

Combining Functions

Let's make some new functions using other functions.

10.1

Are Book Sales Improving?

| t (years since 2010) | number of books sold in the US (millions) | population of the US (millions) |
|------------------------|---|---------------------------------|
| 0 | 2,530 | 309.35 |
| 1 | 2,400 | 311.64 |
| 2 | 2,730 | 313.99 |
| 3 | 2,720 | 316.23 |
| 4 | 2,700 | 318.62 |
| 5 | 2,710 | 321.04 |
| 6 | 2,700 | 323.41 |

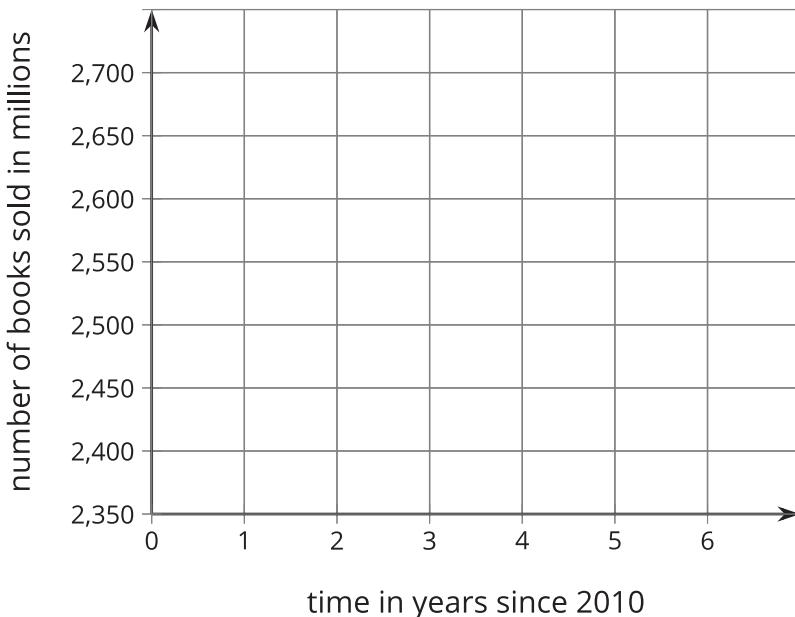
10.2

How Many Books Can One Person Have?

The table shows the values of two functions, P and B , where $P(t)$ is the population of the US, in millions, t years after 2010, and $B(t)$ is the number of books sold per year, in millions, t years after 2010.

| t (years since 2010) | $B(t)$ (millions) | $P(t)$ (millions) | $R(t)$ |
|------------------------|-------------------|-------------------|--------|
| 0 | 2,530 | 309.35 | |
| 1 | 2,400 | 311.64 | |
| 2 | 2,730 | 313.99 | |
| 3 | 2,720 | 316.23 | |
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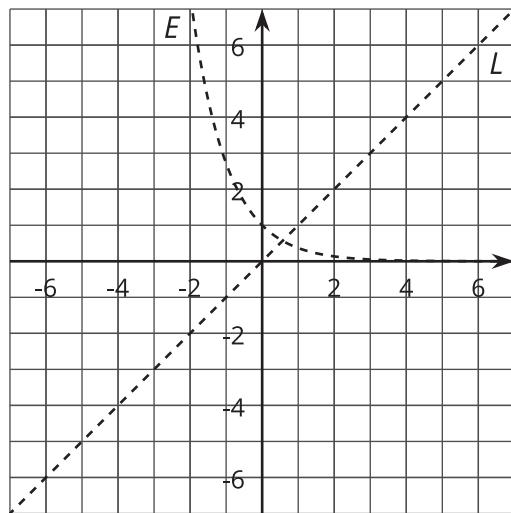
1. Plot the values of B as a function of t . What does the plot tell you about book sales?



