Unit 1, Lesson 8

Rotation Patterns

Goals

- Draw and label rotations of 180 degrees of a line segment from centers of the midpoint, a point on the segment, and a point not on the segment.
- Generalize (orally and in writing) the outcome when rotating a line segment 180 degrees.
- Identify (orally and in writing) the rigid transformations that can build a diagram from one starting figure.

Learning Targets

- I can describe how to move one part of a figure to another using a rigid transformation.

Lesson Narrative

The purpose of this lesson is for students to observe properties of figures that have been rotated 90 degrees or 180 degrees. They generalize that a line segment rotated 180 degrees about a point is either parallel to the original or lies along the same line (MP8). They also identify rigid transformations on a triangle involving rotations of multiples of 90 degrees. Students explain that lengths on the composite figure must be equal using the property of rigid transformations that corresponding side lengths are the same (MP3). The composite figure in this activity arises in a later unit to support thinking about the Pythagorean theorem.

Math Community
Today, students use sticky notes to document actions in the "Doing Math" sections of the Math Community Chart that they see or hear throughout the lesson. During the Lesson Synthesis, students share what they noticed, and then they suggest additions for the chart as part of the Cool-down. The work today continues to build a foundation for developing math community norms in a later exercise and is the start of students identifying strengths in the actions of their peers.

Standards

Addressing	**8.G.A.1.a, 8.G.A.1.b**
Building Toward	**8.G.A.1.c**

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Materials to Gather

- Math Community Chart: Lesson, Activity 1, Cool-down
- Sticky notes: Activity 1
- Geometry toolkits: Activity 2, Activity 3

Student Facing Learning Goals

Let's rotate figures in a plane.

8.1 Notice and Wonder: Building a Quadrilateral

5 min

Warm-up

Activity Narrative

The purpose of this *Warm-up* is to identify transformations used to build a shape, which will be useful when students perform transformations in a later activity. While students may notice and wonder many things about these figures, identifying transformations used to build a shape and using knowledge of the original figure to classify the new shape are the important discussion points.

This prompt gives students opportunities to see and make use of structure (MP7). The specific structure they might notice is the properties of the isosceles right triangle needed to classify the final shape.

Standards

Addressing **8.G.A.1.a, 8.G.A.1.b**

Instructional Routines

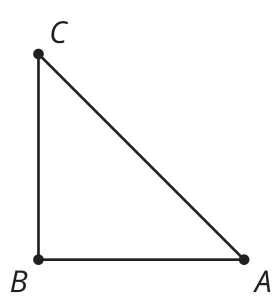
- Notice and Wonder

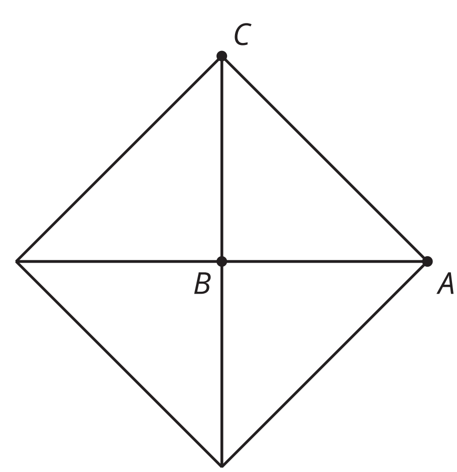
Launch

Arrange students in groups of 2. Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time, and then 1 minute to discuss with their partner.

Student Task Statement

What do you notice? What do you wonder?





Student Response

Students may notice:

- The quadrilateral is made of four triangles.
- Triangle ABC can be rotated to line up with the other triangles.
- Triangle ABC can be reflected to line up with the other triangles.
- A 90-degree rotation of the quadrilateral would line up with the current quadrilateral (the quadrilateral is a square).

