Unit 6, Lesson 17

# Lines in Triangles

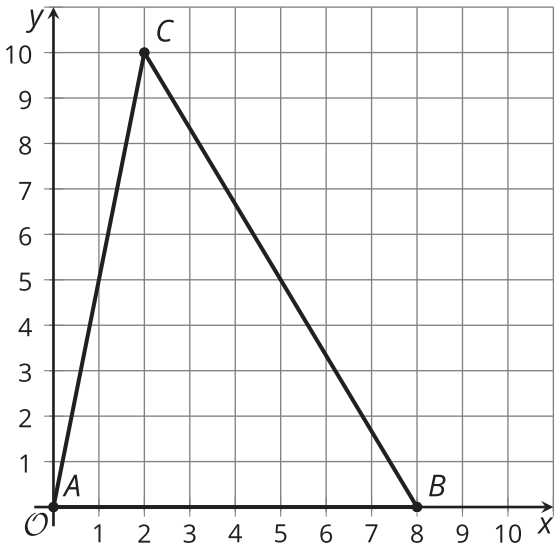
Let's investigate more special segments in triangles.

## 17.1 Folding Altitudes

Draw a triangle on tracing paper. Fold the altitude from each vertex.

## 17.2 Altitude Attributes

Triangle $ABC$ is graphed.




1. Find the slope of each side of the triangle.

2. Find the slope of each altitude of the triangle.

3. Sketch the altitudes. Label the point of intersection, $H$.
4. Write equations for all 3 altitudes.

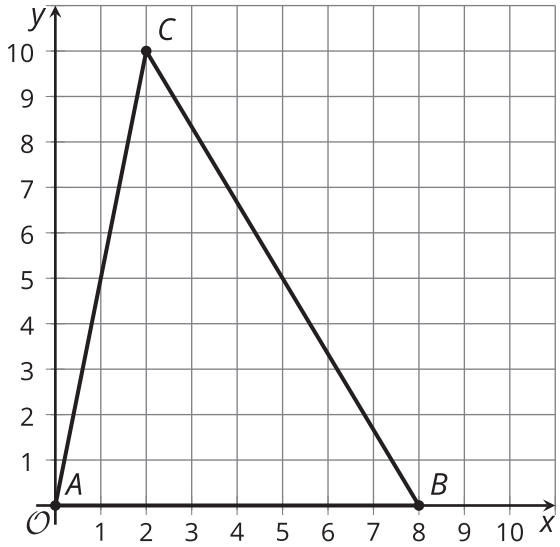
5. Use the equations to find the coordinates of $H$, and verify algebraically that the altitudes all intersect at $H$.

## Are you ready for more?

Any triangle $ABC$ can be translated, rotated, and dilated so that the image $A'$ lies on the origin, $B'$ lies on the point $(1, 0)$, and $C'$ has position $(a, b)$. Use this as a starting point to prove that the altitudes of any triangle will all meet at the same point.

# 17.3 Percolating on Perpendicular Bisectors

Triangle $ABC$ is graphed.




1. Find the midpoint of each side of the triangle.

2. Sketch the perpendicular bisectors, using an index card to help draw 90-degree angles. Label the intersection point as $P$.
3. Write equations for all 3 perpendicular bisectors.

4. Use the equations to find the coordinates of $P$, and verify algebraically that the perpendicular bisectors all intersect at $P$.

# 17.4 Perks of Perpendicular Bisectors

Consider triangle $ABC$ from an earlier activity.

1. What is the distance from $A$ to $P$, the intersection point of the perpendicular bisectors of the triangle's sides? Round to the nearest tenth.

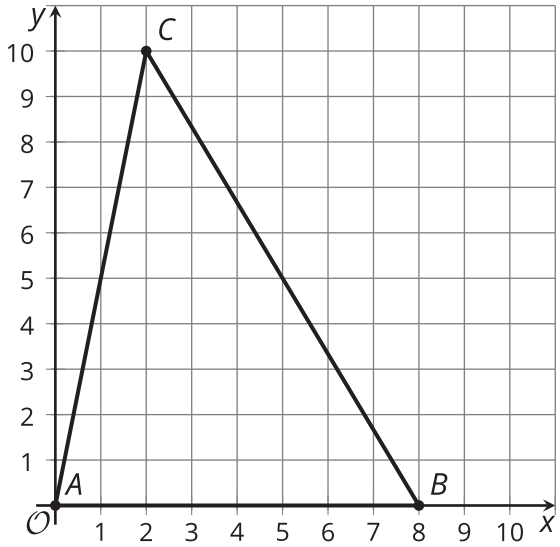
2. Write the equation of a circle with center $P$ and radius $AP$.

