

Adding and Subtracting Decimals with Many Non-Zero Digits

Goals

- Choose a strategy for adding and subtracting decimals efficiently, especially decimals with multiple non-zero digits.
- Interpret a description (in written language) of a real-world situation involving decimals, and write an addition or subtraction problem to represent it.
- Use the standard algorithm to add or subtract decimals with multiple non-zero digits, and explain (orally) the solution method.

Learning Targets

- I can solve problems that involve addition and subtraction of decimals.

Lesson Narrative

This lesson strengthens students' ability to add and subtract decimals, enabling them to work toward fluency. Students encounter longer decimals (beyond thousandths), find unknown addends, and work with decimals in the context of situations. They decide which operation (addition or subtraction) to perform and which strategy to use when finding sums and differences.

The lesson also reinforces the idea that we can express a decimal in different but equivalent ways, and that writing additional zeros after the last digit to the right of the decimal point in a number does not change its value. Students use this understanding to practice subtracting numbers with more decimal places from those with fewer decimal places (such as, $1.9 - 0.4563$).

To solve these problems, students must lean heavily on their understanding of base-ten numbers and make use of structure (MP7). Given problems such as $7 - ? = 3.4567$ and $0.404 + ? = 1$, they need to think carefully about the meaning of each place value, the meaning of addition and subtraction, and potential paths toward the solution. In thinking about what symbols represent and how to represent base-ten values, students reason abstractly and quantitatively (MP2).

Standards

Addressing 6.NS.B.3

Instructional Routines

- 5 Practices
- MLR8: Discussion Supports
- Notice and Wonder


Required Materials

Materials to Gather

- Graph paper: Activity 2



Student Facing Learning Goals

 Let's practice adding and subtracting decimals.

15.1

The Cost of a Photo Print

Warm-up

 10 min

Activity Narrative

This *Warm-up* prompts students to review the alignment of the digits when using a standard algorithm to subtract two numbers in base-ten. The *Notice and Wonder* routine in the launch gives students a chance to think about whether or how different placements of the 5 (the first number) affects the subtraction. It also gives the teacher insight about how students interpret the 5 and its value. For instance:

- Would students place the decimal point directly after the 5?
- When placing 5 above 7 (in the tenths place), do they see it as 5 ones or as 5 tenths?
- Do they wonder if the 5 is missing a decimal point in the second and third case?

In the *Student Task Statement*, students see the same subtraction in the context of a situation, allowing them to see more clearly the equivalence of 5 and 5.00.

Standards

Addressing 6.NS.B.3

Instructional Routines

- Notice and Wonder

Launch

Tell students to close their books or devices (or to keep them closed). Display the three ways of writing a subtraction calculation for all to see. Give students 1 minute of quiet think time, and ask them to be prepared to share at least one thing they notice and one thing they wonder about. Record and display responses without editing or commentary. If possible, record the relevant reasoning on or near the calculation setup referred to.

If the question of whether the placement of the 5 affects the result of subtraction does not come up during the conversation, ask students to discuss it.

Tell students to open their books or devices. Give students 2 minutes of quiet work time, and follow that with a whole-class discussion.

Student Task Statement

1. Clare bought a photo for 17 cents and paid with a \$5 bill. Which of these three ways of writing the numbers could Clare use to find the change she should receive? Be prepared to explain your reasoning.

$$\begin{array}{r} 5 \\ - 0.17 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ - 0.17 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \\ - 0.17 \\ \hline \end{array}$$



2. Find the amount of change that Clare should receive. Explain or show your reasoning.

Student Response

1. The first setup (in which 5 and 0 line up vertically) is most conducive to correct subtraction. Sample reasoning: The 5 means \$5.00, and it helps line up the dollars (the ones) and the cents (the tenths and hundredths) when subtracting.
2. \$4.83. Students may or may not use vertical calculation. Sample reasoning:
 - Adding 0.17 to 4.83 gives 5.
 - Subtracting \$0.10 (10 cents) from \$5 leaves \$4.90. Subtracting another \$0.07 (7 cents) leaves \$4.83.

Activity Synthesis

Invite students to share which way they think Clare could write the calculation to find the amount of change. Clarify that in this particular case, the first setup is the most conducive to correct computation, but there is not one correct answer. Students could find the answer with any one of the setups as long as they understand that the 5 represents \$5.00.

If any students choose the second or third setup because they can mentally subtract the values without lining them up by place values, invite them to share their reasoning. Ask if they would use the same strategy for dealing with longer decimals (such as $5.23 - 0.4879$) and, if not, ask them what approach might be more conducive to correct calculation in those cases.

