Unit 4, Lesson 17

Two Related Quantities, Part 2

Goals

- Create a table, graph, and equation to represent the relationship between distance and time for an object moving at a constant speed.
- Identify (in writing) the independent and dependent variable in an equation.
- Interpret (orally and in writing) an equation that represents the relationship between distance and time for an object moving at a constant speed.

Learning Targets

- I can create tables and graphs to represent the relationship between distance and time for something moving at a constant speed.
- I can write an equation with variables to represent the relationship between distance and time for something moving at a constant speed.

Lesson Narrative

In this lesson, students use tables, graphs, and equations to represent the relationships between time and distance, in the context of objects moving at a constant rate. They use their representations to compare rates and consider how each of the representations would change if the independent and dependent variables were switched.

Standards

Addressing **6.EE.C.9, 6.RP.A.3.a, 6.RP.A.3.b**

Required Materials

Materials to Gather

- Colored pencils: Activity 2

Student Facing Learning Goals

Let's use equations and graphs to describe stories with constant speed.

17.1 Which One Travels Faster?

Warm-up

10 min

Activity Narrative

In this *Warm-up*, students reason about the relationship between distance, rate, and time to solve a problem. The purpose is to activate the idea that constant speed can be represented by a set of equivalent ratios associating distance traveled and elapsed time. In the longer activity that follows, students represent the relationship between these two

quantities using a table, equations, and graphs.

As students work, monitor for different representations used (particularly tables) as well as for students who calculate each wheelchair's speed in meters per second or each wheelchair's pace in seconds per meter.

Standards

Addressing **6.EE.C.9, 6.RP.A.3.a, 6.RP.A.3.b**

Launch

Arrange students in groups of 2. Give students 3–4 minutes of quiet work time and 1–2 minutes to share their thinking with a partner. Follow with a whole-class discussion.

If needed, remind students of tools or strategies that may be appropriate for solving this problem, including double number line diagrams or tables of equivalent ratios. Consider allowing students to use calculators to ensure inclusive participation in the activity.

Student Task Statement

A wheelchair user is considering two electric wheelchairs. Wheelchair A can travel 25 meters in 10 seconds. Wheelchair B can travel 13 meters in 5 seconds.

1. Which wheelchair can travel at a faster rate? Be prepared to explain how you know.
2. How many seconds will it take each wheelchair, moving at a constant speed, to travel 195 meters? Show your reasoning.

Student Response

1. Wheelchair B can travel faster. Sample reasoning:
   - It can travel 26 meters in 10 seconds. Wheelchair A can travel 25 meters in the same amount of time.
   - Wheelchair A can travel 2.5 meters in 1 second. Wheelchair B can travel 2.6 meters in 1 second.
2. 78 seconds for Wheelchair A and 75 seconds for Wheelchair B. Sample reasoning:

Wheelchair A

distance (meters)	time (seconds)
25	10
1	$\frac{10}{25}$ or $\frac{2}{5}$
195	$195 \cdot \frac{2}{5}$ or 78

Wheelchair B

distance (meters)	time (seconds)
13	5
1	$\frac{5}{13}$
195	$195 \cdot \frac{5}{13}$ or 75

Activity Synthesis

Invite students to share how they know which wheelchair can travel faster. If one of the following ways of reasoning is not mentioned by students, bring it to students' attention:

- Comparing the distances traveled in the same amount of time: Doubling the given distance and time for Wheelchair B shows that it can cover a longer distance in 10 seconds than Wheelchair A can. Calculating the distance that each wheelchair can travel in 1 second also shows the same result.

- Comparing the amounts of time for travel the same distance: Wheelchair A takes more time ($\frac{5}{13}$ or $\frac{10}{26}$ second) to cover 1 meter than Wheelchair B does ($\frac{10}{25}$ second).

Next, discuss how students found the time it would take each wheelchair to travel 195 meters. Ask students who used different reasoning strategies to share. If no student used a table of equivalent ratios, as shown in the *Student Response*, display a pair of blank tables and discuss how to use the table to reason about the distance each wheelchair can travel in 1 second and the time it takes to travel 1 meter.

