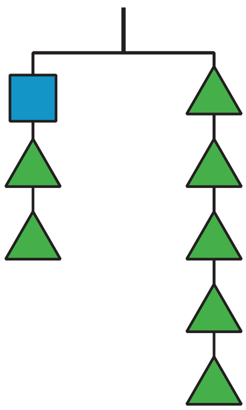
Unit 4 Family Support Materials

Linear Equations and Linear Systems

Section A: Equivalent Equations

This week your student will work on solving linear equations. We can think of a balanced hanger as a metaphor for an equation. An equation says that the expressions on either side have equal value, just like a balanced hanger has equal weights on either side.



If we have a balanced hanger and add or remove the same amount of weight from each side, the result will still be in balance. For example, we could remove 2 triangles from each side of this hanger and it would still balance. We could also add a square to each side and it would still balance.

We can do this with equations as well: Adding or subtracting the same amount from both sides of an equation keeps the sides equal to each other. For example, if $4x + 20$ and $-6x + 10$ have equal value, we can write an equation $4x + 20 = -6x + 10$. We could add -10 to both sides of the equation or divide both sides of the equation by 2 and keep the sides equal to each other. Using these moves in systematic ways, we can find that $x = -1$ is a solution to this equation.

Here is a task to try with your student:

Elena and Noah work on the equation $\frac{1}{2}(x+4) = -10 + 2x$ together. Elena's solution is $x = 24$ and Noah's solution is $x = -8$. Here is their work:

Elena:

$$\begin{aligned}\frac{1}{2}(x+4) &= -10 + 2x\\ x + 4 &= -20 + 2x\\ x + 24 &= 2x\\ 24 &= x\\ x &= 24\end{aligned}$$

Noah:

$$\begin{aligned}\frac{1}{2}(x+4) &= -10 + 2x\\ x + 4 &= -20 + 4x\\ -3x + 4 &= -20\\ -3x &= -24\\ x &= -8\end{aligned}$$

Do you agree with their solutions? Explain or show your reasoning.

Solution:

No, they both have errors in their solutions.

Elena multiplied both sides of the equation by 2 in her first step, but forgot to multiply the $2x$ by the 2. We can also check Elena's answer by replacing x with 24 in the original equation and seeing if the equation is true.

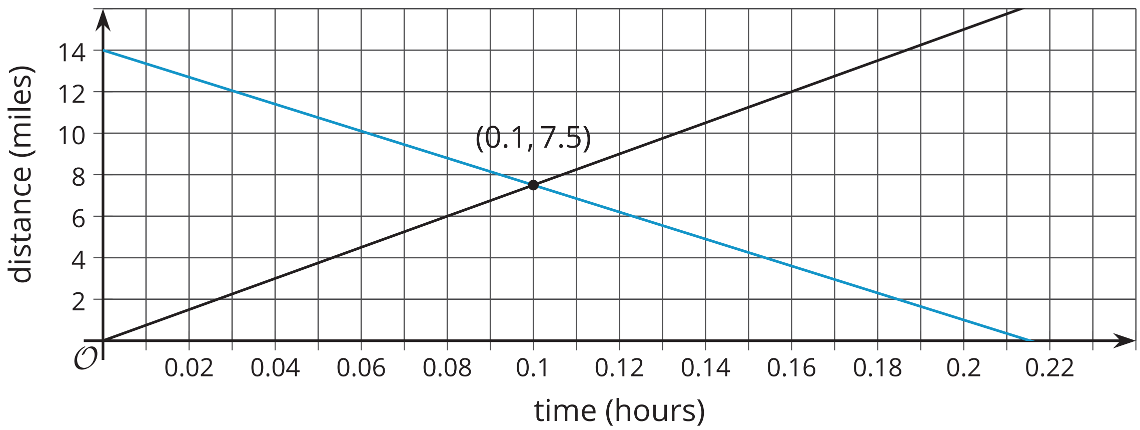
$$\frac{1}{2}(x+4) = \text{-}10 + 2x$$
$$\frac{1}{2}(24+4) = \text{-}10 + 2(24)$$
$$\frac{1}{2}(28) = \text{-}10 + 48$$
$$14 = 38$$

Because 14 is not equal to 38, Elena's answer is not correct.

Noah divided both sides by -3 in his last step, but wrote -8 instead of 8 for $\text{-}24 \div \text{-}3$. We can also check Noah's answer by replacing x with -8 in the original equation and seeing if the equation is true. Noah's answer is not correct.

Section C: Systems of Linear Equations

This week your student will work with systems of equations. A system of equations is a set of 2 (or more) equations where the letters represent the same values. For example, say Car A is traveling 75 miles per hour and passes a rest area. The distance in miles it has traveled from the rest area after t hours is $d = 75t$. Car B is traveling toward the rest area, and its distance from the rest area at any time is $d = 14 - 65t$. We can ask if there is ever a time when the distance of Car A from the rest area is the same as the distance of Car B from the rest area. If the answer is "yes," then the solution will correspond to one point that is on both lines, such as the point $(0.1, 7.5)$ shown here. This means that 0.1 hours after Car A passes the rest area, both cars will be 7.5 miles from the rest area.



We could also answer the question without using a graph. Because we are asking when the d values for each car will be the same, we are asking for what t value, if any, makes $75t = 14 - 65t$ true. Solving this equation for t, we find that $t = 0.1$ is a solution, and at that time the cars are 7.5 miles away because $75t = 75 \cdot 0.1 = 7.5$. This finding matches the graph.

Here is a task to try with your student:

Lin and Diego are biking the same direction on the same path, but start at different times. Diego is riding at a constant speed of 18 miles per hour, so his distance traveled in miles can be represented by d and the time he has traveled in hours by t, where $d = 18t$. Lin started riding a quarter hour before Diego at a constant speed of 12 miles per hour, so her total distance traveled in miles can be represented by d, where $d = 12\left(t + \frac{1}{4}\right)$. When will Lin and Diego meet?

Solution:

To find when Lin and Diego meet, that is, when they have traveled the same total distance, we can set the two equations equal to one another: $18t = 12\left(t + \frac{1}{4}\right)$. Solving this equation for t, $18t = 12t + 3$, $6t = 3$, $t = \frac{1}{2}$.

They meet after Diego rides for one half hour and Lin rides for three quarters of an hour. The distance that they each travel before meeting is 9 miles, because we can use $\frac{1}{2}$ for t in Diego's equation and see that $9 = 18 \cdot \frac{1}{2}$. Another way to find a solution would be to graph both $d = 18t$ and $d = 12\left(t + \frac{1}{4}\right)$ on the same coordinate plane and interpret the point where these lines intersect.