



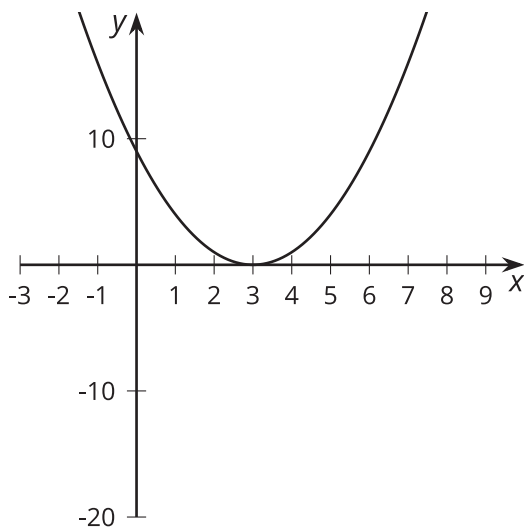
# Multiplicity

Let's sketch some polynomial functions.

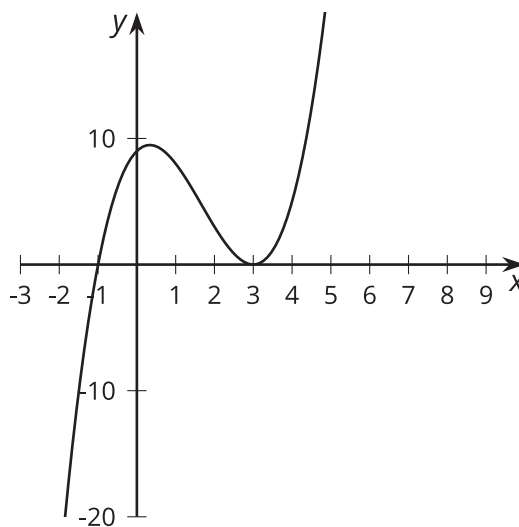
## 10.1 Notice and Wonder: Duplicate Factors

What do you notice? What do you wonder?

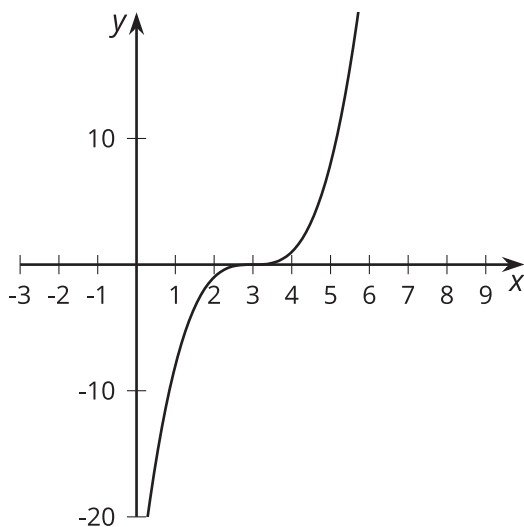
$$y = (x - 3)^2$$



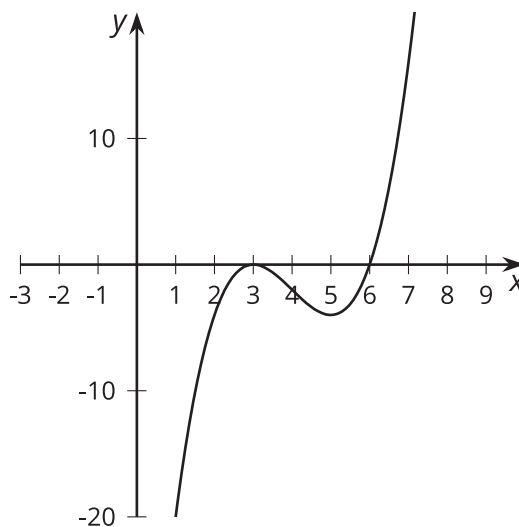
$$y = (x + 1)(x - 3)^2$$



$$y = (x - 3)^3$$



$$y = (x - 6)(x - 3)^2$$



## 10.2 Sketching Polynomials

1. For polynomials  $A$ – $F$ :

- Write the degree, all zeros, and complete the sentence about the end behavior.
- Sketch a possible graph.
- Check your sketch using graphing technology.

Pause here for your teacher to check your work.