17.2 The Robot Race

25 min

Activity Narrative

In this activity, students calculate and compare the unit rates in meters per minute for three robotics teams and consider the graphs and equations that describe the distance–time relationship. As students interpret the data values, graphs, and equations in context, they reason abstractly and quantitatively (MP2).

Standards

Addressing **6.EE.C.9, 6.RP.A.3.a**

Launch

Arrange students in groups of 2. Give students access to colored pencils and 5–8 minutes of quiet work time, followed by a whole-class discussion. Invite students to share what they know about robotics, including any experience they may have with building and programming robots.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the table about the competition, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organization, Attention

Student Task Statement

Diego, Elena, and Andre joined different teams for a robotics competition. Each team programmed a robot to travel at a constant rate.

1. Complete each table to show how far each team's robot traveled during the competition.

Diego's Team

time (minutes)	distance traveled (meters)
	1
1	
2	6
4	
5	

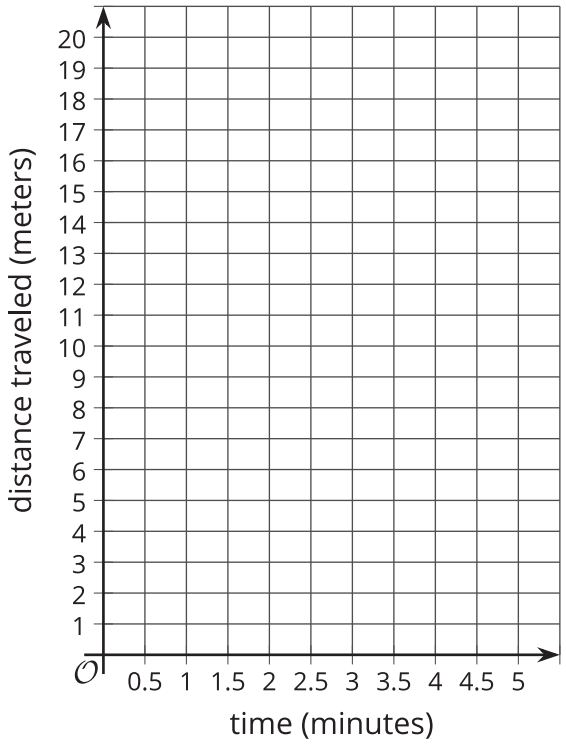
Elena's Team

time (minutes)	distance traveled (meters)
	1
1	
2	
4	11
5	

Andre's Team

time (minutes)	distance traveled (meters)
	1
1	
2	
4	
5	17.5

2. Graph points to show the progress of each robot. Use a different color or symbol for each robot.



3. Let's say that d represents the distance a robot traveled in meters and t represents the time in minutes.
   a. Explain why $d = 3t$ relates the distance and time that Diego's robot traveled.
   b. In this equation, which variable is independent and which one is dependent? Record your answer in the table. Be prepared to explain how you know.
4. Complete the table with equations that can relate distance and time for the other two robots. Make sure that in each equation, the independent and dependent variables are as shown. Be prepared to explain your reasoning.

	equation	independent variable	dependent variable
Diego's team	$d = 3t$		
Elena's team		time	distance
Andre's team		distance	time

Student Response

1. Diego's Team

time (minutes)	distance traveled (meters)
$\frac{1}{3}$	1
1	3
2	6
4	12
5	15

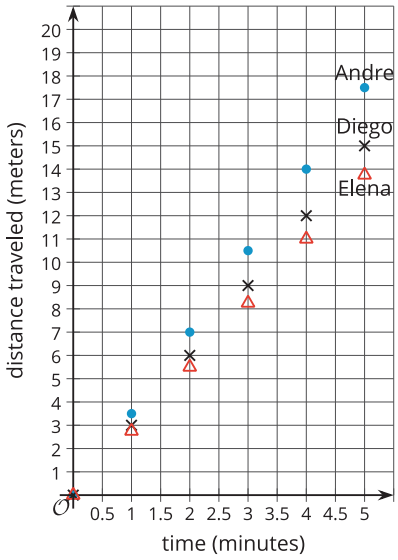
Elena's Team

time (minutes)	distance traveled (meters)
$\frac{4}{11}$	1
1	2.75 or $2\frac{3}{4}$
2	5.5 or $5\frac{1}{2}$
4	11
5	13.75 or $13\frac{3}{4}$