Encourage more students to be involved in the conversation, by asking questions such as:

- “Do you agree or disagree? Why?”
- “Can anyone explain _____’s reasoning in their own words?”
- “What is important for us to think about when subtracting this way?”
- “How could we have solved this problem mentally?”

15.2

Decimals All Around

🕒 15 min

Activity Narrative

This activity has two parts and serves two purposes. The first set of problems aims to help students see the limits of using base-ten diagrams to add and subtract numbers and to think about choosing more-efficient methods. The latter two questions prompt students to reason, in the context of situations, about addition and subtraction of numbers with more decimal places. In all problems, the numbers have enough decimal places that using base-ten representations (physical blocks, drawings, or digital representations) would be cumbersome, making numerical ways of reasoning appealing.

When students work on the first set of problems, monitor for those who use different strategies to subtract. Here are some likely strategies, from more elaborate to more streamlined:

- Using base-ten blocks or diagrams to represent the first number in each expression and removing pieces to represent subtraction.
- Writing each number in expanded form (decomposing each number by place value), subtracting the values in each place separately, and then adding them back at the end. (See sample reasoning for $318.8 - 94.63$ in *Student Response*.)



- Using vertical calculation or standard algorithm for subtraction.
- Reasoning about equivalence and place value, such as thinking of 0.02 and 0.0116 as 200 ten-thousandths and 116 ten-thousandths, and then subtracting the whole numbers, $200 - 116$.

When working on the last two questions, students need to determine the appropriate operations for the given situations, perform the calculations, and then relate their answers back to the contexts. In doing so, they practice reasoning abstractly and quantitatively (MP2).

Standards

Addressing **6.NS.B.3**

Instructional Routines

- 5 Practices

Launch

Arrange students in groups of 2. Give students 3–5 minutes of quiet time to find the differences in the first question. Provide access to graph paper in case students wish to use it for aligning the digits when using vertical calculations.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student voices, especially from students who haven't shared recently.

Before students work on the last two questions, pause for a brief whole-class discussion. Invite previously selected students to share their answers and methods. Sequence the discussion of strategies in the order listed in the *Activity Narrative*. If possible, record and display the students' work for all to see.

Connect the different responses to the learning goals by asking questions such as:

- “Did you use a different strategy for each subtraction? If so, why? If not, how did you decide which strategy to use for both problems?”
- “What are some benefits of the strategy you chose? What is inconvenient about the strategy, if anything?”
- “What challenges did you face when using vertical calculations to find $0.02 - 0.0116$?” (The first number is two digits shorter than the second, which meant some extra steps.)

Give students 2–3 minutes to complete the remaining questions independently or with their partner.

Access for Students with Disabilities

- | *Representation: Internalize Comprehension.* Represent the same information in the word problems through different modalities, by creating diagrams or using physical objects.
- | *Supports accessibility for: Conceptual Processing, Visual-Spatial Processing*

Student Task Statement

1. Find the value of each expression. Show your reasoning.
 - a. $318.8 - 94.63$
 - b. $0.02 - 0.0116$
2. Lin's grandmother ordered a set of screws that were 0.3125 inches long to fix a shelf, but the store sent her screws that were 0.6875 inches long. How much longer were these screws than the ones she ordered? Show your reasoning.
3. There is 0.162 liter of water in a 1-liter bottle. How much more water should be put in the bottle so it contains exactly 1 liter? Show your reasoning.

Student Response

1. a. 224.17. Sample reasoning:

$$\begin{array}{r} 200 \quad 110 \quad 0.7 \quad 0.10 \\ \cancel{300} + \cancel{10} + 8 + \cancel{0.8} \\ - \quad \quad 90 + 4 + 0.6 + 0.03 \\ \hline 200 + 20 + 4 + 0.1 + 0.07 = 224.17 \end{array}$$

- b. 0.0084. Sample reasoning: 0.02 is 0.0200 or 200 ten-thousandths and 0.0116 is 116 ten-thousandths. Subtracting 116 from 200 gives 84, so the answer is 84 ten-thousandths.
2. 0.375 inches. Sample reasoning: $0.6875 - 0.3125 = 0.375$
3. 0.838. Sample reasoning: $0.838 + 0.162 = 1$



Are You Ready for More?



One micrometer is 1 millionth of a meter. A red blood cell is about 7.5 micrometers in diameter. A coarse grain of sand is about 70 micrometers in diameter. Find the difference between the two diameters in meters. Show your reasoning.