Students may wonder:

- Is the quadrilateral a square?
- Is triangle ABC an isosceles right triangle?
- Would rotating any triangle 90 degrees repeatedly create a square?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses without editing or commentary for all to see. If possible, record the relevant reasoning on or near the image. Next, ask students, “Is there anything on this list that you are wondering about now?” Encourage students to observe what is on display and respectfully ask for clarification, point out contradicting information, or voice any disagreement.
If using transformations to build a shape does not come up during the conversation, ask students to discuss this idea.

Math Community
After the *Warm-up*, display the revised Math Community Chart created from student responses in Exercise 3. Tell students that today they are going to monitor for two things:

- “Doing Math” actions from the chart that they see or hear happening.
- “Doing Math” actions that they see or hear that they think should be added to the chart.

Provide sticky notes for students to record what they see and hear during the lesson.

8.2 Rotating a Segment

15 min

Activity Narrative

The purpose of this activity is to allow students to explore special cases of rotating a line segment 180°. In general, rotating a segment 180° produces a parallel segment the same length as the original. This activity also addresses two special cases:

- When the center of rotation is the midpoint, the rotated segment is the same segment as the original, except the vertices are switched.
- When the center of rotation is an endpoint, the segment together with its image form a segment twice as long as the original.

Students experiment with a particular line segment, then make conclusions about what happens when a line segment is rotated 180° in order to generalize for any line segment (MP8).

Watch for how students explain that the 180° rotation of segment CD in the second part of the question is parallel to

CD. Some students may say that they "look parallel" while others might try to reason using the structure of the grid. Students will continue to investigate this in a future lesson, so formal language is not necessary at this time.

Standards

Addressing **8.G.A.1.a**

Building Toward **8.G.A.1.c**

Instructional Routines

- MLR8: Discussion Supports

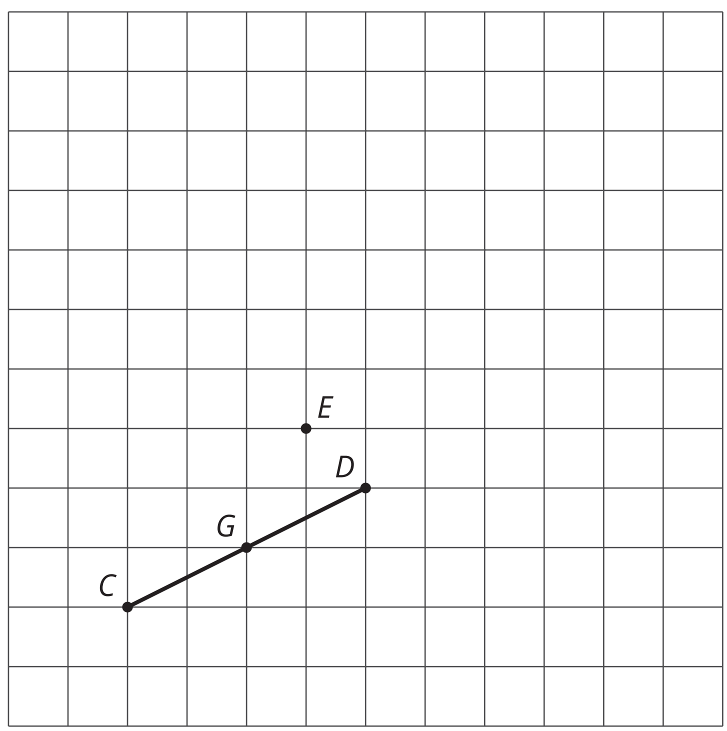
Launch

Arrange students in groups of 2. Provide access to geometry toolkits. Give 3 minutes of quiet work time, followed by sharing with a partner and a whole-class discussion.

