Unit 8, Lesson 15

Keeping Track of All Possible Outcomes

Let's explore sample spaces for experiments with multiple parts.

15.1 How Many Different Bracelets?

Bracelets made by a tribe of Native American jewelers in Arizona and New Mexico are available with several different options. How many different bracelets are possible if each one includes 1 stone, 1 design, and 1 size?

stones

- turquoise
- pink coral
- lapis lazuli

designs

- animal
- floral

sizes

- small
- medium
- large

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15.2 Lists, Tables, and Trees

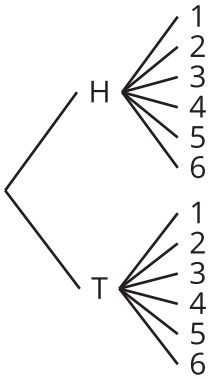
Consider the experiment: Flip a coin, and then roll a number cube.

Elena, Kiran, and Priya each use a different method for finding the sample space of this experiment.

- Elena carefully writes a list of all the options: Heads 1, Heads 2, Heads 3, Heads 4, Heads 5, Heads 6, Tails 1, Tails 2, Tails 3, Tails 4, Tails 5, Tails 6.
- Kiran makes a table:

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

- Priya draws a tree with branches in which each pathway represents a different outcome:



1. Compare the three methods. What is the same about each method? What is different? Be prepared to explain why each method produces all the different outcomes without repeating any.

2. Which method do you prefer for this situation?

Pause here so your teacher can review your work.

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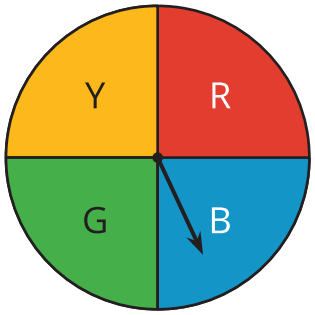
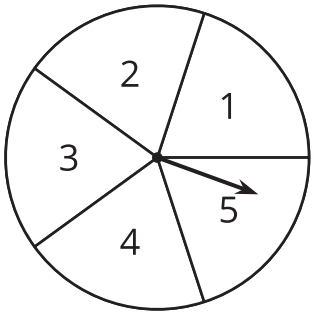
3. Find the sample space for each of these experiments using any method. Make sure you list every possible outcome without repeating any.

   a. Flip a dime, then flip a nickel, and then flip a penny. Record whether each lands heads or tails up.

   b. Han's closet has: a blue shirt, a green shirt, a white shirt, blue pants, khaki pants, and black pants. He must select one shirt and one pair of pants to wear for the day.

   c. Spin a color, then spin a number.





   d. Spin the hour hand on an analog clock, then choose a.m. or p.m.

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15.3 How Many Necklaces?

1. A store that sells Native American jewelry offers beaded necklaces with 1 bead color, 1 pendant option, 1 length, and 1 style. How many different necklaces are possible? Explain your reasoning. You do not need to write out the sample space.
   - color: red, blue, black, pink, turquoise
   - pendant: large stone, silver design, none
   - length: 18 inches, 21 inches, 25 inches, 27 inches
   - style: one strand, two strand, three strands

2. Andre knows he wants a necklace that is red and 21 inches in length. He isn't sure about the other choices. How many of the different necklaces could Andre consider to fit his two requirements?

3. If a necklace is made by randomly choosing each of the options, what is the probability it will be a necklace that fits Andre's requirements?

Are you ready for more?

Describe a situation that involves three parts and has a total of 24 outcomes in the sample space.

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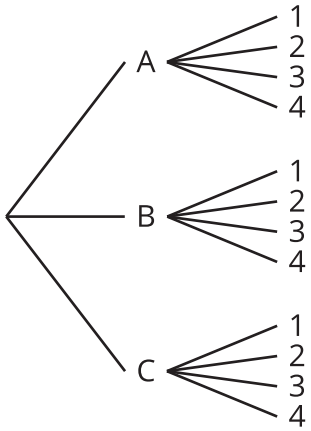
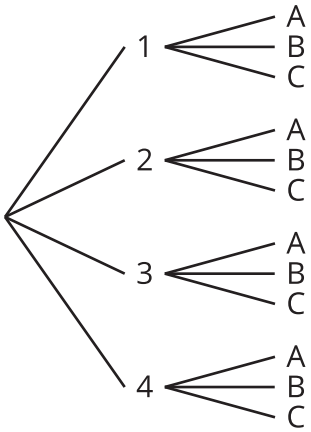
Lesson 15 Summary

Sometimes we need a systematic way to count the number of outcomes that are possible in a given situation. For example, suppose there are 3 people (A, B, and C) who want to run for the president of a club and 4 different people (1, 2, 3, and 4) who want to run for vice president of the club. We can use a tree, a table, or an ordered list to count how many different combinations are possible for a president to be paired with a vice president.

With a tree, we can start with a branch for each of the people who want to be president. Then for each possible president, we add a branch for each possible vice president, for a total of $3 \cdot 4 = 12$ possible pairs. We can also start by counting vice presidents first and then adding a branch for each possible president, for a total of $4 \cdot 3 = 12$ possible pairs.





A table can show the same result:

	1	2	3	4
A	A1	A2	A3	A4
B	B1	B2	B3	B4
C	C1	C2	C3	C4

So does this ordered list:

A1, A2, A3, A4, B1, B2, B3, B4, C1, C2, C3, C4

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