Converse of the Pythagorean Theorem

Let's look at the converse of the Pythagorean Theorem.

15.1 An Angle and Side

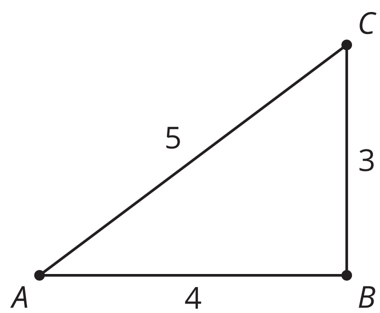
1. Select two positive lengths to represent two side lengths of a triangle.
   a. $a =$
   b. $b =$

   Use a ruler to draw several different possibilities for triangle ABC so that the length of $BC = a$ and the length of $AC = b$.

2. What connection do you see between angle BCA and the length of segment AB in the triangles that you drew?

3. What else do you notice about the length of segment AB?

15.2 Classifying Triangles Based on Side Lengths



1. Is this triangle a right triangle? Explain or show your reasoning.

2. Triangle DEF is constructed so that $DE = 4$, $EF = 3$, and $DF = 6$. What can you say about the angles in triangle DEF? Explain or show your reasoning.

3. Triangle LMN is constructed so that $LM = 4$, $MN = 3$, and $LN = 4.5$. What can you say about the angles in triangle LMN? Explain or show your reasoning.

4. A triangle has sides of length a, b, and c so that $a \le b \le c$.
   a. Make a conjecture about a triangle for which $c^2 < a^2 + b^2$.

   b. Make another conjecture about a triangle for which $c^2 > a^2 + b^2$.

Are you ready for more?

Here is the first part of a proof of the converse of the Pythagorean Theorem.

Step 1: For the sake of contradiction, assume triangle ABC is not a right triangle, but it has sides of length a, b, and c so that $a^2 + b^2 = c^2$.

Step 2: Construct triangle XYZ so that the length of $XY = a$, the length of $YZ = b$, and angle Y is 90 degrees.

Step 3: By the Pythagorean Theorem, $a^2 + b^2 = (XZ)^2$.

Step 4: From our assumption, $a^2 + b^2 = c^2$, too, so $XZ = c$. This means that triangle XYZ has sides of length a, b, and c.

If you can use this information to show that triangle ABC must be a right triangle, then we have arrived at a contradiction to the assumption in step 1. This means that we have proven the converse of the Pythagorean Theorem.

How can you use the information in steps 1 through 4 to show that triangle ABC must be a right triangle?

15.3 Triangle Inequality

1. Triangle ABC is constructed so that $BC = 5$ and $AC = 3$. Is there a maximum length of AB? Explain your reasoning.

2. Is it possible for a triangle to have sides of length 1, 16, and 20? Explain or show your reasoning.

3. A triangle has sides of length a, b, and c. Write an inequality that shows the maximum length for c.

Lesson 15 Summary

The Pythagorean Theorem describes a relationship among the side lengths of a right triangle. In particular, if the sides are labeled a, b, and c with c, as the longest side, then $a^2 + b^2 = c^2$.

The converse of this theorem is also true. In fact, if we know the lengths of all three sides, we can classify the triangle as either acute, right, or obtuse.

- If $c^2 = a^2 + b^2$, then the triangle has a right angle.
- If $c^2 > a^2 + b^2$, then the triangle has an obtuse angle.
- If $c^2 < a^2 + b^2$, then the triangle has only acute angles.

A key reason this is true is due to the relationship between an angle and the side opposite the angle in a triangle. If the two segments making up the angle are constant, then when the angle is made greater, the opposite side must also be greater.

Another relationship among the sides of a triangle is due to limitations from the ways the ends of the segments can be connected. The length of any side of a triangle must be less than the sum of the other two side lengths in the triangle and must be greater than the positive difference between the other two side lengths. For a triangle with side lengths a, b, and c, this can be written as

- $a + b > c$
- $|a - b| < c$

If the third side is too long, then putting the other two sides end to end will not be enough to reach both ends of the third side. If the third side is too short, then putting it at the end of one of the other sides will not be enough to reach the end of the other side.