Extension Student Response

0.0000625 meters. Sample reasoning:

- The blood cell is 7.5 micrometers, and the grain of sand is 70 micrometers. The difference is 62.5 micrometers. Since 1 micrometer is 0.000001 meters, this is 0.0000625 meters.
- The bacteria is 0.0000075 meters, and the grain of sand is 0.00007 meters. The difference is 0.0000625 meters.

Activity Synthesis

Focus the discussion on how students interpreted the last two questions, decided on what operations to perform, and found the sum or differences (including how they handled any regroupings). Select a student to share the response and reasoning for each question. Discuss questions such as:

- “When solving the word problems, how did you know whether to use addition or subtraction?”
- “What method did you use to find the sums or differences? Why did you choose that method?”

If some students chose base-ten diagrams for the calculations and if time permits, consider contrasting its efficiency with that of numerical calculations.

15.3

Unknown Numbers

🕒 10 min

Activity Narrative

In this activity, students find differences of pairs of numbers with different numbers of decimal places, prompting them to look for and make use of structure (MP7). For instance, to find the difference of a whole number and a decimal, such as $1 - 0.256$, they may:



- Rely on what they know about finding a difference of two whole numbers, such as $1,000 - 256$.
- Find the difference of $0.999 - 0.256$, and then add 0.001 to the difference.
- Subtract by the value of one place at a time, so first find $1 - 0.2 = 0.8$, then $0.8 - 0.05 = 0.75$, and finally $0.75 - 0.06 = 0.744$.

The first two problems are presented as unknown-addend problems to activate what students know about the relationship between addition and subtraction to find differences (for instance, to think of $1 - 0.256$ in terms of $0.256 + ? = 1$).

Select students who reason in different ways, and invite them to share later.

Standards

Addressing **6.NS.B.3**

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Give students 4–5 minutes of quiet work time and 1–2 minutes to discuss their answers with a partner. If time is limited, consider asking students to choose one unknown-addend problem (one of the first two questions) and one unknown-subtrahend problem (one of the last three questions). Allow at least 2–3 minutes for a whole-class discussion.

Student Task Statement

Write the missing digits in each calculation so that the value of each sum or difference is correct. Be prepared to explain your reasoning.

1.

$$\begin{array}{r} 0.404 \\ + \square\square\square\square \\ \hline 1 \end{array}$$

2.

$$\begin{array}{r} 9.8765 \\ + \square\square\square\square\square\square \\ \hline 10 \end{array}$$

3.

$$\begin{array}{r} 0.7 \\ - \square\square\square\square \\ \hline 0.012 \end{array}$$

4.

$$\begin{array}{r} 7 \\ - \square\square\square\square\square\square \\ \hline 3.4567 \end{array}$$

5.

$$\begin{array}{r} 70 \\ - \square\square\square\square\square\square \\ \hline 0.0089 \end{array}$$

Student Response

1.

$$\begin{array}{r} 0.404 \\ + 0.596 \\ \hline 1.000 \end{array}$$

2.

$$\begin{array}{r} 9.8765 \\ + 0.1235 \\ \hline 10.0000 \end{array}$$

3.

$$\begin{array}{r} 0.700 \\ - 0.688 \\ \hline 0.012 \end{array}$$

4.

$$\begin{array}{r} 7.0000 \\ - 3.5433 \\ \hline 3.4567 \end{array}$$

5.

$$\begin{array}{r} 70.0000 \\ - 69.9911 \\ \hline 0.0089 \end{array}$$

Building on Student Thinking

Students may be unsure how to begin adding or subtracting a number in the thousandths from a whole number, where there are no digits in the tenths, hundredths, or thousandths place, as in $0.404 + ? = 1$. Consider asking them, “What is the value of tenths (or hundredths, or thousandths) in the number 1?” and prompting them to write “0” in each of those places.



Are You Ready for More?

In a cryptarithmic puzzle, the digits 0-9 are represented using the first 10 letters of the alphabet. Use your understanding of decimal addition to determine which digits go with the letters A, B, C, D, E, F, G, H, I, and J. How many possibilities can you find?

$$\begin{array}{r} I H F . I J \\ + J I I . F I \\ \hline E J I . I E \end{array}$$

Extension Student Response

- E: 4, F: 0, H: 2, I: 1, J: 3
- E: 9, F: 0, H: 5, I: 2, J: 7

Activity Synthesis

Ask previously selected students to share their responses and discuss their approaches: whether they work backward, write additional zeros, or use other strategies to find the unknown numbers.

