Unit 2, Lesson 2

Circular Grid

Goals

- Create dilations of polygons using a circular grid given a scale factor and center of dilation.
- Explain (orally) how a dilation affects the size, side lengths and angles of polygons.

Learning Targets

- I can apply dilations to figures on a circular grid when the center of dilation is the center of the grid.

Lesson Narrative

This lesson formally defines the term **dilation** as a transformation in which each point on a figure moves along a line and changes its distance from a fixed point, called the center of dilation. The scale factor determines how far each point moves.

First, students are introduced to the circular grid as an effective tool for performing a dilation. By using the structure of the grid in the next activities, they find that each grid circle maps to a grid circle, line segments map to line segments, and the image of a polygon is a scaled copy of the polygon (MP7). In addition, students determine scale factors that are used and explain their reasoning (MP3).

Standards

Building On **3.G.A, 4.MD.C.5**

Addressing **8.G.A**

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Materials to Gather

- Straightedges: Activity 2, Activity 3, Activity 4

Required Preparation

Activity 2:

Provide access to straightedges. For the digital version of the activity, acquire devices that can run the applet.

Activity 3:

Provide access to straightedges. For the digital version of the activity, acquire devices that can run the applet.

Activity 4:

Provide access to straightedges. For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's dilate figures on circular grids.

2.1 Notice and Wonder: Concentric Circles

5 min

Warm-up

Activity Narrative

This *Warm-up* introduces the circular grid, which students will use in a later activity. While students may notice and wonder many things about this image, the fact that the circles in the grid all have the same center and that the distance between consecutive circles is the same are the important discussion points. This prompt gives students opportunities to see and make use of this specific structure (MP7).

Standards

Building On **3.G.A, 4.MD.C.5**

Instructional Routines

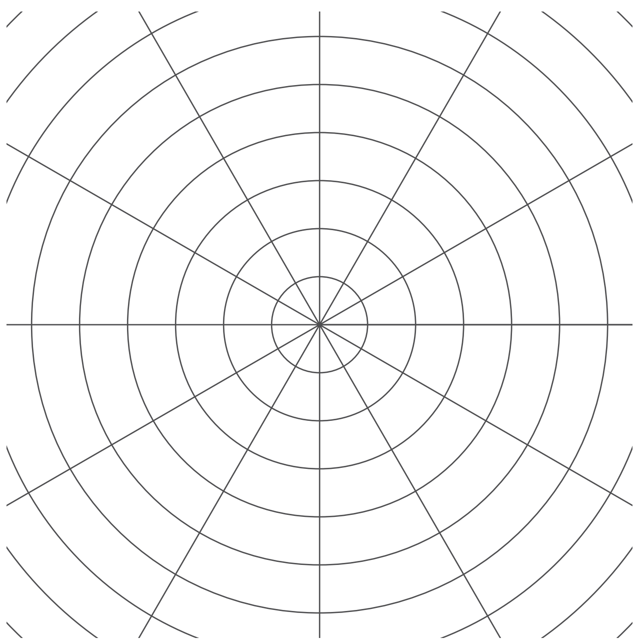
- Notice and Wonder

Launch

Arrange students in groups of 2. Display the image for all to see. Give students 1 minute of quiet think time and ask them to be prepared to share at least 1 thing they notice and 1 thing they wonder. Give students another minute to discuss their observations and questions.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- The circles share the same center.
- The center of the circles is the point where the lines meet.
- The distance from one circle to the next is always the same.

Things students may wonder:

- When is this grid useful?
- Why are the circles equally spaced?
- Why are there 6 lines meeting in the center?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary. If possible, record the relevant reasoning on or near the image. Next, ask students, “Is there anything on this list that you are wondering about now?” Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.

2.2 A Droplet on the Surface

15 min

Activity Narrative

There is a digital version of this activity.

The term "dilation" is formally defined in this activity. Students measure the distance between the center of a circular grid and corresponding points on two circles. They observe that the distances to the smaller circle are all multiplied by the same scale factor to get the distance to the larger circle. Students need to explain their reasoning when finding the scale factor (MP3).

In the digital version of the activity, students use an applet to find the scale factor between a smaller circle and a larger circle. The applet allows students to practice using the digital tools to visualize both circles and the scale factor. The digital version may be helpful as a tutorial for the digital tools. If students don't have individual access, displaying the applet for all to see would be helpful in the *Launch*.

Standards

Addressing **8.G.A**

Instructional Routines

- MLR8: Discussion Supports

Launch

Ask students if they have ever seen a pebble dropped in a still pond, and discuss how the pebble becomes the center of a sequence of circular ripples.

Then, display the image from the task statement, and ask students to think about how it is like a pebble dropped in a still pond. Demonstrate how distance on the circular grid is measured by counting units along one of the rays that start at the center, P. Remind students that a ray starts at a point and goes forever in one direction, and so any rays they draw should start at point P and be drawn to the edge of the grid using a straightedge.