Andre's Team

time (minutes)	distance traveled (meters)
$\frac{2}{7}$	1
1	3.5
2	7
4	14
5	17.5

2. Sample response:



3. a. Sample response: Diego's robot traveled 3 meters in 1 minute, so multiplying the number of minutes by 3 gives the distance in meters.

   b. Time is independent and distance is dependent. Sample reasoning: The distance that Diego's robot travels depends on how long it travels at a constant speed.

4.

	equation	independent variable	dependent variable
Diego's team	$d = 3t$	time	distance
Elena's team	$d = 2.75t$ (or equivalent)	time	distance
Andre's team	$t = \frac{2}{7}d$ (or equivalent)	distance	time

Are You Ready for More?

1. Two trains are traveling toward each other, on parallel tracks. Train A is moving at a constant speed of 70 miles per hour. Train B is moving at a constant speed of 50 miles per hour. The trains are initially 320 miles apart. How long will it take them to meet?

   If you get stuck, consider using the table and adding as many rows as needed.

elapsed time (hours)	distance apart (miles)
0	
1	
2	

2. How long will it take a train traveling at 120 miles per hour to go 320 miles?
3. How are the two problems related?

Extension Student Response

1. $2\frac{2}{3}$ hours, or 2 hours and 40 minutes
2. 2 hours and 40 minutes
3. Sample response: Since trains A and B are moving toward each other, we can add their two speeds to find the rate at which the distance between them decreases. $70 + 50$ is 120, so the combined distance the two trains cover is the same as that of a train going at 120 miles per hour.

Activity Synthesis

The goal of the discussion is to ensure that students understand how the table, graph, and equations represent the situation and make connections across the representations.

Invite students to share their responses to the last two sets of questions and explain their reasoning. In particular, ask them how they identified the independent and dependent variables for the equation for Diego's robot, and how they figured out how to write equations when the independent and dependent variables are specified.

Point out the standard convention of showing the independent variable on the horizontal axis of a graph.

If time permits, consider asking questions such as:

- "How can you tell from the table whose robot traveled the fastest or the slowest?"
- "How can you tell from the graph whose robot traveled the fastest or the slowest?"
- "How can you tell from the equations whose robot traveled the fastest or the slowest?"
- "If distance was the independent variable, how would the equations and graphs be different?"

Lesson Synthesis

In this lesson, students interpret equations with two variables in context. To summarize the key ideas from the lesson, consider asking students:

- "In general, how can we tell from an equation which variable is independent and which is dependent?"
- "How might an equation change if we switched the dependent and independent variables?"
- "Why might we want to rewrite an equation to change the dependent and independent variables?"

17.3 Interpret the Point

Cool-down

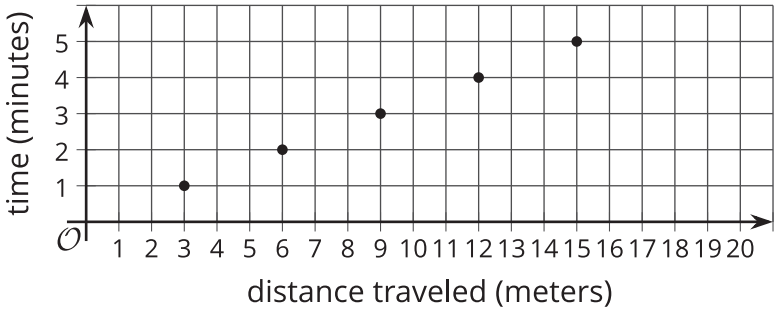
5 min

Standards

Addressing **6.EE.C.9**

Student Task Statement

Noah built a robot that travels at a constant rate. The equation $\frac{1}{3}d = t$ and the graph both represent the relationship between the distance traveled in meters, d, and the travel time in minutes, t.



