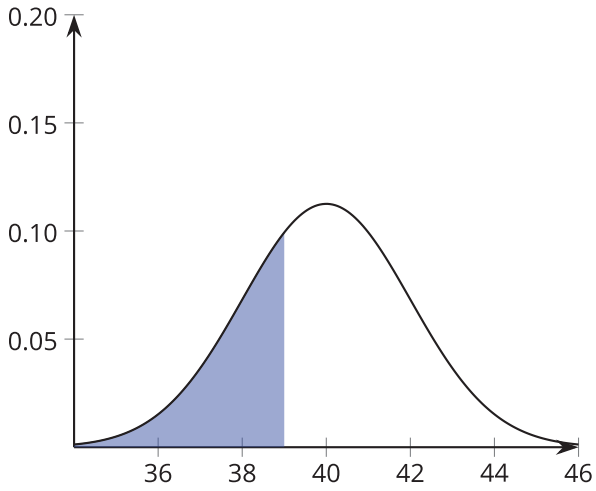
Areas under a Normal Curve

Let's use the normal distribution to estimate the proportion of data values falling within given intervals.

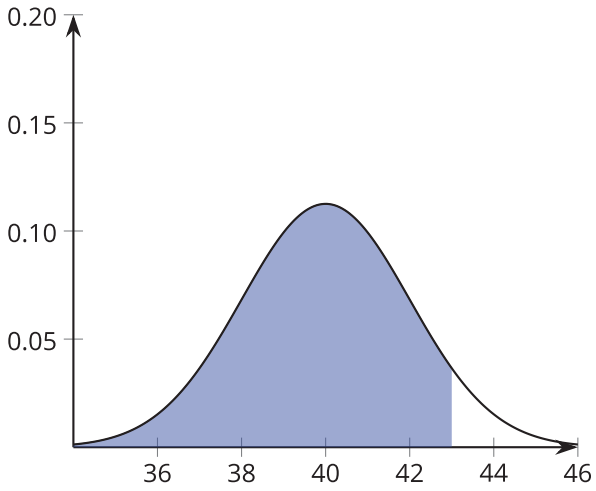
7.1 Find the Areas

The images show a normal curve with mean of 40 and standard deviation of 2.

The area under the curve to the left of 39 is 0.3085.



The area under the curve to the left of 43 is 0.9332.



Since it is a normal curve, we know that the total area under the curve is 1. Use the given areas to find the areas in question. Explain your reasoning for each.

1. Find the area under the curve to the right of 39.

2. Find the area under the curve between 39 and 43.

3. Find the area under the curve to the left of 40.

4. Find the area under the curve between 39 and 40.

7.2 Life of Lights

The lifespan of light bulbs is approximately normally distributed. Some statistics about lifespans of two different types of light bulbs are listed.

- LED bulbs: mean: 2,300 days, standard deviation: 230 days
- Incandescent bulbs: mean: 100 days, standard deviation: 10 days

To estimate the proportion of bulbs that burn out in a certain interval of time, use technology to find the area under the normal curve and above the appropriate interval.

1. Estimate the proportion of LED bulbs that are expected to burn out before getting within 1 standard deviation of the mean (before 2,070 days).
2. Estimate the proportion of incandescent bulbs that are expected to burn out before getting within 1 standard deviation of the mean (before 90 days).
3. Estimate the proportion of LED bulbs that are expected to burn out after getting more than 1 standard deviation greater than the mean (after 2,530 days).
4. Estimate the proportion of incandescent bulbs that are expected to burn out after getting more than 1 standard deviation greater than the mean (after 110 days).
5. Estimate the proportion of LED bulbs that are expected to burn out in the interval between 1 standard deviation less than the mean and 1 standard deviation greater than the mean (between 2,070 and 2,530 days).
6. Estimate the proportion of incandescent bulbs expected to burn out in the interval between 1 standard deviation less than the mean and 1 standard deviation greater than the mean (between 90 and 110 days).
7. Estimate the proportion of LED bulbs that are expected to burn out in the interval between 2 standard deviations less than the mean and 2 standard deviations greater than the mean (between 1,840 and 2,760 days).
8. Estimate the proportion of LED bulbs that are expected to burn out in the interval between 1,900 days and 2,100 days.
9. Estimate the proportion of incandescent bulbs that are expected to burn out in the interval between 107 and 118 days.

7.3 Waiting for a Waiter

The wait times at a popular restaurant are approximately normally distributed. The mean wait time is 24.3 minutes with a standard deviation of 3.2 minutes.

Use technology to estimate the wait times for the described groups of customers.

1. Describe the number of minutes customers have to wait if their wait times are in the longest 10% of wait times for customers at this restaurant.

2. Describe the number of minutes customers have to wait if their wait times are in the shortest 15% of wait times for customers at this restaurant.

3. To find the wait times for the middle 50% of wait times for customers:
   a. Draw an example of a normal distribution, and shade approximately the middle 50% of the area under the curve.

   b. What percentage of the total area is the unshaded region to the left of the region you shaded? What value marks the line between the unshaded and shaded parts?

   c. What percentage of the total area is the unshaded region to the right of the region you shaded? What value marks the line between the unshaded and shaded parts?

   d. The shaded region is between which two values?

