# More Balanced Moves

Let's rewrite some more equations while keeping the same solutions.

## 4.1 Different Equations?

Equation 1

$$x - 3 = 2 - 4x$$

Which of these have the same solution as Equation 1? Be prepared to explain your reasoning.

| Equation A | Equation B | Equation C | Equation D |
|---|---|---|---|
| $2x - 6 = 4 - 8x$ | $x - 5 = \text{-}4x$ | $2(1 - 2x) = x - 3$ | $\text{-}3 = 2 - 5x$ |

## 4.2 Step by Step by Step by Step

Here is an equation and the steps that Clare wrote to solve it:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 3(5x + 9) \\ 3(4x + 1) &= 3(5x + 9) \\ 4x + 1 &= 5x + 9 \\ 1 &= x + 9 \\ -8 &= x \end{aligned}$$

Here is the same equation, and the steps Lin wrote to solve it:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 3(5x + 9) \\ 12x + 3 &= 15x + 27 \\ 12x &= 15x + 24 \\ -3x &= 24 \\ x &= -8 \end{aligned}$$

1. Are both of their solutions correct? Explain your reasoning.

2. Describe some ways the steps they took are alike and different.

3. Mai and Noah also solved the equation, but some of their steps have errors. Find the incorrect step in each solution and explain why it is incorrect.

   Mai:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 3(5x + 9) \\ 7x + 3 &= 3(9) \\ 7x + 3 &= 27 \\ 7x &= 24 \\ x &= \frac{24}{7} \end{aligned}$$

   Noah:

$$\begin{aligned} 14x - 2x + 3 &= 3(5x + 9) \\ 12x + 3 &= 3(5x + 9) \\ 12x + 3 &= 15x + 27 \\ 27x + 3 &= 27 \\ 27x &= 24 \\ x &= \frac{24}{27} \end{aligned}$$

## 4.3 Make Your Own Steps

Solve these equations for $x$.

1. $\frac{12+6x}{3} = \frac{5-9}{2}$

2. $x - 4 = \frac{1}{3}(6x - 54)$

3. $-(3x - 12) = 9x - 4$

### Are you ready for more?

I have 24 pencils and 3 cups. The second cup holds one more pencil than the first. The third holds one more than the second. How many pencils does each cup contain?

  

## Lesson 4 Summary

How do we make sure that the solution we find for an equation is correct? Accidentally adding when we meant to subtract, missing a negative when we distribute, forgetting to write an $x$ from one line to the next are some of the many possible mistakes to watch out for!

Fortunately, each valid step we take to solve an equation results in a new equation with the same solution as the original. This means that we can check our work by substituting the value of the solution into the original equation. For example, suppose we solve the following equation:

$$\begin{aligned} 2x &= \text{-}3(x+5) \\ 2x &= \text{-}3x + 15 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

Because the last equation shows that $x$ equals 3, and because valid steps make equivalent equations, we can use the equivalence in the original equation to check that all of the steps are valid. Substituting 3 in place of $x$ into the original equation,

$$\begin{aligned} 2(3) &= \text{-}3(3+5) \\ 6 &= \text{-}3(8) \\ 6 &= \text{-}24 \end{aligned}$$

we get a statement that isn't true! This tells us we must have made a mistake somewhere. Checking our original steps carefully, we made a mistake when distributing -3. Fixing it, we now have

$$\begin{aligned} 2x &= \text{-}3(x+5) \\ 2x &= \text{-}3x - 15 \\ 5x &= \text{-}15 \\ x &= \text{-}3 \end{aligned}$$

Substituting -3 in place of $x$ into the original equation to make sure we didn't make another mistake:

$$\begin{aligned} 2(\text{-}3) &= \text{-}3(\text{-}3+5) \\ \text{-}6 &= \text{-}3(2) \\ \text{-}6 &= \text{-}6 \end{aligned}$$

This equation is true, so $x = \text{-}3$ is the solution.

  