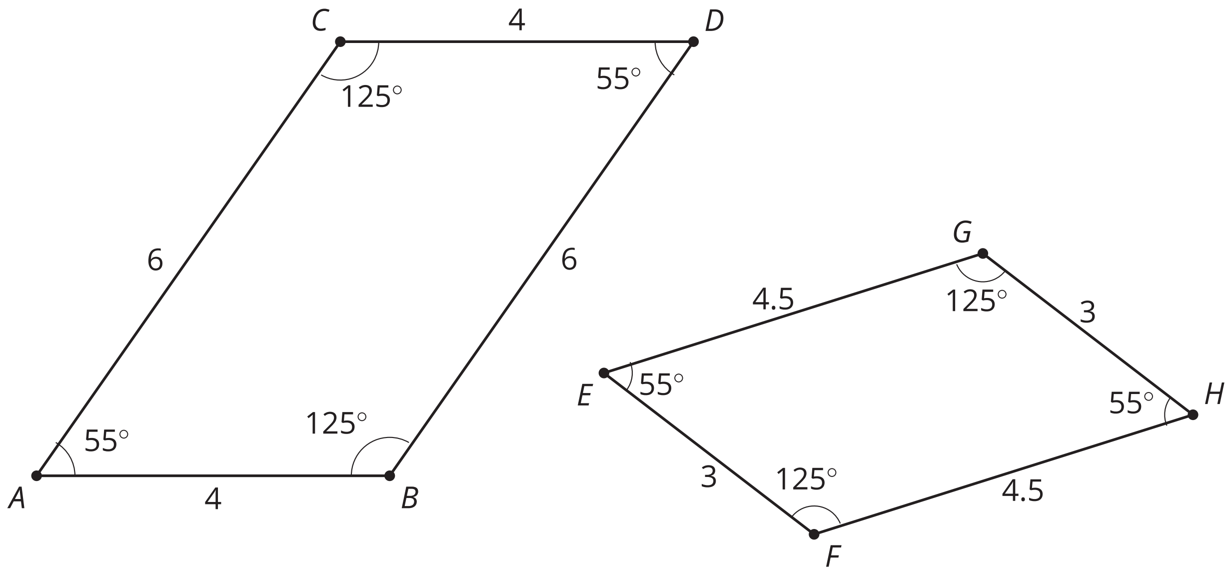
1. If the scale factor is $\frac{1}{2}$, what are the dimensions of Rectangle B?
2. If the scale factor is 3, what are the dimensions of Rectangle B?
3. If Rectangle B has dimensions 15 cm by 36 cm, what is the scale factor?

Solution:

1. Rectangle B has dimensions 5 cm by 12 cm, since $10 \cdot \frac{1}{2} = 5$ and $24 \cdot \frac{1}{2} = 12$.
2. Rectangle B has dimensions 30 cm by 72 cm, since $10 \cdot 3 = 30$ and $24 \cdot 3 = 72$.
3. The scale factor is $\frac{3}{2}$, since $15 \div 10 = \frac{3}{2}$ and $36 \div 24 = \frac{3}{2}$.

Section D: Similarity

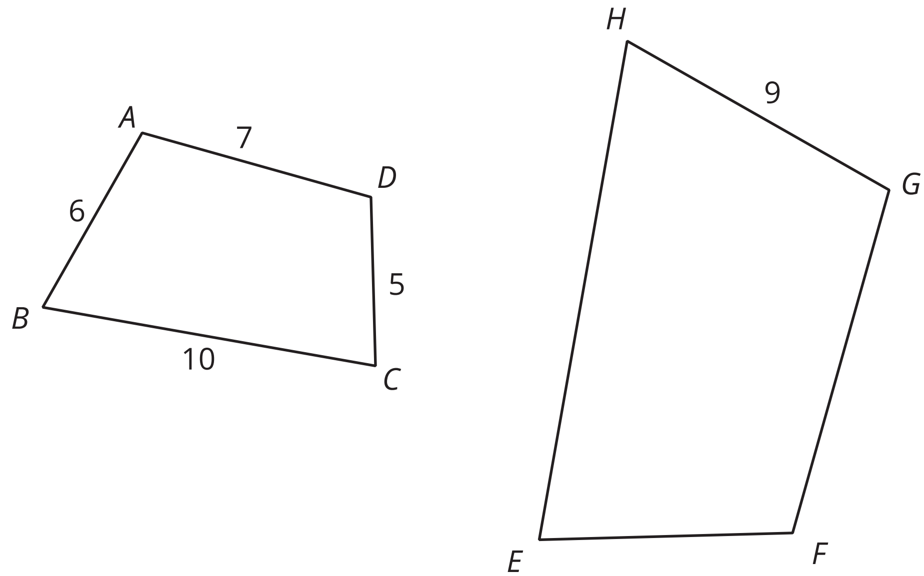
This week your student will investigate what it means for two figures to be **similar**. Similarity in mathematics means there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other. There are always many different sequences of transformations that can show that two figures are similar. Here is an example of two similar figures:



One way to show that these two figures are similar is to first identify the scale factor to go from $ABDC$ to $EFHG$, which is $\frac{3}{4}$, since $3 \div 4 = 4.5 \div 6 = \frac{3}{4}$. Then, using a dilation with scale factor $\frac{3}{4}$, a translation, and a rotation, we can line up an image of $ABDC$ perfectly on top of $EFHG$.