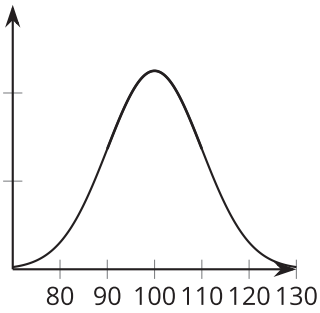
4. The customers who have wait times in the middle 70% are between which two values?

Are you ready for more?

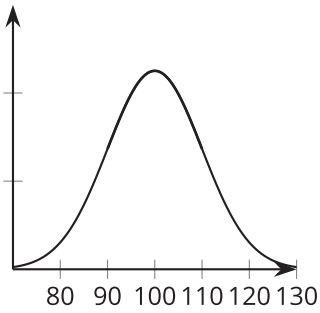
A normal curve has a mean of 100 and a standard deviation of 10. For each value given, find two different regions that have approximately the given area, and shade them in the graphs provided.

1. 0.68
2. 0.16
3. 0.10

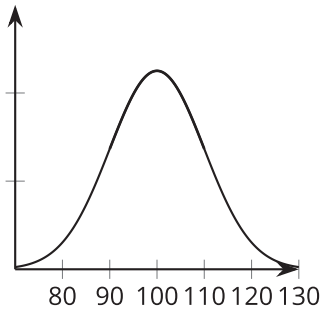
Area: 0.68



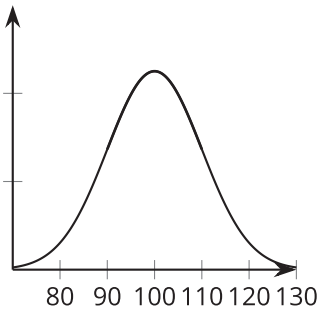
Area: 0.16



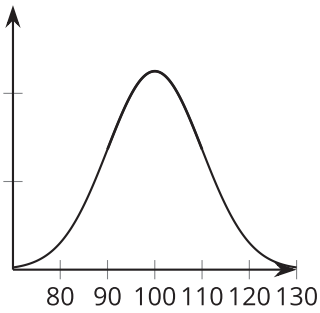
Area: 0.10



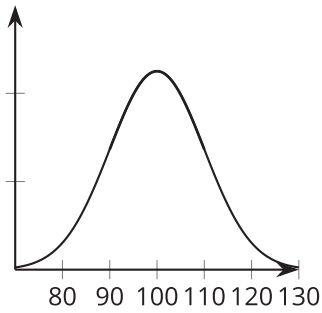
Area: 0.68



Area: 0.16



Area: 0.10

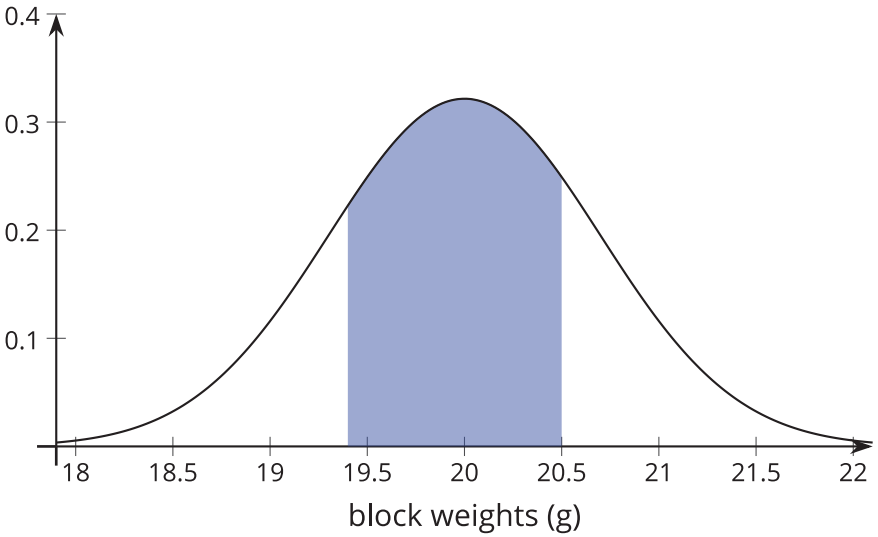


Lesson 7 Summary

The normal distribution can be used to estimate the proportion of values expected in a certain interval by finding the area under the normal curve and between the bounds that define the interval. Since the total area under a normal curve is 1, the area within any particular interval can be interpreted as the proportion of values that are in that interval.

To find the area of a region, technology or reference tables can be used. When using technology, the system will need to know the mean and standard deviation for the data as well as the boundary values for the region.

For example, the weights of large plastic building blocks are approximately normally distributed. The mean weight is 20 grams, and the standard deviation is 0.7 gram. Let's estimate the proportion of all plastic building blocks that weigh between 19.4 and 20.5 grams. This proportion is represented by the shaded area in the figure.



By adding the mean, standard deviation, and bounds defining the region to a technological tool, we find that the proportion of values in the region is 0.5668, or 56.68%. This also means that, when selecting a building block at random, the probability of selecting a block with a weight between 19.4 and 20.5 grams is approximately 0.5668.