2. Create your own polynomial for your partner to figure out.

- Create a polynomial with degree greater than 2 and less than 8, and write the equation in the space given.
- Trade papers with a partner, then fill out the information about their polynomial and complete a sketch.
- Trade papers back. Check your partner's sketch using graphing technology.

$$A(x) = (x + 2)(x - 2)(x - 8)$$

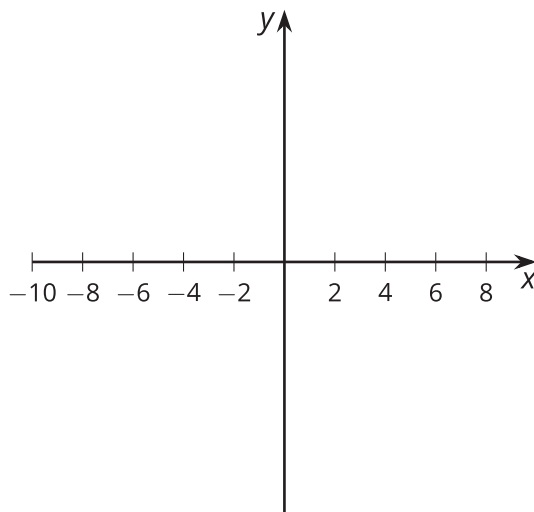
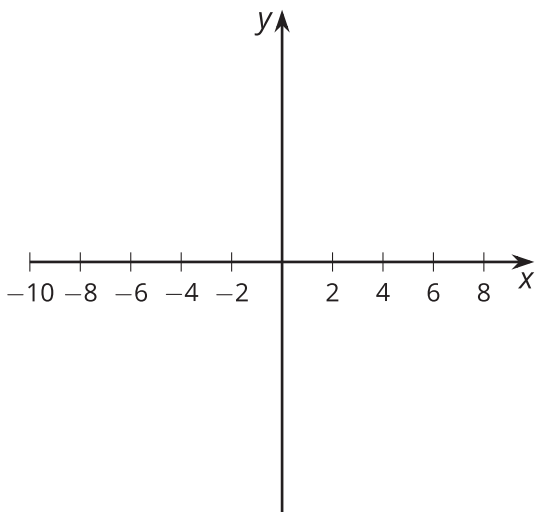
Degree:                      Zeros:

End behavior: As  $x$  gets larger and larger in the negative direction,

$$B(x) = -(x + 2)(x - 2)^2$$

Degree:                      Zeros:

End behavior: As  $x$  gets larger and larger in the negative direction,

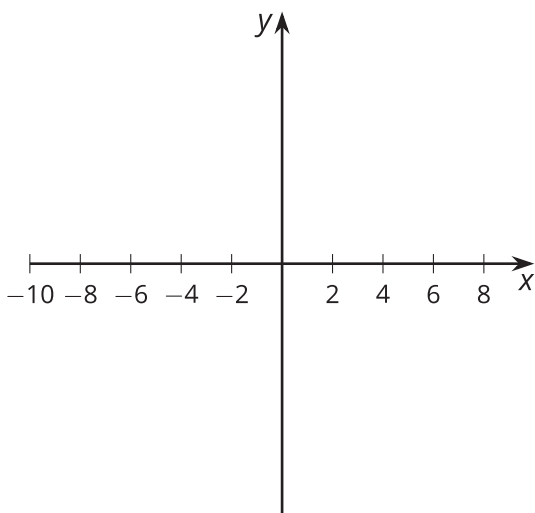


$$C(x) = (x + 6)(x + 2)^2$$

Degree:

Zeros:

End behavior: As  $x$  gets larger and larger in the negative direction,

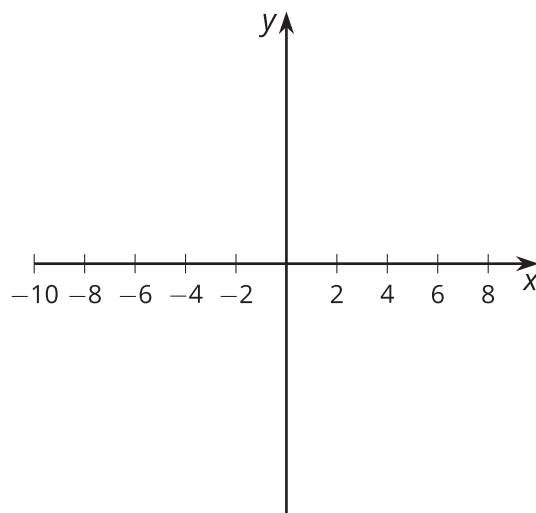


$$D(x) = -(x + 6)^2(x + 2)$$

Degree:

Zeros:

End behavior: As  $x$  gets larger and larger in the negative direction,

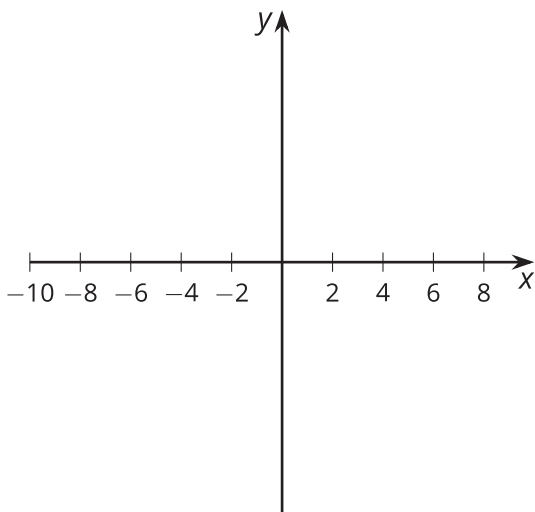


$$E(x) = (x + 4)(x - 2)^3$$

Degree:

