Unit 2, Lesson 11

Side-Side-Angle (Sometimes) Congruence

Goals

- Generate examples and counterexamples of Side-Side-Angle triangle congruence (using words and other representations).

Learning Targets

- I know Side-Side-Angle does not guarantee triangles are congruent.

Lesson Narrative

This lesson is optional because it is additional practice not all students will need.

In this lesson, students study the ambiguous case of triangle congruence. Students know that two pairs of corresponding sides are congruent and a pair of corresponding angles not between the two sides are congruent. They create triangles with this ambiguous information and notice that multiple triangles can be produced with the same information. They then study the case in which the longer side is known to be across from the given angle, which is not ambiguous. Finally, students practice recognizing situations in which they can or can't determine if two triangles are congruent given information about two pairs of corresponding sides and one pair of corresponding angles. Students are looking for structure both as they build cases and as they apply their reasoning to new problems (MP7).

Technology isn't required for this lesson, but there are opportunities for students to choose to use appropriate technology to solve problems. We recommend making technology available.

Standards

Building On **HSG-CO.B.8**

Instructional Routines

- 5 Practices
- Draw It
- MLR2: Collect and Display
- Notice and Wonder

Required Materials

Materials to Gather

- 1-inch strips cut from card stock with evenly spaced holes: Activity 2
- Dried linguine pasta : Activity 2, Activity 3
- Metal paper fasteners: Activity 2
- Tools for creating a visual display: Activity 3

Materials to Copy

- Ambiguously Ambiguous Handout (1 copy for every 30 students): Activity 3
- Ambiguously Ambiguous Answer Key (1 copy for every 0 students): Activity 3

Student Facing Learning Goals

Let's explore triangle congruence criteria that are ambiguous.

11.1 Notice and Wonder: Congruence Fail

Warm-up

5 min

Activity Narrative

The purpose of this *Warm-up* is to elicit the idea that not every set of three pairs of congruent corresponding parts will guarantee triangle congruence, which will be useful when students explore Side-Side-Angle Triangle Congruence in a later activity. While students may notice and wonder many things about these statements and images, the fact that the triangles are not congruent despite having so many corresponding congruent parts is the important discussion point.

Standards

Building On **HSG-CO.B.8**

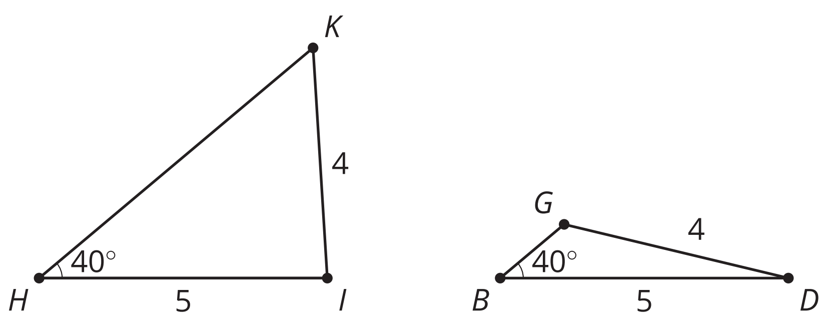
Instructional Routines

- Notice and Wonder

Launch

First, display the congruence statements without the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time and then 1 minute to discuss with their partner the things they notice.

Next, display the image for all to see. Give students 1 minute to add to their lists.



Student Task Statement

What do you notice? What do you wonder?

In triangles GBD and KHI:

- Angle GBD is congruent to angle KHI.
- Segment BD is congruent to segment HI.
- Segment DG is congruent to segment IK.

Student Response

Things students may notice:

- Two pairs of corresponding sides are congruent.
- One pair of corresponding angles is congruent.

- The triangles don't look congruent.
- Triangle BGD looks smaller than the other triangle.

Things students may wonder:

- Are the triangles supposed to be congruent?
- Should triangles with two pairs of corresponding sides and one pair of corresponding angles be congruent?
- Why aren't the triangles congruent?
- How did you make those triangles?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After all responses have been recorded without commentary or editing, ask students, "Is there anything on this list that you are wondering about now?" Encourage students to respectfully disagree, ask for clarification, or point out contradicting information. If students don't express surprise that there are so many congruent parts yet the image shows triangles that are not congruent, ask students to discuss this idea.

