Unit 2 Family Support Materials

Congruence

In this unit, your student will be learning about triangles and proof. Triangles are the building blocks of geometric figures. Once students understand triangles, they can apply their understanding to quadrilaterals and other shapes.

Students start out with some experiments. You can recreate these experiments at home with different-sized pieces of linguine.

- If I know 2 side lengths, is that enough to describe a unique triangle?
- How about 3 side lengths?
- If I know 2 side lengths, does that describe a unique quadrilateral?
- How about a unique rectangle?

If a set of information seems to work, make a *conjecture*. One conjecture is: 3 side lengths describes a unique triangle. In other words, if 2 triangles have all 3 sides of the same length, then one triangle fits exactly on top of the other. Any pair of figures (such as segments or triangles) in which we can find transformations that take one figure exactly onto the other figure so every part lines up is called *congruent*. So it seems that one way to create 2 triangles that are congruent is to have all 3 pairs of sides congruent. We can try dozen of triangles, and the triangles always seem to fit on top of each other exactly (even the angles!), but how can we be certain that it will work for every possible triangle anyone could ever make? For that, we need a proof that relies on precise definitions.

Proof is how mathematicians take a conjecture, a claim that seems to be true, and turn it into a theorem, a claim we are certain is true. To prove that something is true, every statement must be backed up with a reason. Students are building a list of reasons they can use for proofs in a reference chart. This list includes definitions, assumptions, and theorems they have already proven. Proofs in geometry work like court cases in which lawyers use evidence and case law to make an argument. They also work like arguments at home. Next time your student says you need to buy them something, ask them to prove it. They could use the definition of need and provide convincing evidence of that need, or they might have to adjust their conjecture and provide convincing evidence they deserve something they want instead.

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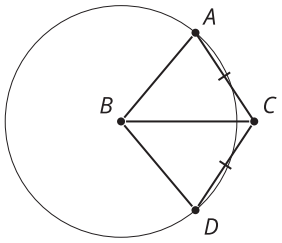
$\overline{AC} \cong \overline{CD}$



Here is a task to try with your student:

1. Write a triangle congruence statement based on the diagram.
2. What information do you know that could help you write a proof?
3. Prove the triangles are congruent.
4. What type of quadrilateral does $ABDC$ have to be?
5. What type of quadrilateral could $ABDC$ possibly be?

Solution:

1. Triangle ABC is congruent to triangle DBC. (Other orders such as triangle BAC is congruent to BDC are okay, but the corresponding letters have to match, so ABC is congruent to BDC is not okay.)
2. Segments AC and DC are congruent, because they're marked on the diagram. Segments AB and DB are congruent, because they're both radii of the same circle.
3. It is given that sides AC and DC are congruent. Sides AB and DB are congruent because they're both radii of the same circle. Side BC is congruent to side BC, because they are the same segment. All 3 pairs of corresponding sides are congruent in triangles ABC and DBC, so the triangles are congruent by the SSS Triangle Congruence Theorem.
4. $ABDC$ has to be a kite since it has 2 pairs of congruent sides and the congruent sides are next to each other.
5. $ABDC$ could be a rhombus if AC and DC are the same length as the radii of the circle.