Zeros:

End behavior: As  $x$  gets larger and larger in the negative direction,

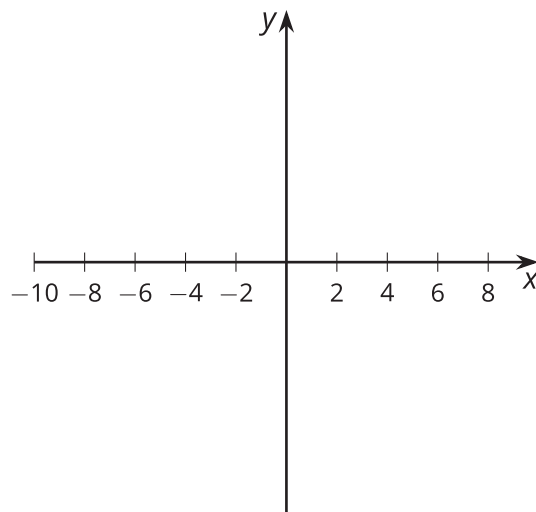


$$F(x) = x^3(x + 4)(x - 3)^2$$

Degree:

Zeros:

End behavior: As  $x$  gets larger and larger in the negative direction,

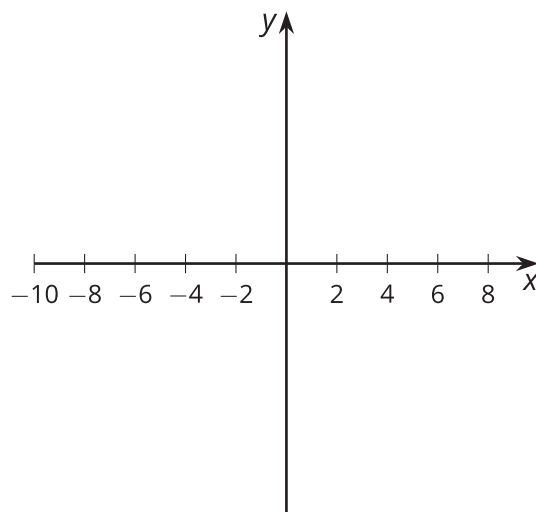


Your polynomial:

Degree:

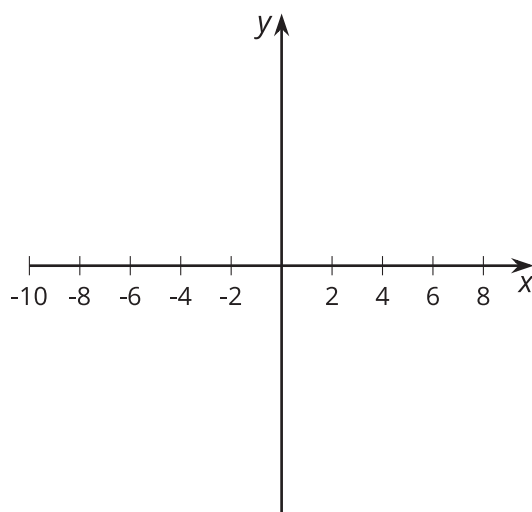
Zeros:

End behavior: As  $x$  gets larger and larger in the negative direction,



### 10.3 Using Knowledge of Zeros

1. Sketch a graph for a polynomial function  $y = f(x)$  that has 3 different zeros and  $f(x) \geq 0$  for all values of  $x$ .



2. What is the smallest degree the polynomial could have?
3. What is a possible equation for the polynomial? Use graphing technology to see if your equation matches your sketch.

## Are you ready for more?

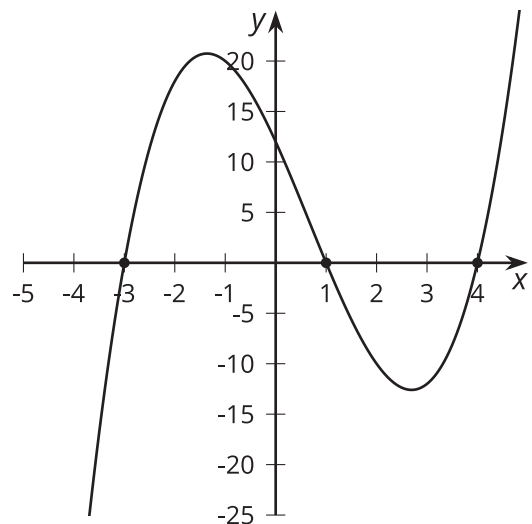
What is a possible equation of a polynomial function that has degree 5 but whose graph has exactly three horizontal intercepts and crosses the  $x$ -axis at all three intercepts? Explain why it is not possible to have a polynomial function that has degree 4 with this property.