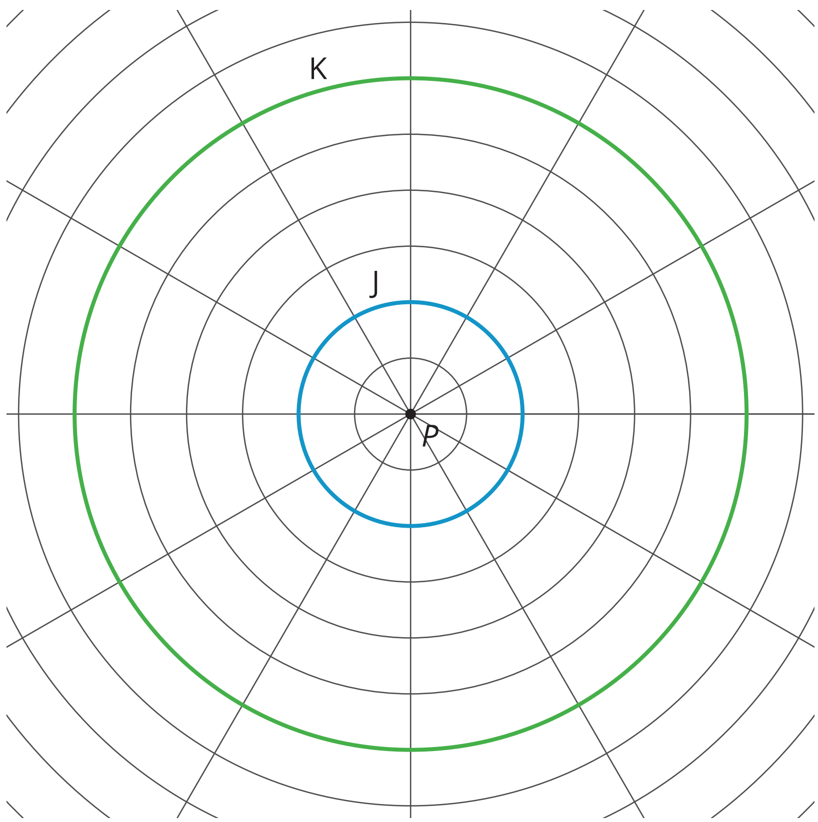
If necessary, remind students that when we say “on the circle,” we mean “on the curve” or “on the edge,” not “on the circle’s interior.”

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of dilations. Terms may include "circular grid," "ray," "scale factor," and "distance." Include a circular grid on the display, give examples, and label the features.
Supports accessibility for: Conceptual Processing, Language

Student Task Statement

Here are two circles drawn on a circular grid with point P at the center.



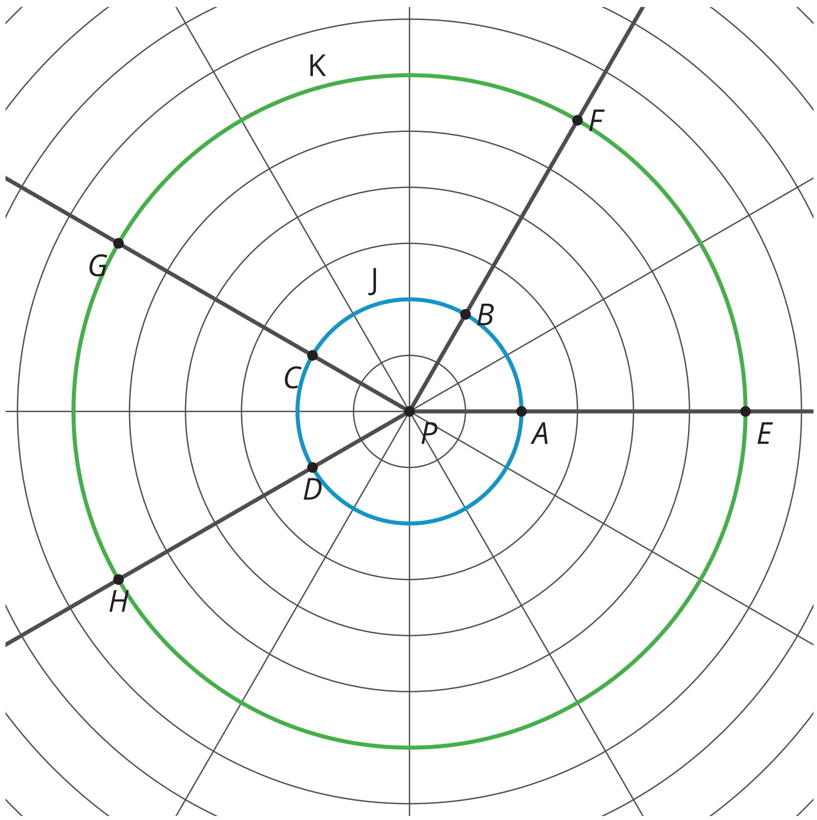
1. Draw four points on Circle J (not inside the circle), and label them A, B, C, and D.
2. Draw a ray from P through each of your four points.
3. Mark the points where the rays intersect Circle K, and label them as E, F, G, and H.
4. In the first table, write the distance between point P and each point on the smaller circle. In the second table, write the distance between point P and each point on the larger circle.

	A	*B*	*C*	*D*
P				

	E	*F*	*G*	*H*
P				

5. What is the scale factor that takes smaller Circle J to larger Circle K? Explain your reasoning.

Student Response



1. See image.
2. See image.
3. See image.
4.

	A	*B*	*C*	*D*
P	2	2	2	2

	E	*F*	*G*	*H*
P	6	6	6	6

5. Scale factor is 3. Sample reasoning: The distance from point P to the all of the points on the smaller circle is multiplied by 3 to get the distance from point P to all of the points on the the larger circle.

Building on Student Thinking

For the last question, some students may think the scale factor is 4, because the distance between the smaller and larger circle for each point increases by 4. If this happens, ask students how many grid units Circle J is from the center (2) and how many grid units Circle K is from the center (6). Remind them that scale factor means a number you multiply by.

Activity Synthesis

The purpose of this discussion is to define dilation. Begin by asking students to share their answers to the last question and what they noticed about the values in the table. Explain to students that this is an example of a dilation and share the following definition:

A **dilation** is a transformation in which each point on a figure moves along a line and changes its distance from a fixed point, called the center of dilation. The scale factor determines how far each point moves.

