Finding Cone Dimensions

Let's figure out the dimensions of cones.

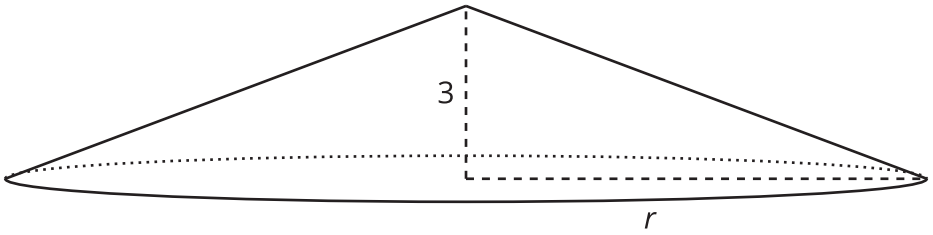
16.1 Math Talk: Thirds

Solve each equation mentally.

- $27 = \frac{1}{3}h$
- $27 = \frac{1}{3}r^2$
- $12\pi = \frac{1}{3}\pi a$
- $12\pi = \frac{1}{3}\pi b^2$

16.2 An Unknown Radius

The volume V of a cone with radius r is given by the formula $V = \frac{1}{3}\pi r^2 h$.



This cone has a height 3 units and radius r units.

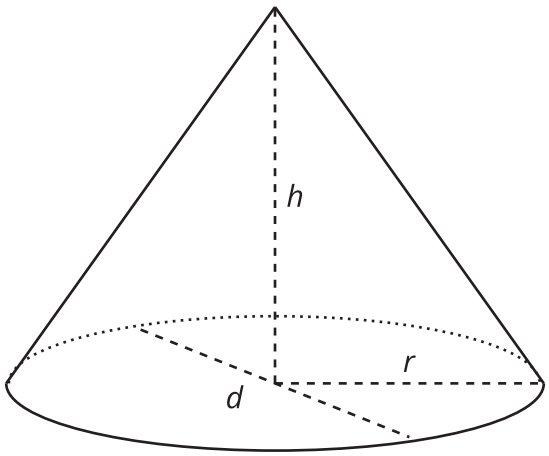
The volume of this cone is given by the equation $V = 64\pi$ cubic units.

This statement is true:

$$64\pi = \frac{1}{3}\pi r^2 \cdot 3.$$

What does the radius of this cone have to be? Explain how you know.

16.3 Cones with Unknown Dimensions

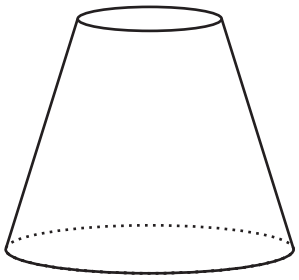


Each row of the table has some information about a particular cone. Complete the table with the missing dimensions.

	diameter (units)	radius (units)	area of the base (square units)	height (units)	volume of cone (cubic units)
a		4		3	
b		8			64π
c			144π	$\frac{1}{4}$	
d	20				200π
e				6	64π
f				3	π

Are you ready for more?

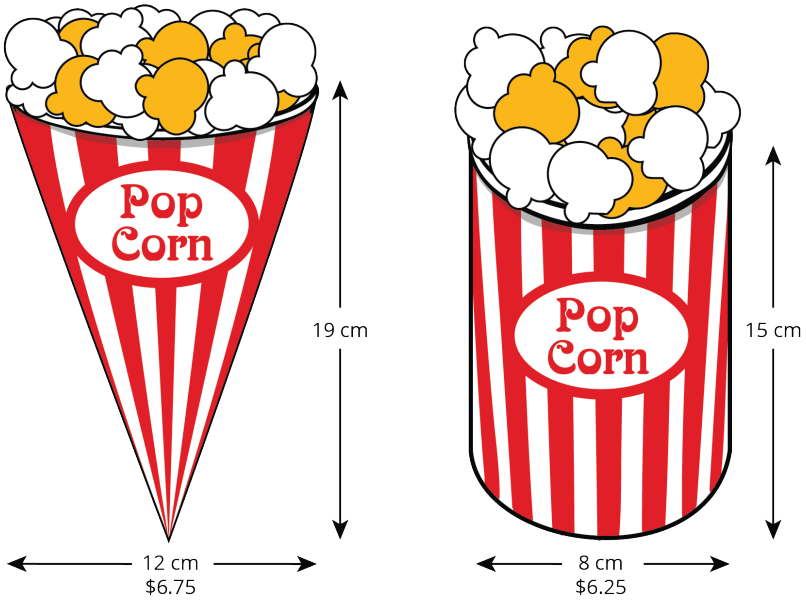
A *frustum* is the result of taking a cone and slicing off a smaller cone using a cut parallel to the base.



Find a formula for the volume of a frustum. (You will have to decide which quantities you are going to include in your formula.)

16.4 Popcorn Deals

A movie theater offers two containers:



Which container is the better value? Use 3.14 as an approximation for π.

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

Lesson 16 Summary

As we saw with cylinders, the volume V of a cone depends on the radius r of the base and the height h:

$$V = \frac{1}{3}\pi r^2 h$$

If we know the radius and height, we can find the volume. If we know the volume and one of the dimensions (either radius or height), we can find the other dimension.

For example, imagine a cone with a volume of 64π cm^3, a height of 3 cm, and an unknown radius r. From the volume formula, we know:

$$64\pi = \frac{1}{3}\pi r^2 \cdot 3$$

Looking at the structure of the equation, we can see that $r^2 = 64$, so the radius must be 8 cm.

Now imagine a different cone with a volume of 18π cm^3, a radius of 3 cm, and an unknown height h. Using the formula for the volume of the cone, we know:

$$18\pi = \frac{1}{3}\pi 3^2 h$$

So, the height must be 6 cm. Can you see why?