Access for Students with Disabilities

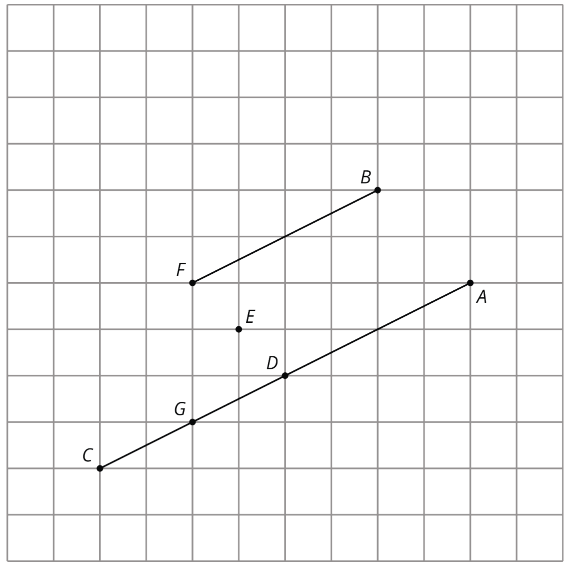
Representation: Access for Perception. Ask students to read each problem aloud to their partner. Students who both listen to and read the information will benefit from extra processing time.
Supports accessibility for: Language, Attention

Student Task Statement



1. Rotate segment CD 180° around point D. Draw its image and label the image of C as A.
2. Rotate segment CD 180° around point E. Draw its image and label the image of C as B and the image of D as F.
3. Rotate segment CD 180° around its midpoint, G. What is the image of C?
4. What happens when you rotate a segment 180° around a point?

Student Response

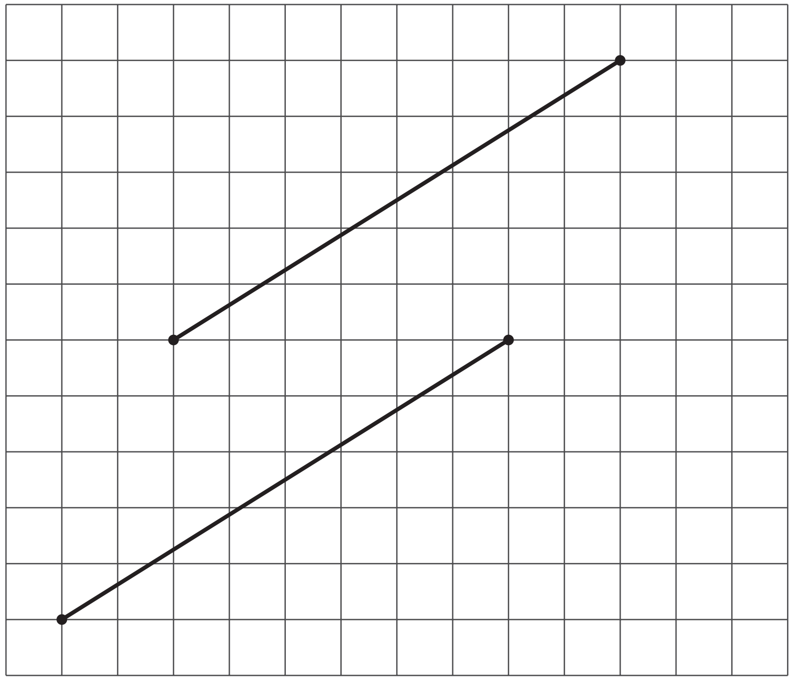
1. 

2. See image.
3. The image of the segment lines up with itself, but the endpoints are switched. D is now where C was and C is where D was.
4. Sample responses:
    - The new segment may change its location, but it remains the same length.
    - The new segment is parallel to the original segment. When the point of rotation is the midpoint of the segment, then the rotated segment is the same as the original (the endpoints trade places).
    - When the point of rotation is an end point of the segment, the image connects to the original to form a segment twice as long.

Building on Student Thinking

Students may be confused when rotating around the midpoint because they think the image cannot be the same segment as the original. Assure students this can occur and highlight that point in the discussion.