In this activity, the larger circle (K) is a dilation of the smaller circle (J). Point P is the center of dilation, and the scale factor is 3 because every point on Circle K is 3 times as far from point P than every point on Circle J.

Here are some questions for discussion:

- "What would happen if Circle J were dilated with point P as the center of dilation and a scale factor of 2?" (The result would be a circle with twice the radius and the same center.)
- "What would happen if Circle J were dilated with point P as the center of dilation and a scale factor of $\frac{1}{2}$?" (The result would be a circle with $\frac{1}{2}$ the radius and the same center.)
- "What would happen if Circle J were dilated about point P with a scale factor of 1?" (The result would be a circle with the same radius and the same center.)

Access for English Language Learners

MLR8 Discussion Supports. Invite students to repeat their reasoning for why the scale factor that takes the smaller circle to the larger circle is 3 using mathematical language: "Can you say that again, using the word 'distance' or 'radius' and the phrase 'center of dilation'?"
Advances: Speaking, Representing

2.3 Quadrilateral on a Circular Grid

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students continue studying dilations on a circular grid by focusing on what happens to points on a polygon. They discover that just as the image of a grid circle is another circle, the dilation of a polygon is another polygon, and the dilated polygon is a scaled copy of the original (MP7). These important properties of dilations are not apparent in the definition.

In the digital version of the activity, students use an applet to visualize the dilation of a polygon. The applet allows students to dilate each vertex by the provided scale factor on a circular grid.

Standards

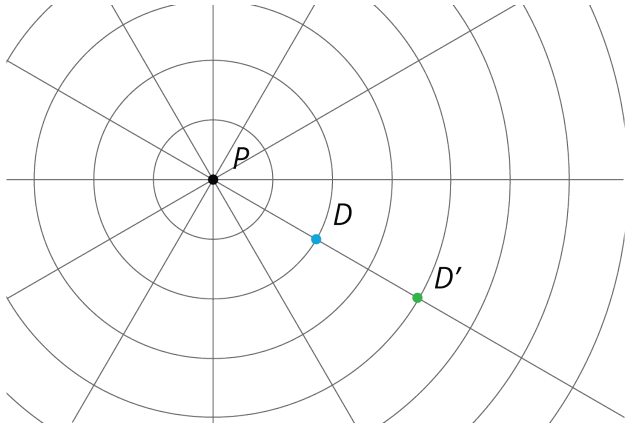
Addressing **8.G.A**

Instructional Routines

- MLR2: Collect and Display

Launch

Display this image for all to see.



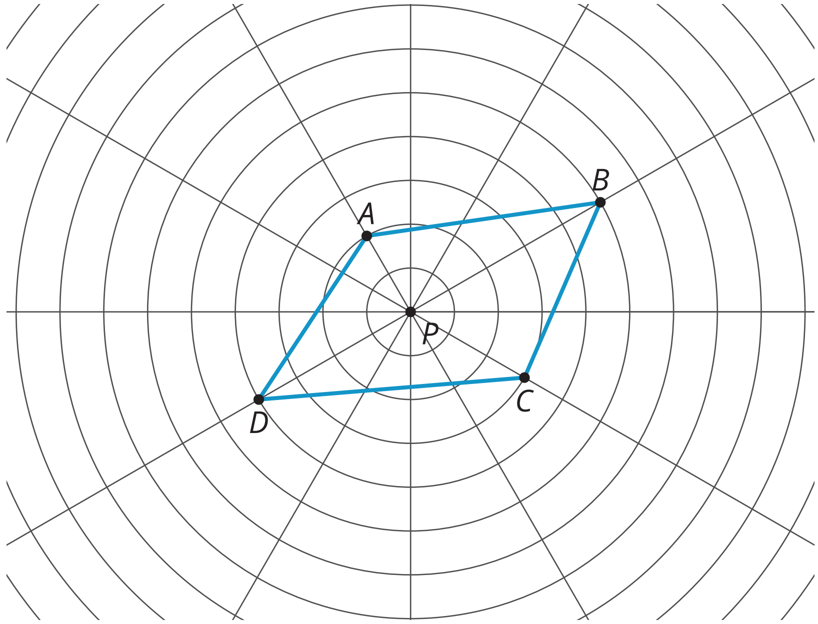
Using a straightedge, demonstrate how point D was dilated using point P as the center of dilation and a scale factor of 2. Explain that the distance from P to D is 2 units and that the new point, labeled D', is on the ray PD, twice as far away from P.

Access for English Language Learners

MLR2 Collect and Display. Direct attention to words collected and displayed about scale factors from a previous lesson. Invite students to borrow language from the display as needed, and update it throughout the lesson. Circulate, listen for, and collect the language students use as they make observations about the polygon with a scale factor of 2 and the polygon with a scale factor of $\frac{1}{2}$. Record words and phrases students use to compare features of the new polygons to the original polygon, such as "The new polygon is the same as the original polygon but bigger (or smaller)." A phrase such as "the polygons have the same angles" can be clarified with the phrase "each angle in the original polygon is the same as the corresponding angle in the new polygon."
Advances: Conversing, Reading