If no students mentioned replacing the 0.7, 7, or 70 in the last three problems with 0.699, 6.9999, or 69.9999, discuss this idea. Ask students how their work to find $7 - ? = 3.4567$ would change if they were to replace 7 with 6.9999. Ask questions such as:

- “How might the number 6.9999 help us find the unknown number?” (The 9s make it easier to subtract, so we can find the difference between 6.9999 and 3.4567, and then add 0.0001 to the result because 6.9999 is 0.0001 less than 7.)
- “How would this method work for a problem such as $9.8765 + ? = 10$?” (We can replace 10 with 9.9999, determine the unknown number, and then add 0.0001 to that number).



This strategy is effective because it eliminates the “extra zeros” and the need to compose or decompose.

Access for English Language Learners

- MLR8 Discussion Supports. For each reasoning strategy that is shared, invite students to turn to a partner and restate what they heard, using precise mathematical language.
- Advances: Listening, Speaking

Lesson Synthesis

Focus the discussion on different strategies for adding and subtracting two decimals of different lengths. Ask students:

- “What are some ways to subtract a number with more decimal places from another number with fewer decimal places, such as $2.4 - 0.1587$?” (We can:
 - Write 2.4 as 2.4000. Decompose 1 of the 4 tenths into 10 hundredths, 1 of the hundredths into 10 thousandths, and 1 of the thousandths into 10 ten-thousandths, and then subtract the value of 0.1587 one digit at a time, starting from the rightmost place.
 - Think of the 2.4 as $2.3999 + 0.0001$, line up the decimal points of 0.1587 and 2.3999, subtract the former from the latter, and then add 0.0001 back to the difference.
 - Think of finding a unknown addend, $0.1587 + ? = 2.4$, set up a vertical calculation, and find a number to add to each digit of 0.1587 to get 2.4000.)

If time permits, invite students to reflect on their preferences or the usefulness of strategies:

- “Which strategy did you find most helpful? Why?”
- “Which strategy did you least prefer? Why?”

15.4

How Much Farther?


Cool-down

 5 min

Standards

Addressing 6.NS.B.3

Student Task Statement

 A runner has run 1.192 kilometers of a 10-kilometer race. How much farther does she need to run to finish the race? Show your reasoning.

Student Response

8.808 kilometers. Sample reasoning:

- $9.999 - 1.192 = 8.807$. Adding 0.001 to 8.807 gives 8.808.



$$\begin{array}{r}
 1.192 \\
 + 8.808 \\
 \hline
 10.000
 \end{array}$$

Responding to Student Thinking

Press Pause

If students struggle to find the difference of the two numbers, make time to revisit different ways to subtract two numbers that have different numbers of decimal places. For instance, ask students to work in small groups, think of as many ways as they can for finding the difference in the practice problem referred to here, and explain to another group how each strategy works.

Accelerated 6, Unit 3, Lesson 15, Practice Problem 2

Lesson 15 Summary

Base-ten diagrams work best for representing subtraction of numbers with few non-zero digits, such as $0.16 - 0.09$. For numbers with many non-zero digits, such as $0.25103 - 0.04671$, it would take a long time to draw the base-ten diagram. With vertical calculations, we can find this difference efficiently.

Thinking about base-ten diagrams can help us make sense of this calculation.

$$\begin{array}{r}
 0.25103 \\
 - 0.04671 \\
 \hline
 0.20432
 \end{array}$$

10
 4 0 10

The thousandth in 0.25103 is decomposed to make 10 ten-thousandths so that we can subtract 7 ten-thousandths. Similarly, one of the hundredths in 0.25103 is decomposed to make 10 thousandths.



Lesson 15 Practice Problems

1 Student Task Statement

For each subtraction problem, circle the correct calculation.

a. $7.2 - 3.67$

$$\begin{array}{r} 7.2 \\ -3.67 \\ \hline 3.05 \end{array}$$

$$\begin{array}{r} 0\ 7.2 \\ -\color{blue}3.67 \\ \hline 3.05 \end{array}$$

$$\begin{array}{r} 7.20 \\ -\color{blue}3.67 \\ \hline 3.53 \end{array}$$

b. $16 - 1.4$

$$\begin{array}{r} 16 \\ -1.4 \\ \hline 0.2 \end{array}$$

$$\begin{array}{r} 1\ 6.0 \\ -\color{blue}1.40 \\ \hline 0.20 \end{array}$$

$$\begin{array}{r} 16.0 \\ -\color{blue}1.4 \\ \hline 14.6 \end{array}$$

Solution

- a. The last calculation is the correct one.
- b. The last calculation is the correct one.

2 Student Task Statement

 Explain or show how you could find the difference of 1 and 0.1978.