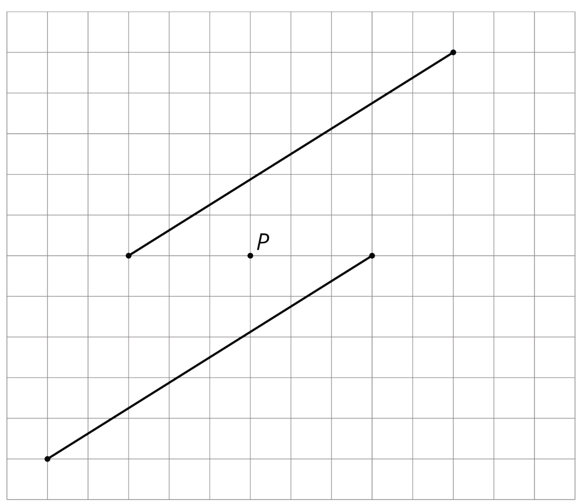
Are You Ready for More?



Here are two line segments. Is it possible to rotate one line segment to the other? If so, find the center of such a rotation. If not, explain why not.

Extension Student Response

Yes



Activity Synthesis

Ask students why it is not necessary to specify the direction of a 180-degree rotation (because a 180-degree clockwise rotation around point P has the same effect as a 180-degree counterclockwise rotation around P). Invite groups to share their responses. Ask the class if they agree or disagree with each response. When there is a disagreement, have students discuss possible reasons for the differences.

Three important ideas that emerge in the discussion are:

- Rotating a segment 180-degrees around a point that is not on the original line segment produces a parallel segment the same length as the original.
- When the center of rotation is the midpoint, the rotated segment is the same segment as the original, except the vertices are switched.
- When the center of rotation is an endpoint, the segment together with its image form a segment twice as long.

If any of the ideas above are not brought up by the students during the class discussion, be sure to make them known.

Access for English Language Learners

MLR8 Discussion Supports. Invite students to repeat their reasoning using mathematical language: "Can you say that again, using the phrase 'line segment'?"

Advances: Speaking, Listening

8.3 A Pattern of Four Triangles

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students use rotations to build a pattern of triangles that form an interesting pattern.

Triangle ABC can be mapped to each of the three other triangles in the pattern with a single rotation. As students work on the first three questions, watch for any students who see that a single rotation can take triangle ABC to CDE. The center for the rotation is not drawn in the diagram (it is the intersection of segment AE and segment CG). For students who finish early, prompt them to look for a single transformation taking ABC to each of the other triangles.

This is the first time Math Language Routine 1: *Stronger and Clearer Each Time* is suggested in this course. In this routine, students are given a thought-provoking question or prompt and asked to create a first draft response in writing. It is not necessary that students finish this draft before moving to the structured partner meetings step. Students then meet with 2–3 partners to share and refine their response through conversation. While meeting, listeners ask questions such as, “What did you mean by . . .?” and “Can you say that another way?” Finally, students write a second draft of their response reflecting ideas from partners, and improvements on their initial ideas. Students should be encouraged to incorporate any good ideas and words they got from their partners to make their second draft stronger and clearer.

Access for English Language Learners

This activity uses the *Stronger and Clearer Each Time* math language routine to advance writing, speaking, and listening as students refine mathematical language and ideas.

Standards

Addressing **8.G.A.1.a, 8.G.A.1.b**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

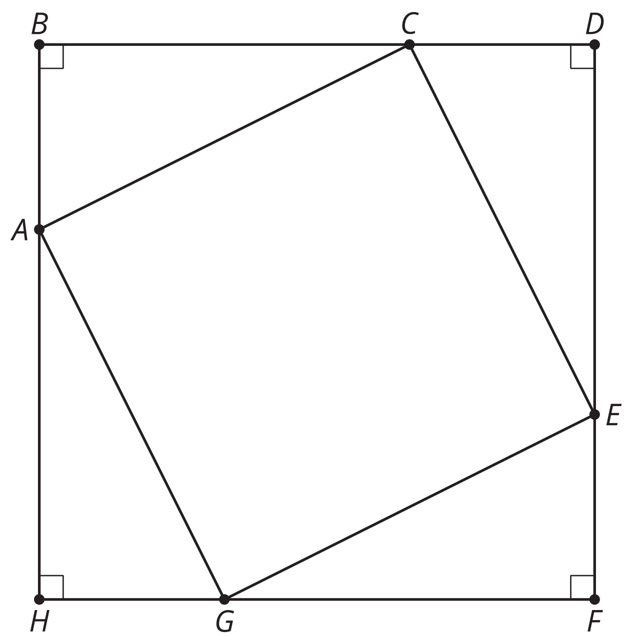
Arrange students in groups of 2–4. Provide access to geometry toolkits.