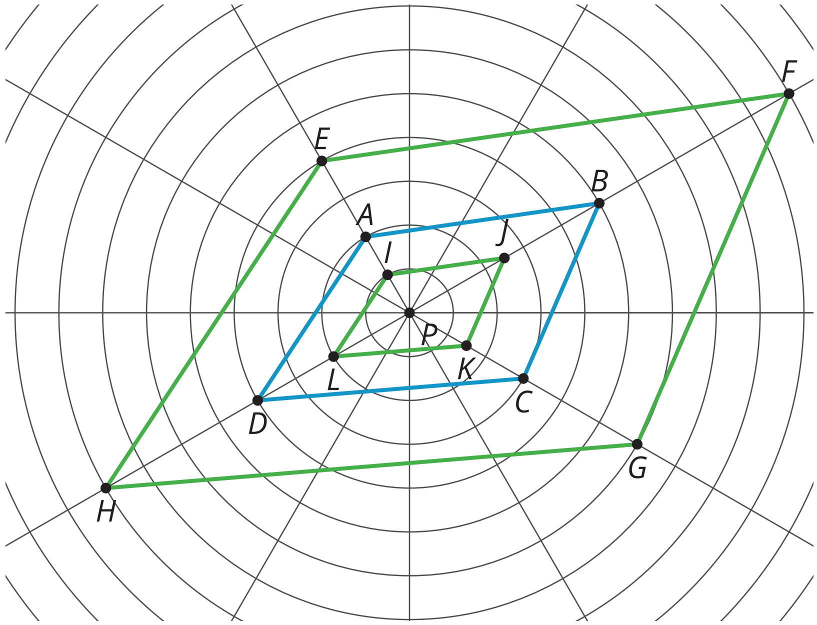
Student Task Statement

Here is a polygon $ABCD$.



1. **Dilate** each vertex of polygon $ABCD$ using P as the center of dilation and a scale factor of 2. Label the image of A as E, the image of B as F, the image of C as G, and the image of D as H. Draw segments between the dilated points to create polygon $EFGH$.
2. What are some things you notice about the new polygon?
3. Choose a few more points on the sides of the original polygon and transform them using the same dilation. What do you notice?
4. Dilate each vertex of polygon $ABCD$ using P as the center of dilation and a scale factor of $\frac{1}{2}$. Label the image of A as I, the image of B as J, the image of C as K, and the image of D as L. Draw segments between the dilated points to create polygon $IJKL$.
5. What do you notice about polygon $IJKL$?

Student Response



1. See image.
2. Sample responses: The new polygon is a scaled copy of the original polygon. Each side of the new polygon is parallel to the corresponding side on the original polygon. Each side of the new polygon is twice the length of the corresponding side in the original polygon.
3. Sample response: The dilation of any point on polygon $ABCD$ results in a point on polygon $EFGH$.
4. See image.
5. Sample responses: The new polygon is a scaled copy of the original polygon. Each side of the new polygon is parallel to the corresponding side on the original polygon. Each side of the new polygon is half the length of the corresponding side in the original polygon.

Are You Ready for More?

Suppose P is a point that is not on line segment WX. Let line segment YZ be the dilation of line segment WX using P as the center with a scale factor of 2. Experiment using a circular grid to make predictions about whether each of the following statements is always true, sometimes true, or never true.

1. Line segment YZ is twice as long as line segment WX.
2. Line segment YZ is 5 units longer than line segment WX.
3. The point P is on line segment YZ.
4. Line segments YZ and WX intersect.

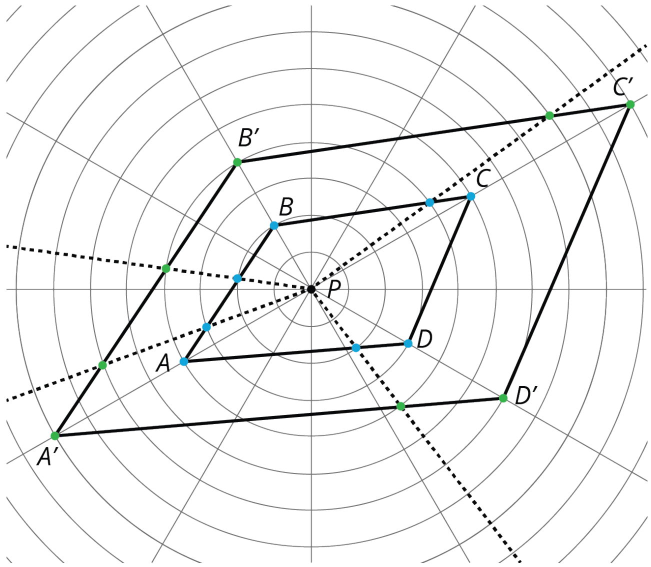
Extension Student Response

1. Always true
2. Sometimes true (if line segment WX has length 5)
3. Never true

4. Sometimes true (if line segment WX and line segment YZ are both on the same line)

Activity Synthesis

The goal of this discussion is to make observations about a figure and its image under dilation. Begin by displaying this figure.



Ask students to share what they notice about the new polygon. Encourage students to verify their responses using geometry tools like tracing paper, a ruler, or a protractor. Here are some questions for discussion:

- “What do you notice about corresponding sides, such as segment BC and segment $B'C'$?” (They look parallel. The side of the new polygon is twice the length of the original.)
- “How can we verify this?” (We can use a ruler to measure each side and the sides of the new figure should be twice the length of the original figure.)
- “Is our new polygon a scaled copy of the original? (Yes, because all the sides of the new polygon are twice the length of the original polygon.)
- “What do you notice about corresponding angles?” (They look the same.)
- “How can we verify this?” (We can measure them with a protractor.)
- “What happened when we dilated additional points that were on the sides of the original polygon?” (They landed on the side of the dilated polygon.)

2.4 A Quadrilateral and Concentric Circles

Optional

10 min

Activity Narrative

There is a digital version of this activity.