11.2 Dare to Be (Even More) Different

10 min

Activity Narrative

There is a digital version of this activity.

In this activity, students are arranged in groups of 3 and given side lengths and an angle measure. They are encouraged to make triangles that do not look like one another's. Given an angle and two sides that are not both adjacent to the given angle, there are three possible triangles that students can make.

As students negotiate how they will make the triangles, listen for different strategies students have for making different triangles. Monitor for students who complete the task in these ways, from least to most efficient:

- Build different triangles by trial and error.
- Draw the arc for the nonadjacent side to see if there are multiple triangles possible.

Standards

Building On **HSG-CO.B.8**

Instructional Routines

- 5 Practices
- Draw It
- MLR2: Collect and Display

Launch

Arrange students in groups of 3.

Select students with different strategies, such as those described in the *Activity Narrative*, to share later.

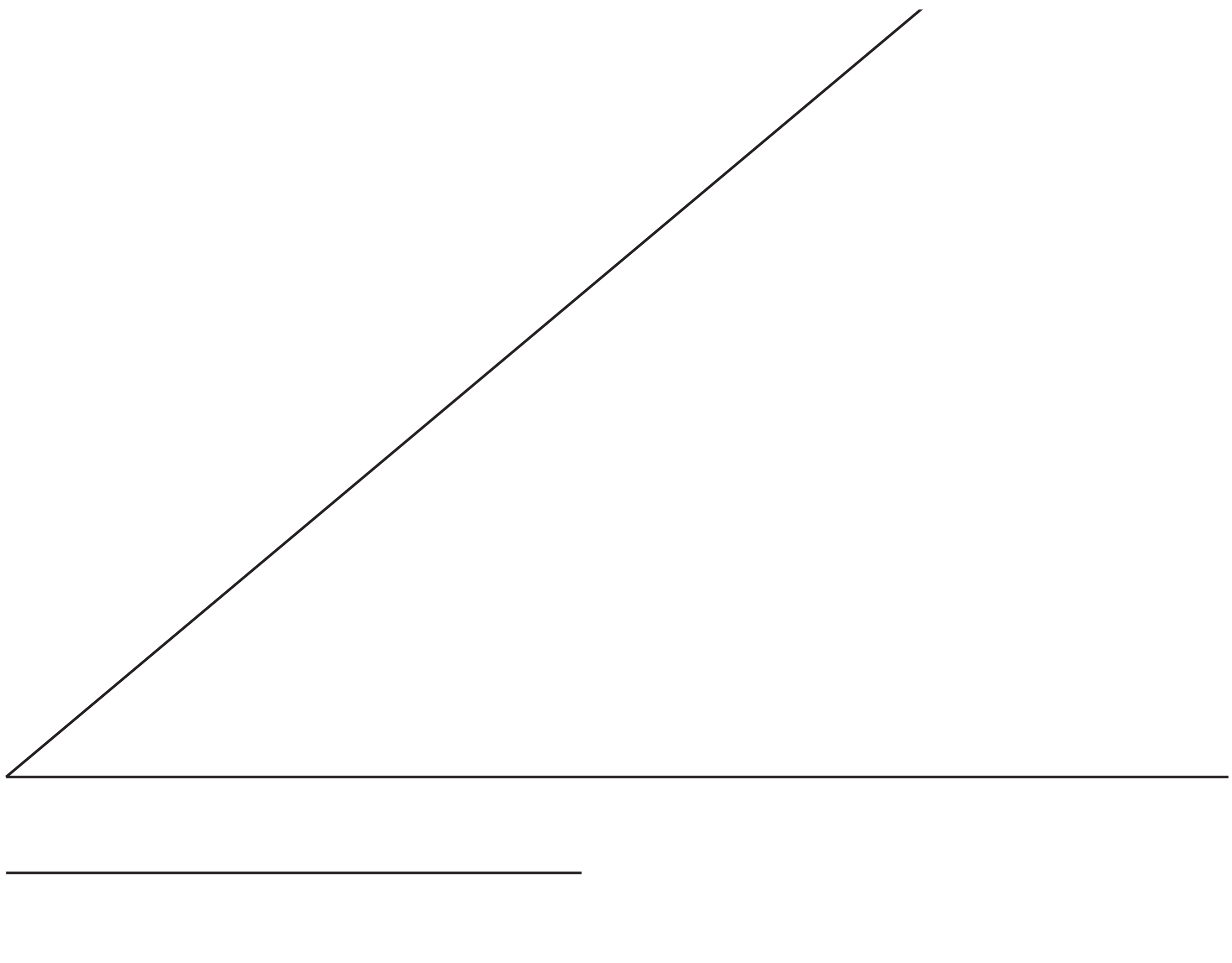
Access for English Language Learners

MLR2 Collect and Display. Circulate to listen for and collect the language students use as they complete the activity. On a visible display, record words and phrases, such as “acute,” “obtuse,” or “adjacent.” Invite students to borrow language from the display as needed, and update it throughout the lesson.
Advances: Conversing, Reading

Access for Students with Disabilities

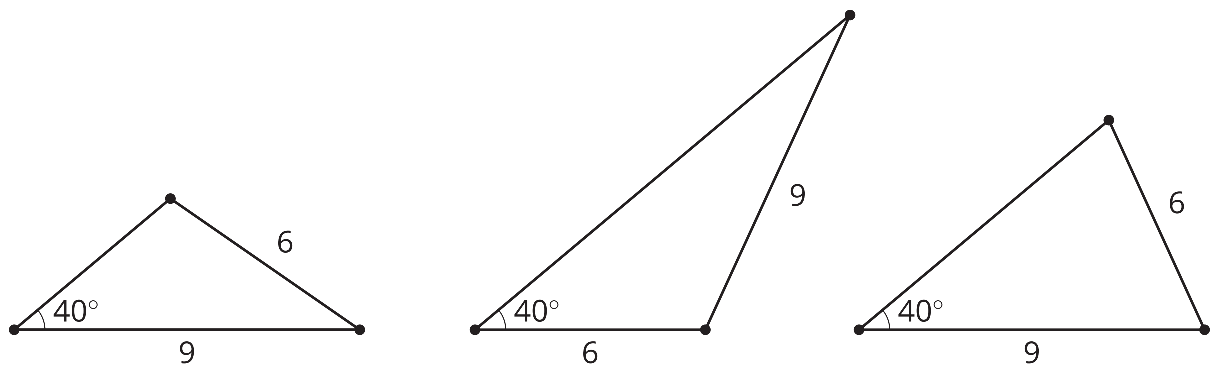
Engagement: Develop Effort and Persistence. Check in and provide each group with feedback that encourages collaboration and community. For example, identify the different strategies students in the group are using to create their triangles.
Supports accessibility for: Social-Emotional Functioning, Organization

Student Task Statement



Copy these segments, and use them to make a triangle using the given angle so that the given angle is *not* between the 2 given sides. Draw your triangle on tracing paper. Try to make your triangle different from the triangles drawn by the other people in your group.

Student Response



Building on Student Thinking

If a group of students decides it is only possible to make one or two different triangles, encourage them to list all the possible ways to order the given pieces and check that they have tried all of them. (angle, short side, long side or angle, long side, short side)

Activity Synthesis

Invite previously selected students to share their strategies. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*. If possible, record and display their work for all to see. If no student drew the arc, demonstrate this strategy to show the two possible triangles.

Connect the different responses to the learning goals by asking questions, such as:

- "Is it possible to prove two triangles are congruent if all you know is two pairs of corresponding sides and one pair of corresponding angles are congruent?" (No, we have three different triangles that fit those criteria.)
- "Is one of these diagrams more convincing?" (No, they all provide counterexamples. Yes, the arcs diagram shows why it's possible to make different triangles with the same measurements.)