Access for Students with Disabilities

Representation: Internalize Comprehension. Use color coding and annotations to highlight connections between representations in a problem. For example, color code triangles ABC and CDE to help make connections to work in a previous activity.
Supports accessibility for: Visual-Spatial Processing

Student Task Statement

You can use rigid transformations of a figure to make patterns. Here is a diagram built with three different transformations of triangle ABC.



1. Describe a rigid transformation that takes triangle ABC to triangle CDE.
2. Describe a rigid transformation that takes triangle ABC to triangle EFG.
3. Describe a rigid transformation that takes triangle ABC to triangle GHA.
4. Do segments AC, CE, EG, and GA all have the same length? Explain your reasoning.

Student Response

1. Sample responses:
   - Translate point B to point D, then rotate 90 degrees clockwise using D as center.
   - Rotate counterclockwise using C as center until segment CA matches up perfectly with segment CE, then rotate 180 degrees using the midpoint of segment CE as center.
2. Sample responses:
   - Translate B to F and then rotate 180 degrees with center F.
   - Translate so segment AC matches up with segment GE, then rotate 180 degrees with the midpoint of segment GE as center of rotation.

3. Sample responses:
    - Translate B to H and then rotate 90 degrees counterclockwise with center H.
    - Rotate with center A so that segment AC matches up with segment AG, then rotate 180 degrees with the midpoint of segment AG as center.
4. Yes. Sample reasoning: The size and shape of triangle ABC did not change under the rigid transformation. Segment AC can be matched up exactly with segments CE, EG, and GA so that the lengths of these segments are all the same.

Activity Synthesis

Use *Stronger and Clearer Each Time* to give students an opportunity to revise and refine their response to "Do segments AC, CE, EG, and GA all have the same length? Explain your reasoning." In this structured pairing strategy, students bring their first draft response into conversations with 2–3 different partners. They take turns being the speaker and the listener. As the speaker, students share their initial ideas and read their first draft. As the listener, students ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing.

If time allows, display these prompts for feedback:

- "_____ makes sense, but what do you mean when you say . . . ?"
- "Can you describe that another way?"
- "How do you know . . . ? What else do you know is true?"

Close the partner conversations and give students 3–5 minutes to revise their first draft. Encourage students to incorporate any good ideas and words they got from their partners to make their next draft stronger and clearer.

As time allows, invite students to compare their first and final drafts. Select 2–3 students to share how their drafts changed and why they made the changes they did.

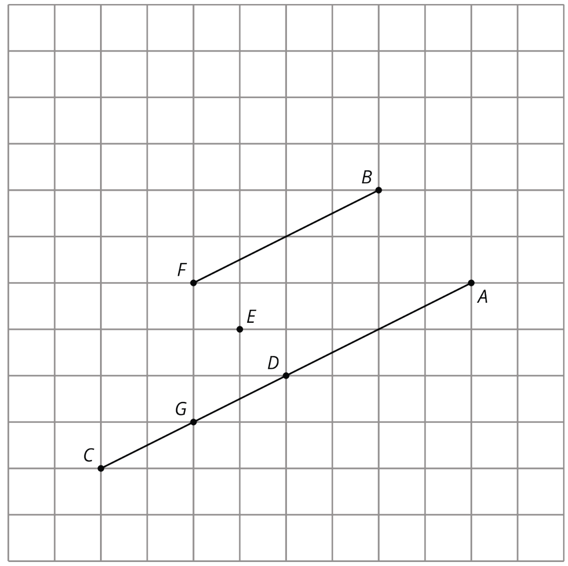
Listen for student thinking that incorporates the following ideas and invite them to share:

- The hypotenuse of each triangle is a corresponding side to the hypotenuse in all of the other triangles.
- Rotations are a type of rigid transformation, which means that lengths of corresponding sides will stay the same.
- Quadrilateral $CAGE$ must be a rhombus because its side lengths are all the same.