The purpose of this optional activity is to work on dilations of polygons on a circular grid but without the radial lines from the center. Students will need to use a ruler or other straightedge to connect each point to the center of the circular grid. If time allows, they can experiment with dilating points other than the vertices and check that the dilation of a side of the polygon is still a line segment (though there may be small deviations due to measurement error).

In the digital version of the activity, students use an applet to visualize the dilation of a polygon. The applet allows students to draw rays and dilate each vertex by the provided scale factor on a circular grid.

Standards

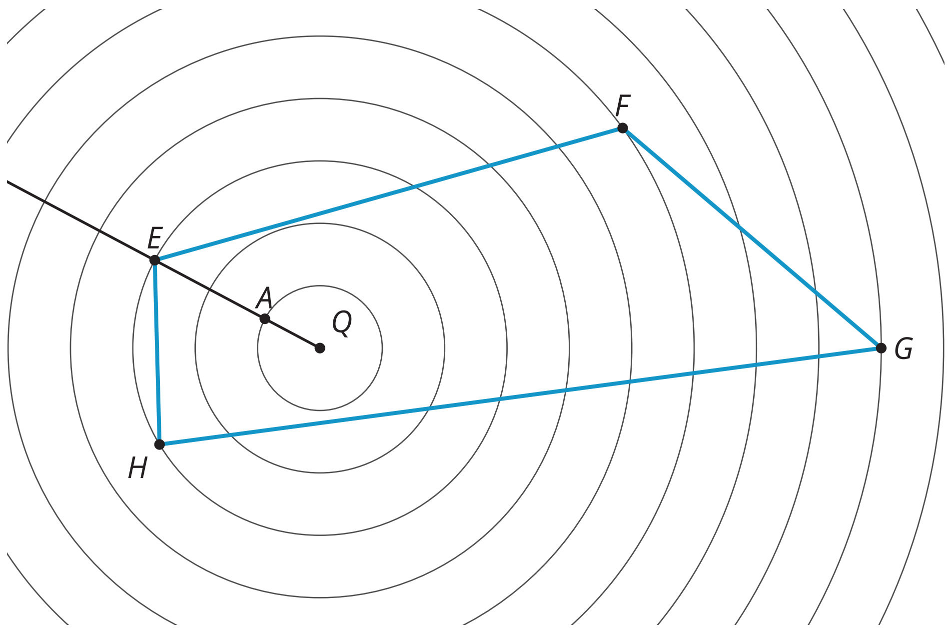
Addressing **8.G.A**

Launch

Display the image and problem stem for all to see. Tell students to quietly read the problem and ask how this problem is alike and different from the previous ones. (It is alike because it shows a quadrilateral and concentric circles, and we are asked to dilate the quadrilateral using the center of the circles as the center of dilation. It is different because there is only one radial line through the center, because the scale factor is now $\frac{1}{3}$, and because one of the points is already dilated.)

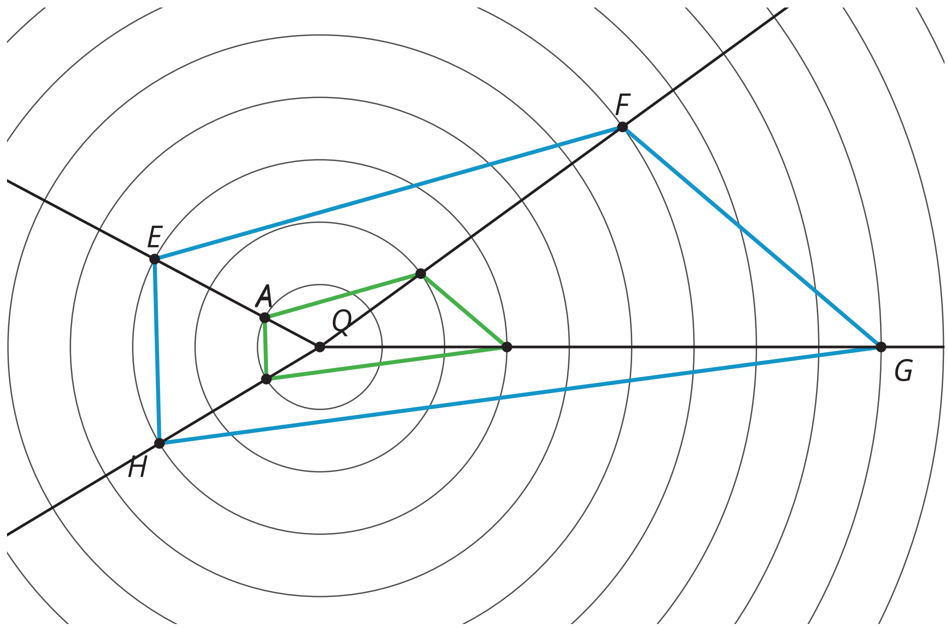
Ask students how the location of point A was determined, and then to dilate the remaining points. Use a ruler or straightedge when modeling student responses.

Student Task Statement



Dilate polygon $EFGH$ using Q as the center of dilation and a scale factor of $\frac{1}{3}$. A, the image of E, is already shown on the diagram. (You may need to use a straightedge to draw more rays from Q in order to find the images of other points.)