3. Construct the circle. What do you notice?

4. Verify your hypothesis algebraically.

# 17.5 Amazing Points

Consider triangle $ABC$ from earlier activities.




1. Plot point $H$, the intersection point of the altitudes.
2. Plot point $P$, the intersection point of the perpendicular bisectors.
3. Find the point where the 3 medians of the triangle intersect. Plot this point and label it $J$.

4. What seems to be true about points $H$, $P$, and $J$? Prove that your observation is true.

# 17.6 Tiling the (Coordinate) Plane

A tessellation covers the entire plane with shapes that do not overlap or leave gaps.

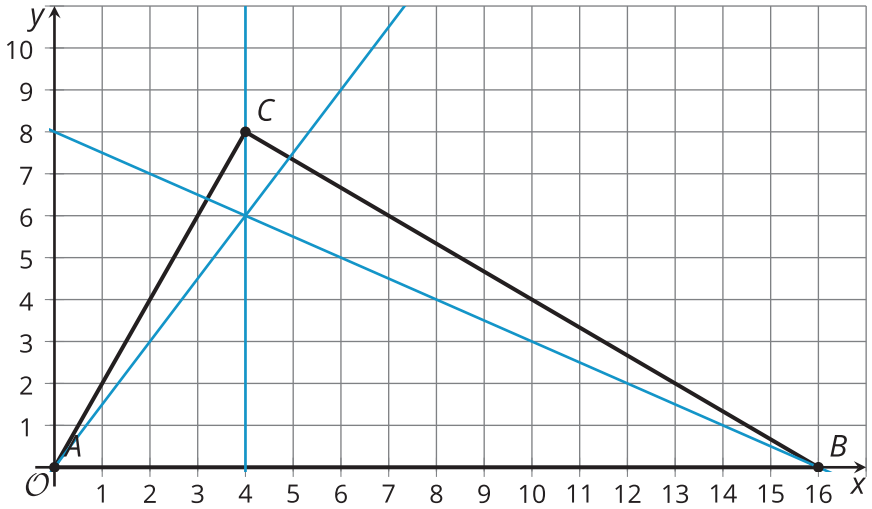
1. Tile the plane with congruent rectangles:
   a. Draw the rectangles on your grid.
   b. Write the equations for lines that outline 1 rectangle.

2. Tile the plane with congruent right triangles:
   a. Draw the right triangles on your grid.
   b. Write the equations for lines that outline 1 right triangle.

3. Tile the plane with any other shapes:
   a. Draw the shapes on your grid.
   b. Write the equations for lines that outline 1 of the shapes.

## Lesson 17 Summary

The 3 medians of a triangle always intersect in 1 point. We can use coordinate geometry to show that the altitudes of a triangle intersect in 1 point, too. The 3 altitudes of triangle $ABC$ are shown here. They appear to intersect at the point $(4, 6)$. By finding their equations, we can prove this is true.




The slopes of sides $AB$, $BC$, and $AC$ are 0, $-\frac{2}{3}$, and 2. The altitude from $C$ is the vertical line $x = 4$. The slope of the altitude from $A$ is $\frac{3}{2}$. Since the altitude goes through $(0, 0)$, its equation is $y = \frac{3}{2}x$. The slope of the altitude from $B$ is $-\frac{1}{2}$. Following this slope over to the $y$-axis we can see that the $y$-intercept is 8. So the equation for this altitude is $y = -\frac{1}{2}x + 8$.

We can now verify that $(4, 6)$ lies on all 3 altitudes by showing that the point satisfies the 3 equations. By substitution we see that each equation is true when $x = 4$ and $y = 6$.