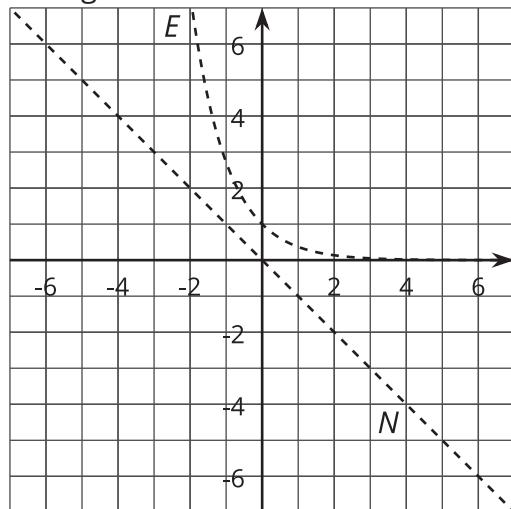
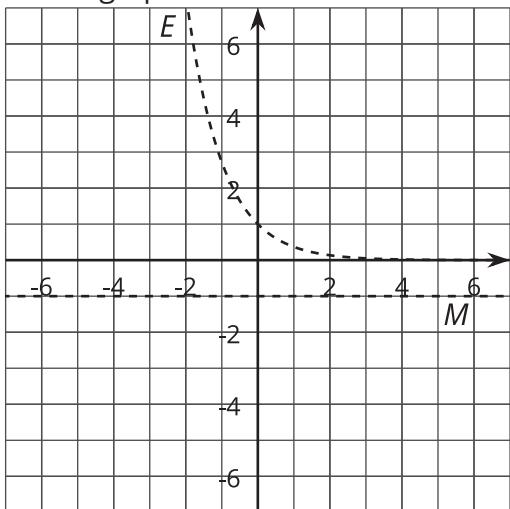
- How many books were sold per person in 2010 and 2016? What do these values tell you about book sales?
- Define a new function R by $R(t) = \frac{B(t)}{P(t)}$. Complete the table and then graph the values of $R(t)$. What do the values of R tell you?

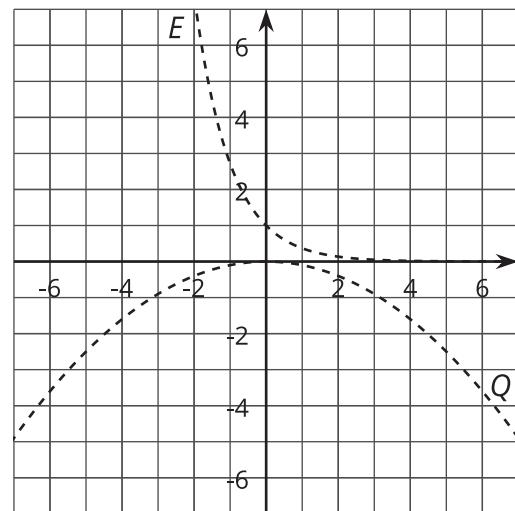
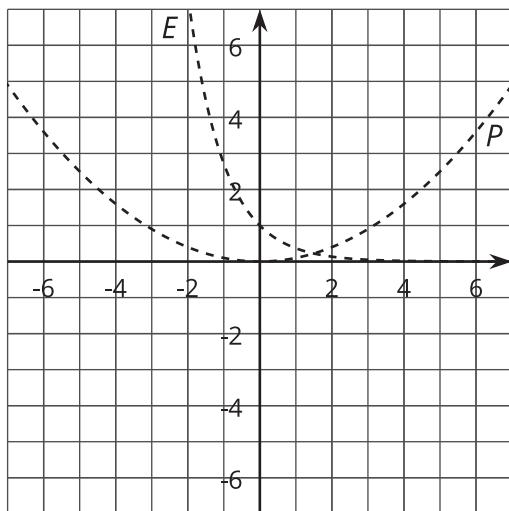
10.3 Adding Functions

1. Here are the graphs of two functions, E and L . Define a new function S by adding E and L , so $S(x) = E(x) + L(x)$. On the same axes, sketch what you think the graph of S looks like.



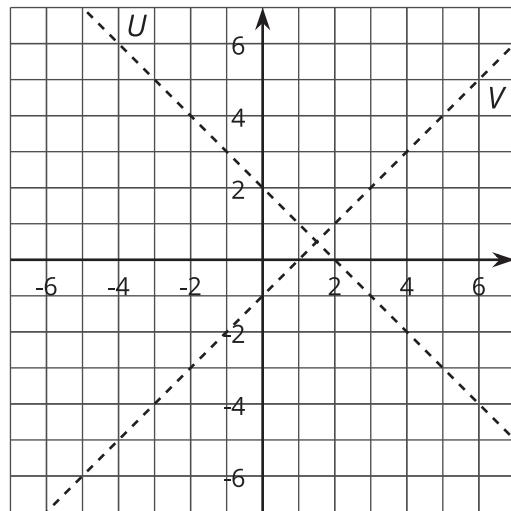
2. Sketch the graph of the sum of E and each of the following functions.