Student Response



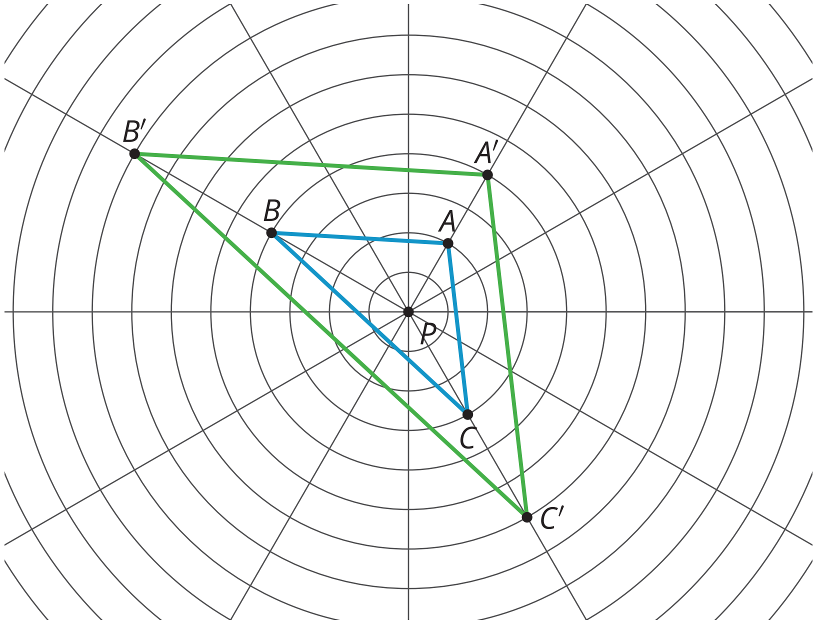
Activity Synthesis

The purpose of this discussion is to highlight the need to draw in the rays going through point Q and each of the vertices of the polygon, and the fact that a scale factor of $\frac{1}{3}$ resulted in an image that was smaller than the original figure. Here are some questions for discussion:

- "What effect did a scale factor of $\frac{1}{3}$ have on the dilation of polygon $EFGH$?" (The resulting figure is smaller than the original and located closer to the center of dilation.)
- "What scale factor would result in an image that was the same size and the same distance from the center of dilation?" (a scale factor of 1)
- "How would you dilate a polygon if you knew the center of dilation but did not have the circles or radial lines printed for you?" (Draw rays starting at the center of dilation and going through each vertex of the polygon, and then measure the appropriate distances along each ray.)

Lesson Synthesis

The goal of this discussion is to make sure students understand how a dilation affects the size, side lengths, and angles of polygons. First ask students to recall the steps for dilating a polygon on the circular grid. (Dilate each vertex by the scale factor and then connect the dilated vertices to create the new side lengths.) Then display the image for all to see.



Here are some questions for discussion:

- “What is the center of dilation that takes triangle ABC to triangle $A'B'C'$?” (point P)
- “What is the scale factor and how do you know?” (The scale factor is 2 because each point of the dilated polygon is twice as far from the center as the corresponding point in the original polygon.)
- “What are some things we know must be true for sides AB and $A'B'$”? (Side $A'B'$ is twice as long as side AB and side $A'B'$ is parallel to side AB.)
- What do we know about corresponding angles in the two triangles?” (Corresponding angles have the same measure.)
- “How is the circular grid useful when performing dilations?” (If the center of dilation is the center of the grid, the circular grid makes it easy to measure distances.)

2.5 Dilating Points on a Circular Grid

Cool-down

5 min

Standards

Addressing **8.G.A**

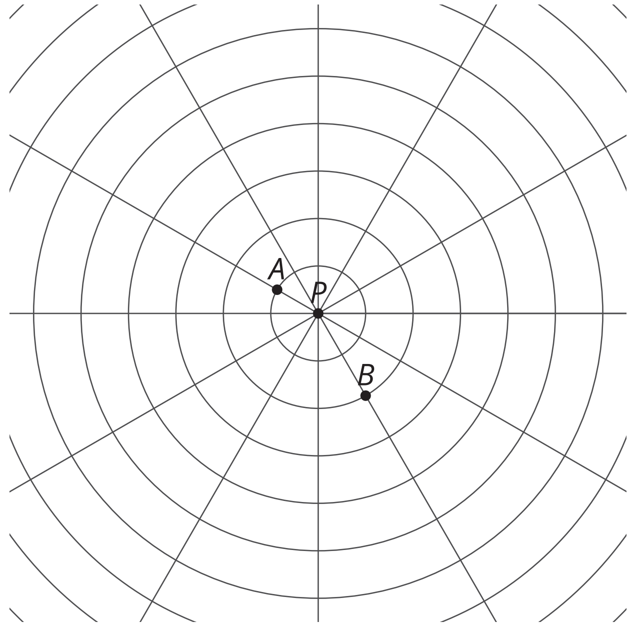
Student Task Statement

1. Dilate A using P as the center of dilation and a scale factor of 3.

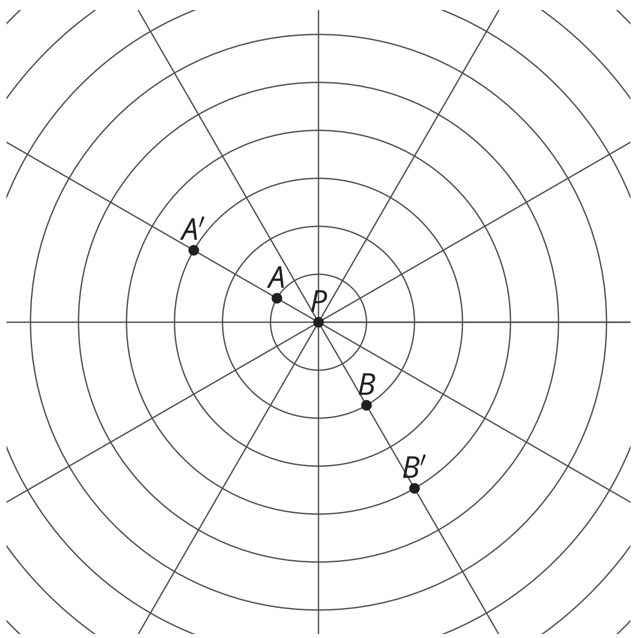
   Label the new point A'.