11.3 Ambiguously Ambiguous?

20 min

Activity Narrative

In the previous activity, students may have noticed that once they decided which side would be adjacent to the given angle, they could sometimes make two triangles and sometimes make one triangle. This may lead to questions about whether knowing an angle, a side, and another side (moving clockwise around the triangle) could be enough information to guarantee we can make a congruent copy of the triangle. This activity formalizes that question and answer by giving students instructions to make different triangles and by exploring which instructions always lead to a single unique triangle and which are ambiguous.

Making dynamic geometry software available gives students an opportunity to choose appropriate tools strategically (MP5).

Standards

Building On **HSG-CO.B.8**

Instructional Routines

- Draw It

Launch

Identify a way for students to compare all the examples of a given triangle. For example, invite students to place all the triangles labeled ABC in a single visual display.

Arrange students in 8 groups. Provide each group with tools to create a visual display. Assign a different card to each group.

Student Task Statement

Your teacher will give you some sets of information.

- For each set of information, make a triangle using that information.
- If you think you can make more than one triangle, make more than one triangle.
- If you think you can't make any triangles, note that.

When you are confident they are accurate, create a visual display.

Student Response

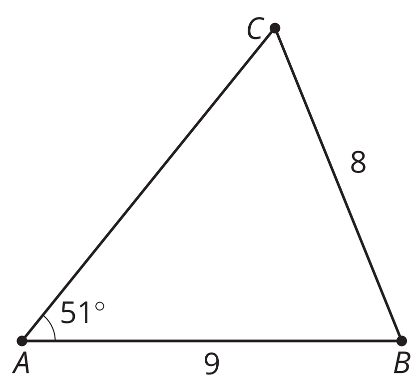
One possible triangle: triangles ABC, GHI, JKL, PQR

Two possible triangles: triangles DEF, MNO, STU, VWX

See the blackline master for copies of the triangles students should construct.

Are You Ready for More?

Triangle ABC is shown. Use your straightedge and compass to construct a new point D on line AC so that the length of segment BD is the same as the length of segment BC.

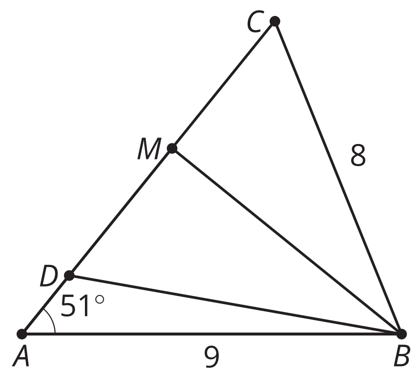


Now use the straightedge and compass to construct the midpoint of CD. Label that midpoint M.

1. Explain why triangle ABM is a right triangle.
2. Explain why knowing the angle at A and the side lengths of AB and BC was not enough to define a unique triangle, but knowing the angle at A and the side lengths of AB and BM would be enough to define a unique triangle.

Extension Student Response

Sample responses:



1. The points M and B are each equidistant from points C and D, so they both lie on the perpendicular bisector of CD. Therefore, line BM is the perpendicular bisector of CD, and thus, angle AMB is a right angle.
2. The circle of radius BC centered at B intersected line AC twice, but the circle of radius BM centered at B intersects line AC only once. Therefore, there is only one position that point M can be at on segment AC, making a unique triangle.

Activity Synthesis

Display the following prompt: "When it are given that two pairs of corresponding sides are congruent and a pair of corresponding angles that are *not* between the sides are congruent, that is enough to guarantee triangle congruence if ________, but not enough information if ________."

Invite students to do a gallery walk and determine how to fill in the blanks. (the longer side is opposite the angle; the shorter side is opposite the given angle)

Note that triangles GHI and STU have the same side lengths and angle measures, just with a different ordering. Comparing these two cases might help students who are struggling to see the difference.

"Only one triangle can be made—and triangle congruence is guaranteed—when we know that the longer of the two given sides is opposite the given angle." Add this theorem to the display of triangle congruence theorems.