Here is a task to try with your student:

Quadrilateral $ABCD$ is similar to quadrilateral $GHEF$.



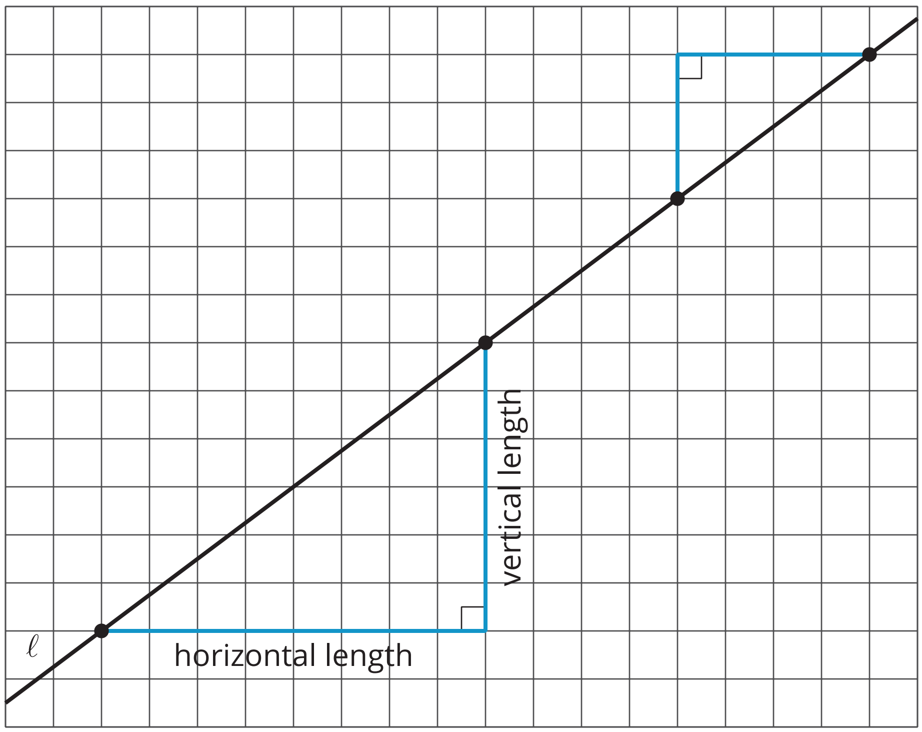
What is the perimeter of quadrilateral $GHEF$?

Solution:

The perimeter is 42. The scale factor is 1.5, since $9 \div 6 = 1.5$. This means the side lengths of $GHEF$ are 9, 15, 7.5, and 10.5, which are the values of the corresponding sides of $ABCD$ multiplied by 1.5. We could also just multiply the perimeter of $ABCD$, to get $28 \cdot 1.5 = 42$.

Section E: Slope

This week your student will use what they have learned about similar triangles to define the **slope** of a line. A slope triangle for a line is a triangle whose longest side lies on the line and whose other two sides are vertical and horizontal. Here are two slope triangles for the line ℓ:

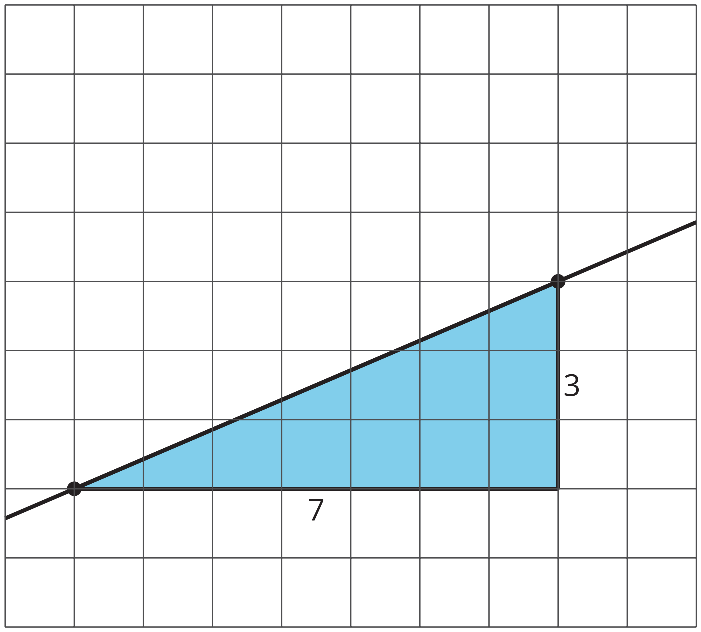


For any given line, it turns out that the quotient of the vertical side length and the horizontal side length of a slope triangle does not depend on the triangle. That is, all slope triangles for a line have the same quotient between their vertical and horizontal side, and this number is called the slope of the line. The slope of line ℓ shown here can be written as $\frac{6}{8}$ (from the larger triangle), $\frac{3}{4}$ (from the smaller triangle), 0.75, or any other equivalent value.

By combining what they know about the slope of a line and similar triangles, students will begin writing equations of lines—a skill they will continue to use and refine throughout the rest of the year.

Here is a task to try with your student:

Here is a line with a slope triangle already drawn in.



1. What is the slope of the line?
2. Draw another line on the grid with a slope of $\frac{4}{3}$. Include a slope triangle for the new line to show how you know this line has a slope of $\frac{4}{3}$.

Solution:

1. The slope of the line is $\frac{3}{7}$.
2. Sample response: