The Addition Rule

Let's learn about and use the addition rule.

6.1 Hats Off, Sneakers On

The table displays information about people at a neighborhood park.

	wearing sneakers	not wearing sneakers	total
wearing a hat	8	2	10
not wearing a hat	3	12	15
total	11	14	25

1. Andre says the number of people wearing sneakers or wearing a hat is 21, because there are a total of 10 people wearing a hat and a total of 11 people wearing sneakers. Is Andre correct? Explain your reasoning.

2. What is the probability that a person selected at random from those in the park is wearing sneakers or wearing a hat?

 CC BY NC 2024 Illustrative Mathematics®

6.2 State Names

Jada has a way to find the probability of a random outcome being in event A or event B. She says, "We use the probability of the outcome being in event A, then add the probability of the outcome being in event B. Now some outcomes have been counted twice, so we have to subtract the probability of the outcome being in both events so that those outcomes are only counted once."

Jada's method can be rewritten as:

$P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$

1. The table of data summarizes information about the 50 states in the United States from a census in the year 2000. A state is chosen at random from the list of 50. Let event A be "the state name begins with A through M" and event B be "the population of the state is less than 4 million."

	population less than 4 million	**population at least 4 million**
name begins with A through M	11	15
name begins with N through Z	13	11

   - Alaska is one of the 11 states in the top left cell of the table.
   - California is one of the 15 states in the top right cell of the table.
   - Nebraska is one of the 13 states in the bottom left cell of the table.
   - New York is one of the 11 states in the bottom right cell of the table.

   For each event, write which of the four states listed here is an outcome in that event.

   a. A or B

   b. A

   c. B

   d. A and B

2. Find each of the probabilities when a state is chosen at random from among all 50 states:
   a. $P(\text{A or B})$

   b. $P(\text{A})$

   c. $P(\text{B})$

   d. $P(\text{A and B})$

3. Does Jada's formula work for these events? Show your reasoning.

4. Seniors at a high school are allowed to go off campus for lunch if they have a grade of A in all their classes or perfect attendance.
   - An assistant principal in charge of academics knows that the probability of a randomly selected senior having A's in all their classes is 0.1.
   - An assistant principal in charge of attendance knows that the probability of a randomly selected senior having perfect attendance is 0.16.
   - The cafeteria staff know that the probability of a randomly selected senior being allowed to go off campus for lunch is 0.18.

   Use Jada's formula to find the probability that a randomly selected senior has all As and perfect attendance.

Are you ready for more?

Priya lists all of the multiples of 3 for whole numbers between 1 and 48, inclusive.

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48

Tyler lists all of the multiples of 4 between 1 and 48, inclusive.

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48

1. Use a Venn diagram to display the multiples of 3 and the multiples of 4 between 1 and 48.

2. If a whole number between 1 and 48, inclusive, is selected at random, find each probability:
   a. $P(\text{multiple of 3})$
   b. $P(\text{multiple of 4})$
   c. $P(\text{multiple of 3 or 4})$
   d. $P(\text{multiple of 3 and 4})$

3. Priya and Tyler extend their lists of multiples to include whole numbers between 1 and 96 inclusive. If a whole number between 1 and 96, inclusive, is selected at random, find each probability and compare it to the probability in the previous problem:
   a. $P(\text{multiple of 3})$
   b. $P(\text{multiple of 4})$
   c. $P(\text{multiple of 3 or 4})$
   d. $P(\text{multiple of 3 and 4})$

4. If a whole number between 1 and 100, inclusive, is selected at random, what do you think $P(\text{multiple of 3 and 4})$ is? Explain your reasoning.

5. If a whole number between 1 and n, inclusive, for any whole number value of n, is selected at random, what is $P(\text{multiple of 3 and 4})$?

6.3 Coffee or Juice?

1. At a cafe, customers order coffee at the bar, and then either go to another table where the cream and sugar are kept or find a seat. Based on observations, a worker estimates that
   - 70% of customers go to the second table for cream or sugar.
   - 60% of all customers use cream for their coffee.
   - 50% of all customers use sugar.

   Use the worker's estimates to find the percentage of all customers who use both cream and sugar for their coffee. Explain or show your reasoning.

2. At the grocery store,
   - 70% of the different types of juice come in a bottle holding less than 400 milliliters (mL).
   - 40% of the different types of juice come in a low-sugar version.
   - 25% of the different types of juice are in bottles holding less than 400 mL and have a low-sugar version.

   What percentage of the different types of juice come in a bottle holding less than 400 mL or are low-sugar? Explain your reasoning.

Lesson 6 Summary

The **addition rule** is used to compute probabilities of compound events. The addition rule states that, given events A and B, $P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})$.

For example, the student council sold 100 shirts that are either gray or blue and in sizes medium and large.

	medium	large	total
gray	20	10	30
blue	15	55	70
total	35	65	100

A student who bought a shirt is chosen at random. One way to find the probability that this student bought a shirt that is blue or medium is to begin with the probabilities for shirts sold in that color and size. The probability that the student bought a blue shirt is 0.70 since 70 out of the 100 shirts sold were blue. The probability that the student bought a medium shirt is 0.35 since 35 out of the 100 shirts sold were medium.

Because we are interested in the probability of a blue shirt or a medium shirt being purchased, we might think we should add these probabilities together to find $0.70 + 0.35 = 1.05$.

This doesn't seem to work since it is saying the probability is greater than 1.

The problem is that the 15 students who bought shirts that are both medium in size and blue are counted twice when the probabilities are added. To fix this double counting, we should subtract the probability that the chosen student is in both categories so that these students are counted only once.

The addition rule then shows that the probability the student bought a medium shirt or a blue shirt is 0.90 because $P(\text{medium or blue}) = P(\text{medium}) + P(\text{blue}) - P(\text{medium and blue})$, or $P(\text{medium or blue}) = 0.35 + 0.70 - 0.15$, which is 0.90.