2. Dilate B using P as the center of dilation and a scale factor of 2.

   Label the new point B'.



Student Response



Responding to Student Thinking

Points to Emphasize

If students struggle with dilating points on a circular grid, plan to revisit dilations of a point when opportunities arise over the next several lessons. For example, in the Activity Synthesis of the activity referred to here, demonstrate the steps for dilating a point not on a grid and emphasize how the scale factor and center of dilation affect the resulting image.

Lesson 2 Summary

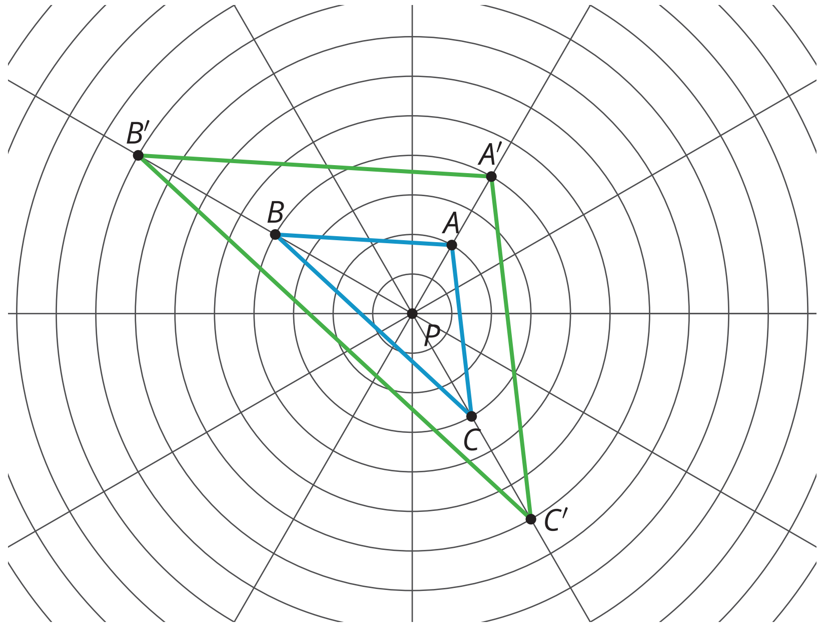
A **dilation** is a transformation in which each point on a figure moves along a line and changes its distance from a fixed point, called the center of dilation.

All of the original distances are multiplied by the same scale factor.

In this diagram, P is the center of dilation and the scale factor is 2.

Each point of triangle ABC stays on the same ray from P, but its distance from P doubles.

Since the circles on a circular grid are the same distance apart, we can simply count units from the center to a given point and use the scale factor to determine where the new point should be located, making the circular grid useful for performing dilations.



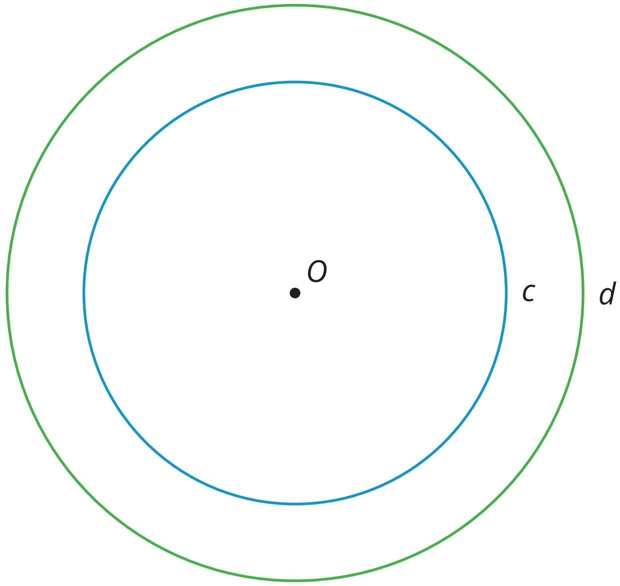
Glossary

- dilation

Lesson 2 Practice Problems

1 Student Task Statement

Here are Circles c and d. Point O is the center of dilation, and the dilation takes Circle c to Circle d.



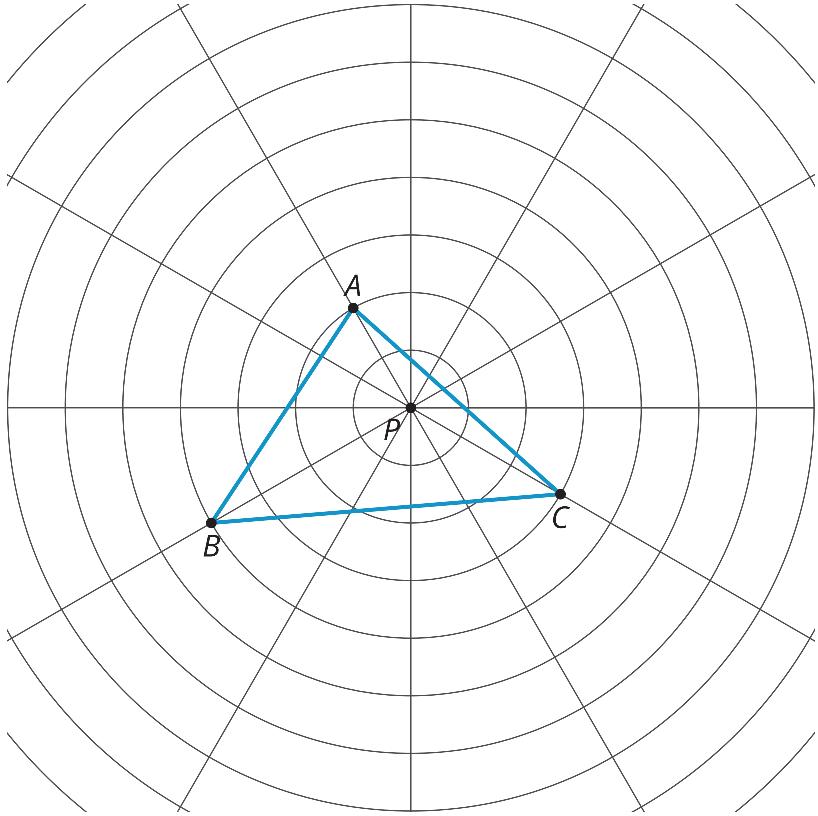
a. Plot a point on Circle c. Label the point P. Plot where P goes when the dilation is applied and label the point P'.
b. Plot a point on Circle d. Label the point Q'. Plot the point that the dilation takes to Q' and label it Q.