1. Which variable is independent variable?
2. What does the point $(12, 4)$ represent in this situation?
3. What does the coefficient $\frac{1}{3}$ tell us about the situation?
4. What point on the graph would represent the time it takes the robot to travel $7\frac{1}{2}$ meters?

Student Response

1. Distance, d, is the independent variable.
2. Sample responses:
   - Noah's robot can travel 12 meters in 4 minutes.
   - It takes Noah's robot 4 minutes to travel 12 meters.
3. Sample response: The $\frac{1}{3}$ tells us that it takes the robot $\frac{1}{3}$ minute to travel 1 meter.
4. $(7\frac{1}{2}, 2\frac{1}{2})$. Sample reasoning: $\frac{1}{3}(7\frac{1}{2}) = t, t = 2\frac{1}{2}$

Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 17 Summary

Equations are very useful for solving problems that involve constant speed. Here is an example.

A boat travels at a constant speed of 25 miles per hour.

1. How far can the boat travel in 3.25 hours?
2. How long does it take for the boat to travel 60 miles?

We can write equations to help us answer questions like these.

Let's use t to represent the time in hours and d to represent the distance in miles that the boat travels.

When we know the time and want to find the distance, we can write:

$$d = 25t$$

In this equation, if t changes, d is affected by the change, so t is the independent variable and d is the dependent variable.

This equation can help us find d when we have any value of t. In 3.25 hours, the boat can travel $25(3.25)$ or 81.25 miles.

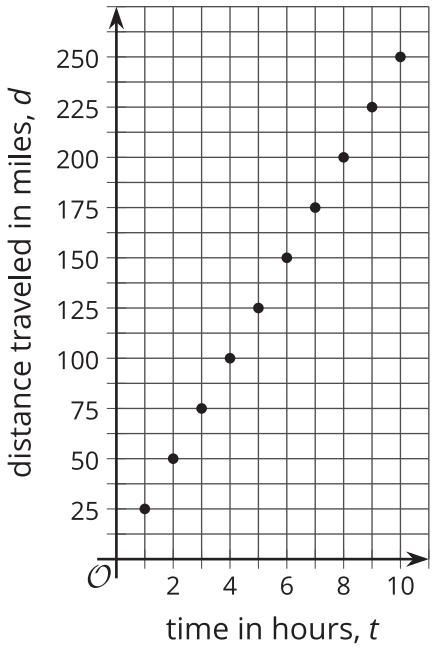
When we know the distance and want to find the time, we can write:

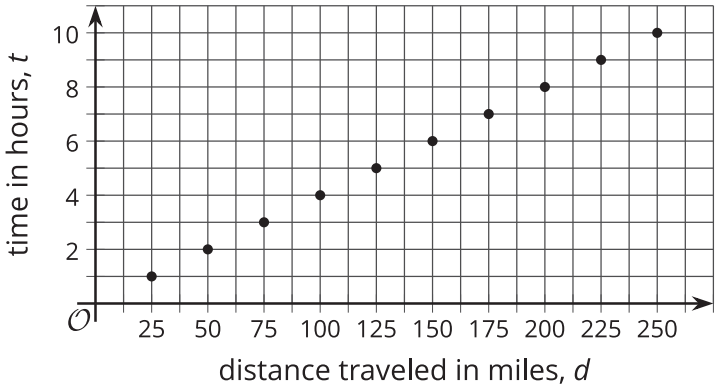
$$t = \frac{d}{25}$$

In this equation, if d changes, t is affected by the change, so d is the independent variable and t is the dependent variable.

This equation can help us find t for any value of d. To travel 60 miles, it will take $\frac{60}{25}$ or $2\frac{2}{5}$ hours.