If no students suggest that quadrilateral $CAGE$ must be a rhombus, ask students what type of quadrilateral $CAGE$ must be. If any students claim that $CAGE$ must be a square, leave this as an open question for now, as this will be revisited in a later lesson.

Lesson Synthesis

Ask students to describe the possible outcomes when a line segment is rotated 180 degrees. Consider displaying this image for all to see to facilitate the discussion:



- When the center of rotation is not on the line segment, the image is a parallel line segment.
- When the center of rotation is an endpoint, the image is a segment that is in line with the original.
- When the center of rotation is the midpoint, the image is the same as the original.

Math Community
Invite 2–3 students to share what “Doing Math” actions they noticed. Record and display their responses for all to see, such as by adding check marks to any already listed items or adding new items near the chart for the class to consider adding. Next, give students 1–2 minutes with a partner to discuss any changes or revisions they think the chart needs. Tell students they can suggest revisions during the *Cool-down*.

8.4 Is It a Rotation?

Cool-down

5 min

Standards

Addressing **8.G.A.1.a, 8.G.A.1.b**

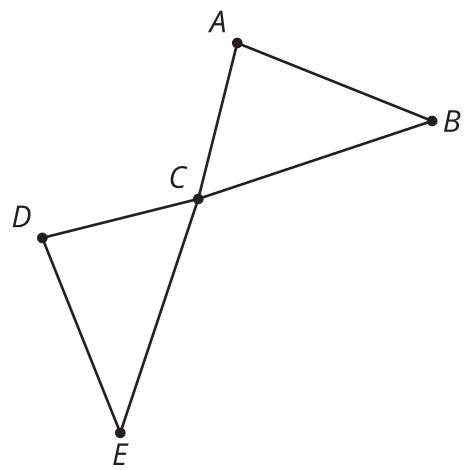
Launch

Provide access to tracing paper.

Math Community
Before distributing the *Cool-downs*, display the Math Community Chart and the community building question “What additions or revisions would you make to the Math Community Chart?” Ask students to respond to the question after completing the *Cool-down* on the same sheet. After collecting the *Cool-downs*, identify themes from the community building question. Use them to add to or revise the Math Community Chart before Exercise 5.

Student Task Statement

Triangle ABC is rotated 180° around point C. Will the image line up with triangle CDE? Explain how you know.



Student Response

No. Sample response: If triangle CDE was a 180° rotation of triangle ABC, then line segment AB would be parallel to line segment DE.

Responding to Student Thinking

Points to Emphasize
If students struggle with rotating a figure 180 degrees about a point, in an upcoming lesson, emphasize the outcomes when rotating a line segment 180 degrees. For example, in the activity referred to here, invite students to share their diagrams after rotating line segment AD around point C by 180 degrees.

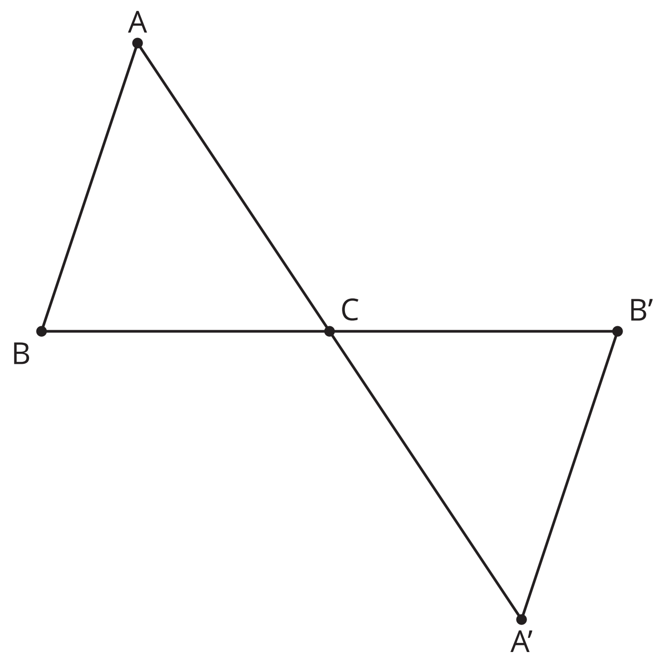
Grade 8, Unit 1, Lesson 9, Activity 3 Let's Do Some 180s