Solution

a. If a ray is drawn from O through P, the point P' is located where this ray intersects circle d.
b. If a ray is drawn from O through Q', the point Q is located where this ray intersects circle c.

2 Student Task Statement

Here is triangle ABC.



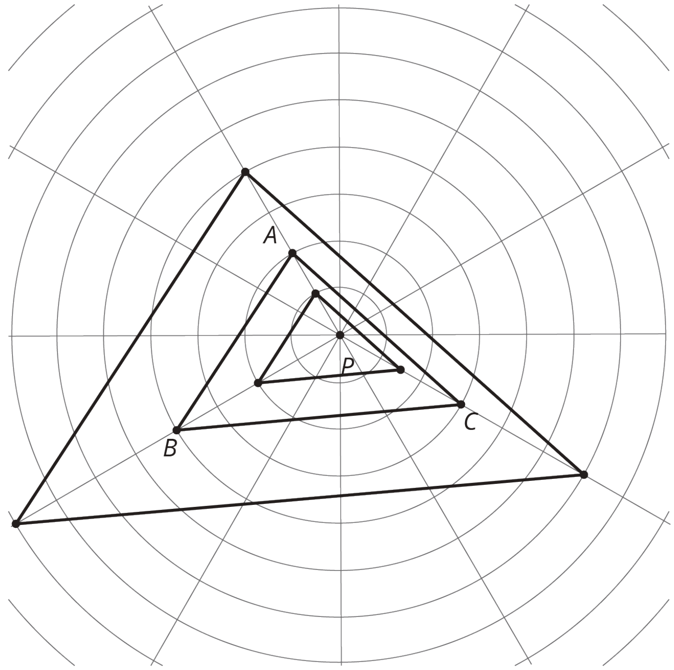
a. Dilate each vertex of triangle ABC using P as the center of dilation and a scale factor of 2. Draw the triangle connecting the 3 new points.

b. Dilate each vertex of triangle ABC using P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the triangle connecting the 3 new points.

c. Measure the longest side of each of the 3 triangles. What do you notice?

d. Measure the angles of each triangle. What do you notice?

Solution



a. See image

b. See image

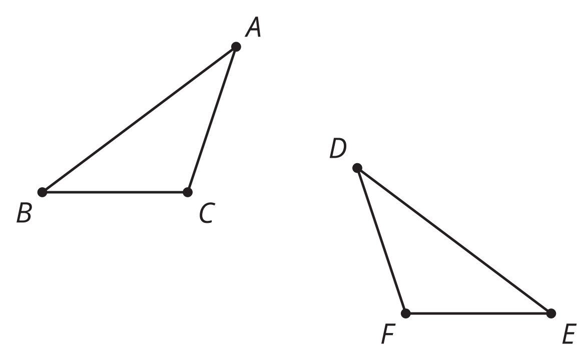
c. Sample response: The longest side of the largest triangle is twice as long as the longest side of triangle ABC, which is twice as long as the smallest triangle.

d. Sample response: The corresponding angles in all three triangles have the same measures.

3 from Unit 1, Lesson 12

Student Task Statement

Describe a sequence of translations, rotations, or reflections that show the triangles are congruent.



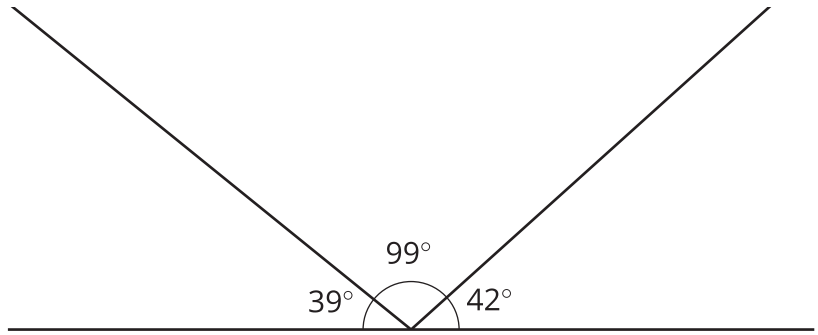
Solution

Sample response: Reflect triangle ABC across a vertical line and translate so that A goes to D.

4 from Unit 1, Lesson 15

Student Task Statement

The line has been partitioned into 3 angles.



Is there a triangle with these 3 angle measures? Explain.

Solution

Yes. Sample reasoning: The 3 angle measures add up to 180 degrees and since the sum of the interior angles of a triangle is always 180 degrees, a triangle can be made with these angle measures.