## Lesson 10 Summary

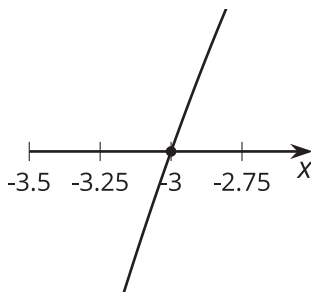
We can combine what we know about factors, degree, end behavior, the sign of the leading coefficient, and multiplicity to sketch polynomials written in factored form. **Multiplicity**, or the power to which a factor occurs in the factored form of a polynomial, tells us the number of times that factor is repeated and affects the shape of the graph near the location of each zero on the horizontal axis.

For example,  $y = (x + 3)(x - 1)(x - 4)$  has three factors with no duplicates. We say that each factor,  $(x + 3)$ ,  $(x - 1)$ , and  $(x - 4)$ , has a multiplicity of 1. This results in a graph that looks a bit like a linear function near  $x = -3$ ,  $x = 1$ , and  $x = 4$  when we zoom in on each of those places.

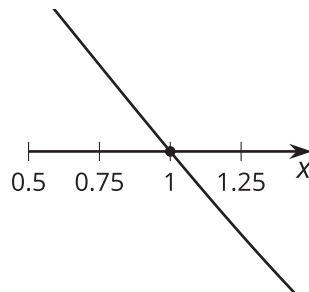
$$y = (x + 3)(x - 1)(x - 4)$$



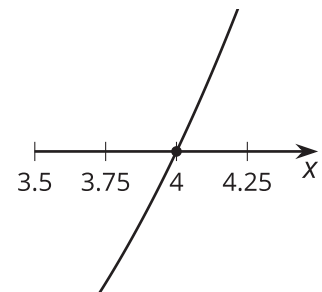
near  $x = -3$



near  $x = 1$

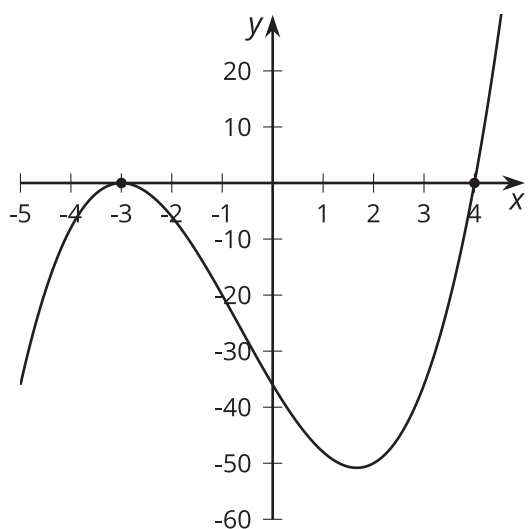


near  $x = 4$

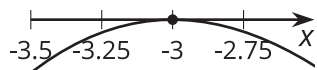


For  $y = (x + 3)^2(x - 4)$ , there are still three factors, but two of them are  $(x + 3)$ . This results in a graph that looks a bit like a quadratic near  $x = -3$  and a bit like a linear function near  $x = 4$ . We say that the factor  $(x + 3)$  has a multiplicity of 2 while the factor  $(x - 4)$  has a multiplicity of 1.

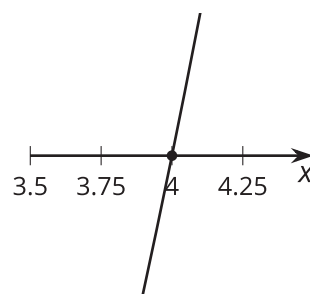
$$y = (x + 3)^2(x - 4)$$



**near  $x = -3$**



**near  $x = 4$**



Now consider what the graph of  $y = (x + 3)(x - 4)^3$  would look like. The factors help us identify that the function has zeros at  $-3$  and  $4$ . We also know that since  $(x + 3)$  has a multiplicity of 1 and  $(x - 4)$  has a multiplicity of 3, the graph looks a bit like a linear polynomial crossing the  $x$ -axis at  $-3$  and a bit like a cubic polynomial crossing the  $x$ -axis at  $4$ . Since this is a 4th-degree polynomial with a positive leading coefficient, we know that as  $x$  gets larger and larger in either the negative or positive direction,  $y$  gets larger and larger in the positive direction.

$$y = (x + 3)(x - 4)^3$$