Lesson 8 Summary

When we apply a 180-degree rotation to a line segment, there are several possible outcomes:

- The image of the segment maps is the same as the original (if the center of rotation is the midpoint of the segment).
- The image of the segment overlaps with the segment and lies on the same line (if the center of rotation is a point on the segment).
- The image of the segment does not overlap with the segment and is parallel to the original segment (if the center of rotation is *not* on the segment).

This can also tell us important information about a figure that has been rotated. In this example, triangle ABC has been rotated 180 degrees with point C as the center of rotation. If we think of side AB as a line segment, then we know that its image $A'B'$ must be parallel to it. If we think of side BC as a line segment, then we know that its image $B'C$ must be along the same line.


A
C
B′
B
A′

Lesson 8 Practice Problems

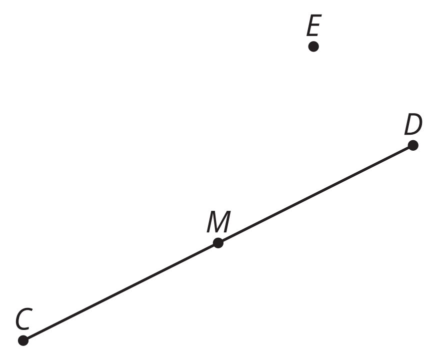
1 Student Task Statement

For the figure shown here,

a. Rotate segment CD 180° around point D.

b. Rotate segment CD 180° around point E.

c. Rotate segment CD 180° around point M.



Solution

a. The segment is attached at point D and is an extension of segment CD.

b. The segment is above point E and is parallel to segment CD.

c. The segment is identical to segment CD.

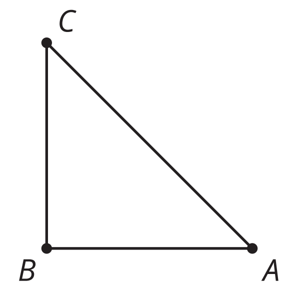
2 Student Task Statement

Here is an isosceles right triangle. Draw these three rotations of triangle ABC together.

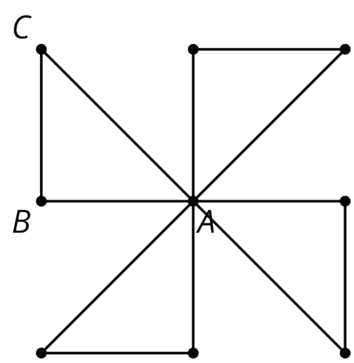
a. Rotate triangle ABC 90° clockwise around A.

b. Rotate triangle ABC 180° around A.

c. Rotate triangle ABC 270° clockwise around A.



