Representing Situations with Inequalities

Let's use inequalities to represent constraints in situations.

1.1 What Do Those Symbols Mean?

1. Match each inequality to the meaning of a symbol within it. Read the inequality from left to right.

   a. $h > 50$
   b. $h \leq 20$
   c. $30 \geq h$

   - less than or equal to
   - greater than
   - greater than or equal to

2. Is 25 a solution to any of the inequalities? Which one(s)?

3. Is 40 a solution to any of the inequalities? Which one(s)?

4. Is 30 a solution to any of the inequalities? Which one(s)?

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1.2 Planning the Senior Ball

Seniors in a student council of a high school are trying to come up with a budget for the Senior Ball. Here is some information they have gathered:

- Last year, 120 people attended. It was a success and is expected to be even bigger this year. Anywhere up to 200 people might attend.
- There needs to be at least 1 chaperone for every 20 students.
- The ticket price can not exceed $20 per person.
- The revenue from ticket sales needs to cover the cost of the meals and entertainment, and also make a profit of at least $200 to be contributed to the school.

Here are some inequalities the seniors wrote about the situation. Each letter stands for one quantity in the situation. Determine what is meant by each letter.

- $t \leq 20$
- $120 \leq p \leq 200$
- $pt - m \geq 200$
- $c \geq \frac{p}{20}$

Are you ready for more?

Kiran says we should add the constraint $t \geq 0$.

1. What is the reasoning behind this constraint?
2. What other "natural constraint" like this should be added?

1.3 Elevator Constraints

An elevator in a skyscraper can hold at most 15 boxes. For safety, the car can carry a maximum of only 1,500 kg. A large box weighs 70 kg and a small box weighs 35 kg. The person riding in the elevator to deliver the boxes also weighs 70 kg.

1. Write as many equations and inequalities as you can think of to represent the constraints in this situation. Be sure to specify the meaning of any letters that you use. (Avoid using the letters z, m, or g.)

2. Trade your work with a partner and read each other's equations and inequalities.
   a. Explain to your partner what you think their statements mean, and listen to their explanation of yours.
   b. Make adjustments to your equations and inequalities so that they are communicated more clearly.

3. Rewrite your equations and inequalities so that they would work for a different building where:
   - an elevator car can hold at most z boxes.
   - each car can carry a maximum of m kilograms.
   - the person delivering the boxes weighs g kg.

Lesson 1 Summary

We have used equations and the equal sign to represent relationships and constraints in various situations. Not all relationships and constraints involve equality, however.

In some situations, one quantity is, or needs to be, greater than or less than another. To describe these situations, we can use inequalities and symbols such as $<, \leq, >$, or $\geq$.

When working with inequalities, it helps to remember what the symbol means, in words. For example:

- $100 < a$ means "100 is less than a."
- $y \leq 55$ means "y is less than or equal to 55," or "y is not more than 55."
- $20 > 18$ means "20 is greater than 18."
- $t \geq 40$ means "t is greater than or equal to 40," or "t is at least 40."

These inequalities are fairly straightforward. Each inequality states the relationship between two numbers ($20 > 18$), or it describes the limit or boundary of a quantity in terms of a number ($100 < a$).

Inequalities can also express relationships or constraints that are more complex. Here are some examples:

- The area of a rectangle, A, with a length of 4 meters and a width of w meters is no more than 100 square meters.

  $$A \leq 100$$
  $$4w \leq 100$$

- To cover all the expenses of a musical production each week, the number of weekday tickets sold, d, and the number of weekend tickets sold, s, must be greater than 4,000.

  $$d + s > 4{,}000$$

- Elena would like the number of hours she works in a week, h, to be more than 5 but no more than 20.

  $$h > 5$$
  $$h \leq 20$$

- The total cost, T, of buying a adult shirts and c child shirts must be less than 150. Adult shirts are \$12 each, and child shirts are \$7 each.

  $$T < 150$$
  $$12a + 7c < 150$$

In upcoming lessons, we'll use inequalities to help us solve problems.