💡 Are you ready for more?

Here are the graphs of two functions, U and V . Define a new function W by multiplying U and V , so $W(x) = U(x)V(x)$. On the same axes, sketch what you think the graph of W looks like.



Lesson 10 Summary

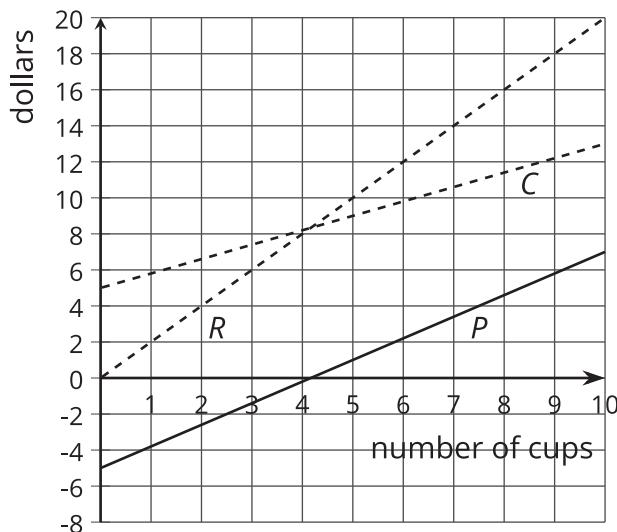
We can add, subtract, multiply, and divide functions to get new functions. For example, the cost in dollars of producing n cups of lemonade at a lemonade stand is $C(n) = 5 + 0.8n$. The revenue (amount of money collected) from selling n cups is $R(n) = 2n$ dollars. The profit $P(n)$ from selling n cups is the revenue minus the cost, so

$$P(n) = R(n) - C(n) = 2n - (5 + 0.8n) = 1.2n - 5$$

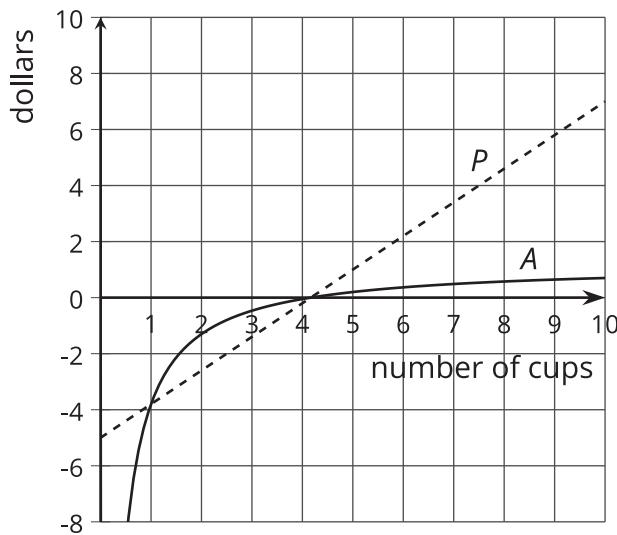
Here are the graphs of C , R , and P . Can you see how each value on P is the result of the difference between the corresponding points on R and C ?

The average profit per cup, $A(n)$, from selling n cups, is the quotient of the profit and the number of cups, so

$$A(n) = \frac{P(n)}{n} = \frac{1.2n - 5}{n} = 1.2 - \frac{5}{n}$$



Here are the graphs of P and A . Can you see how the value of $A(n)$ is the result of the quotient of $P(n)$ and n ? Why does it make sense that both functions are negative when $n < 4\frac{1}{6}$ and positive when $n > 4\frac{1}{6}$?



Since n can only be positive, $P(n)$ and $A(n)$ always have the same sign for a given n value. Notice that for the average profit to be positive, the seller has to sell at least 5 cups (since $4\frac{1}{6}$ is not in the domain, we must round up). It is also true that for a large number of cups, the average profit is close to \$1.20 per cup.