Solution

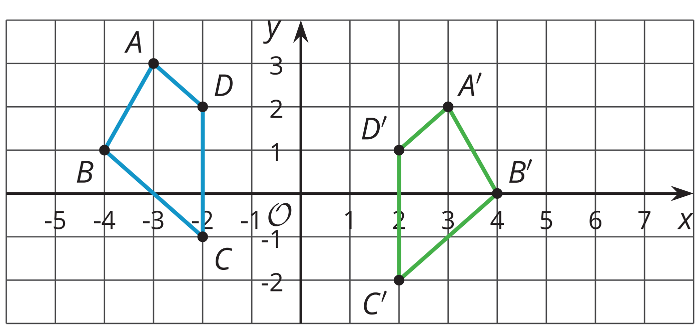


3 from Unit 1, Lesson 5

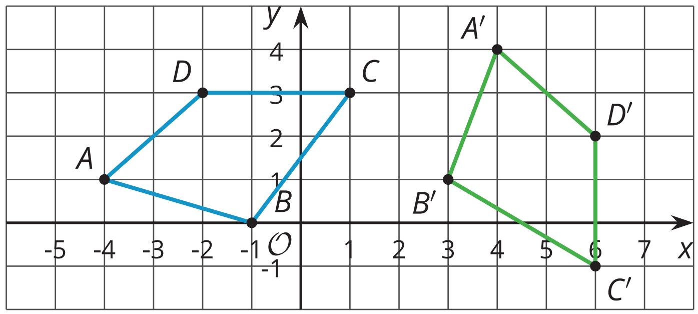
Student Task Statement

Each graph shows two polygons $ABCD$ and $A'B'C'D'$. In each case, describe a sequence of transformations that takes $ABCD$ to $A'B'C'D'$.

a.



b.



Solution

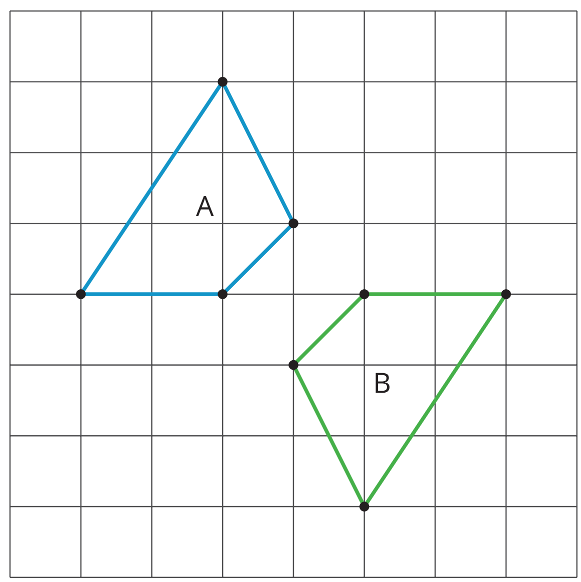
a. Sample response: Reflect $ABCD$ over the y-axis, and then translate down 1.

b. Sample response: Rotate $ABCD$ 90 degrees clockwise with center B: $(-1, 0)$ and then translate $(-1, 0)$ to $(3, 1)$.

4 from Unit 1, Lesson 4

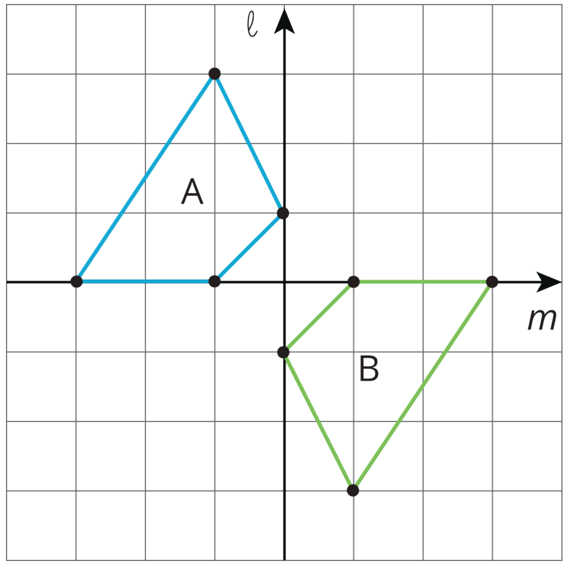
Student Task Statement

Lin says that she can map Polygon A to Polygon B using *only* reflections. Do you agree with Lin? Explain your reasoning.



Solution

I agree with Lin. Sample reasoning: If Polygon A is reflected first over the vertical line ℓ and then over the horizontal line m, this takes Polygon A to Polygon B.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A
m
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