Solution

Sample responses:

- 1 can be decomposed into 10,000 ten-thousandths.
0.1978 is 1,978 ten-thousandths.
Subtract: $10,000 - 1,978 = 8,022$

$$\begin{array}{r} \color{red}{9\ 9\ 9} \\ \color{red}{0\ 10\ 10\ 10\ 10} \\ \color{red}{1.} \color{blue}{\cancel{0}\ \cancel{0}\ \cancel{0}\ \cancel{0}} \\ - 0.1978 \\ \hline 0.8022 \end{array}$$

3 Student Task Statement

 A chemist orders a jar labeled to contain 0.384 kilogram of a chemical she needs for an experiment. The actual weight of the chemical sample she receives is 0.3798 kilogram.





- a. Is the chemical sample heavier or lighter than the weight stated on the label? Explain how you know.
 b. How much heavier or lighter is the sample than stated on the label? Show your reasoning.

Solution

- a. Lighter. Sample reasoning: 0.3798 is 3,798 ten-thousandths. 0.384 is 384 thousandths, which is equal to 3,840 ten-thousandths, so 0.384 is greater than 0.3798.
 b. 0.0042 ounce lighter. Sample reasoning:

- 3,840 ten-thousandths subtracted by 3,798 ten-thousandths is 42 ten-thousandths, because $3,840 - 3,798 = 42$.
- 0.3798 is 0.0002 away from 0.3800, and 0.3800 is 0.004 away from 0.384, so 0.3798 is $(0.0002 + 0.004)$ or 0.0042 away from 0.384.
- $0.384 - 0.3798 = 0.0042$.

$$\begin{array}{r} \\ \\ - \\ \hline 0.0042 \end{array}$$

¹³
~~7~~ ~~3~~ ~~10~~

4



Student Task Statement



Complete the calculations so that each shows the correct sum.

a.

$$\begin{array}{r} 1.036 \\ + \\ \hline 4 \end{array}$$

b.

$$\begin{array}{r} 0.738 \\ + \\ \hline 1 \end{array}$$

c.

$$\begin{array}{r} 0.5137 \\ + \\ \hline 1 \end{array}$$

Solution

- a. 2.964
 b. 0.262
 c. 0.4863

a.

$$\begin{array}{r} \\ \\ + \\ \hline 4.000 \end{array}$$

b.

$$\begin{array}{r} \\ \\ + \\ \hline 1.000 \end{array}$$

c.

$$\begin{array}{r} \\ \\ + \\ \hline 1.0000 \end{array}$$

5

from Unit 3, Lesson 10



Student Task Statement



A shipping company is loading cube-shaped crates into a larger cube-shaped container. The smaller cubes have side lengths of $2\frac{1}{2}$ feet, and the larger shipping container has side lengths of 10 feet. How many crates will fit in the large shipping container? Explain your reasoning.

Solution

64 crates. Sample reasoning:



- Four crates can fit in a length of 10 feet because $4 \cdot 2\frac{1}{2} = 10$. So the container can fit $4 \cdot 4 \cdot 4$ or 64 crates.
- The volume of the larger container is 1000 cubic feet because $10 \cdot 10 \cdot 10 = 1000$. The volume of a crate is $15\frac{5}{8}$, since $2\frac{1}{2} \cdot 2\frac{1}{2} \cdot 2\frac{1}{2} = 15\frac{5}{8}$. Then 64 crates fit inside the container because $1000 \div 15\frac{5}{8} = 64$.

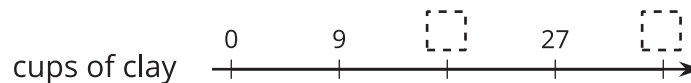
6

from Unit 2, Lesson 11

Student Task Statement

For every 2 students, the art teacher prepares 9 cups of clay.

- a. Here is a double number line diagram showing varying numbers of students and the cups of clay prepared. Complete the missing information.



- b. The same information is shown on a table. Complete the missing information.

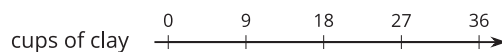
cups of clay	number of students
9	2
	4
27	
	14
	1

Then, answer these questions:

- How many students are there if the teacher prepares 63 cups of clay?
- How many cups of clay are needed for 20 students?
- How many cups of clay are prepared for each student?

Solution

a.



b.

cups of clay	number of students
9	2
18	4
27	6
63	14
$\frac{9}{2}$ (or equivalent)	1

- 14 students
- 90 cups
- $\frac{9}{2}$ (or equivalent) cups