We can also graph the two equations we wrote to get a picture of the relationship between the two quantities. By convention, the independent variable is represented on the horizontal axis.





Problems about constant speed can also be solved using other strategies, such as by making a table of equivalent ratios.

Lesson 17 Practice Problems

1 Student Task Statement

A car is traveling at a constant speed of 50 miles per hour.

a. Complete the table with the amounts of time it takes the car to travel certain distances, or the distances traveled for certain amounts of time.

b. Write an equation that represents the relationship between the travel distance in miles, d, and the amount of travel time in hours, t.

c. In your equation, which is the dependent variable and which is the independent variable?

time (hours)	distance (miles)
2	
1.5	
t	
	50
	300
	d

Solution

a.

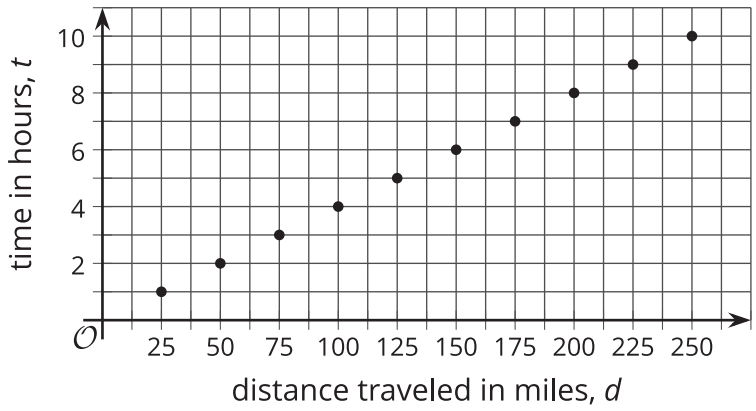
time (hours)	distance (miles)
2	100
1.5	75
t	$50t$
1	50
6	300
$\frac{1}{50}d$	d

b. $d = 50t$

c. t is the independent variable and d is the dependent variable. (If student write $t = \frac{1}{50}d$ in part b, these identification should be switched.)

2 Student Task Statement

The graph represents the amount of time in hours it takes a ship to travel various distances in miles.



a. Write the coordinates of one point on the graph. What does the point represent?

b. What is the speed of the ship in miles per hour?

c. Write an equation that relates the time, t, it takes to travel a given distance, d.

Solution

a. Sample response: $(75, 3)$. This point represents that the ship travels 75 miles in 3 hours.

b. 25 miles per hour

c. $d = 25t$ or $t = \frac{d}{25}$ (or equivalent)

3 from Unit 4, Lesson 15

Student Task Statement

Find a solution to each equation in the list that follows (not all numbers will be used):

a. $2^x = 8$

b. $2^x = 2$

c. $x^2 = 100$

d. $x^2 = \frac{1}{100}$

e. $x^1 = 7$

f. $2^x \cdot 2^3 = 2^7$

g. $\frac{2^x}{2^3} = 2^5$

List: $\frac{1}{10}$ $\frac{1}{3}$ 1 2 3 4 5 7 8 10 16

Solution

a. 3

b. 1
c. 10
d. $\frac{1}{10}$
e. 7
f. 4
g. 8

4 from Unit 4, Lesson 10

Student Task Statement

Select **all** the expressions that are equivalent to $5x + 30x - 15x$.

A. $5(x + 6x - 3x)$
B. $(5 + 30 - 15) \cdot x$
C. $x(5 + 30x - 15x)$
D. $5x(1 + 6 - 3)$
E. $5(x + 30x - 15x)$

Solution

A, B, D

5 from Unit 4, Lesson 15

Student Task Statement

Find the value of each expression if x is 3.

a. $7x^2 - 20$
b. $(x + 4)^3$
c. $2(x + 3^3)$
d. $(7 - x)^2$
e. $0.241 + x^3$

Solution

a. 43
b. 343
c. 60
d. 16

e. 27.241