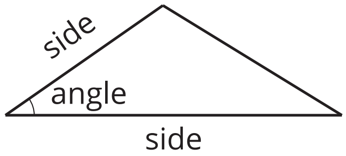
Lesson Synthesis

The goal of this discussion is for all students to understand this statement: "When it is given that two pairs of corresponding sides are congruent and a pair of corresponding angles that are *not* between the sides are congruent, that is enough to guarantee triangle congruence if the longer side is opposite the given angle, but not enough information if the shorter side is opposite the given angle."

Encourage students to sketch pictures of the ambiguous case, and the unambiguous case, and label them in ways that help them understand. Invite students to share their sketches with the class, and work as a class to annotate and understand the sketches. Make sure to discuss the case of the right triangle and how we know the hypotenuse must be the longest side.

A possible sketch might look like this:

Unique!



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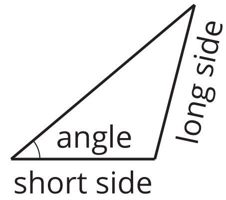

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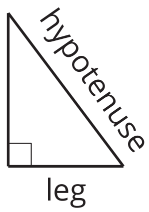

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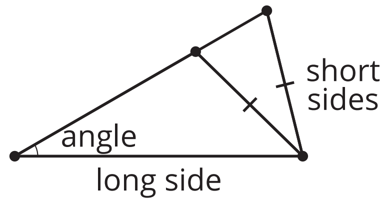


11.4 Are They Ambiguous?

Cool-down

5 min

Standards

Building On HSG-CO.B.8

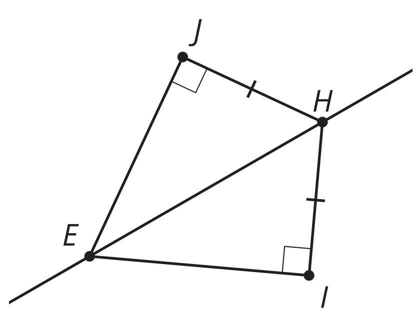
Student Task Statement

Label each example with one of the statements:

- The triangles must be congruent.
- The triangles might not be congruent.

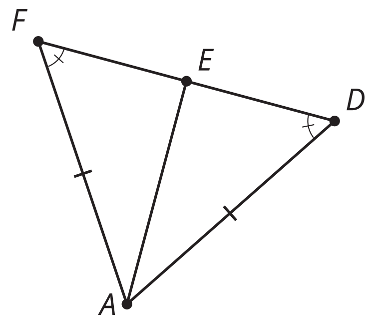
1. triangle EJH and triangle EIH

   $\overline{HJ} \perp \overline{JE}, \overline{HI} \perp \overline{IE}, \overline{HJ} \cong \overline{HI}$



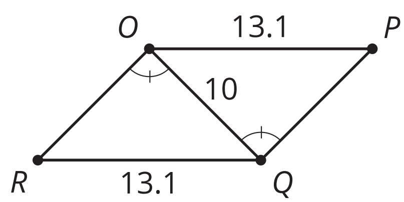
2. triangle AFE and triangle ADE

$\overline{AF} \cong \overline{AD}, \angle F \cong \angle D$



3. triangle OQR and triangle QOP

   $\angle ROQ \cong \angle PQO$



Student Response

1. The triangles must be congruent.
2. The triangles might not be congruent.
3. The triangles must be congruent.

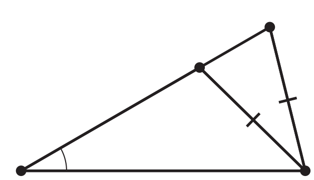
Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

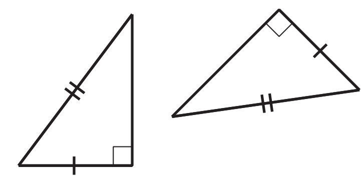
Lesson 11 Summary

Imagine we know triangles have 2 pairs of corresponding, congruent side lengths and 1 pair of corresponding, congruent angles that is not between the given sides. What can we conclude?

Sometimes this is not enough information to determine that the triangles made with those measurements are congruent. These triangles have 2 pairs of congruent sides and 1 pair of congruent angles, but they are not congruent triangles.



If the longer of the 2 given sides is opposite the given angle, though, that does guarantee congruent triangles. In a right triangle, the longest side is always the hypotenuse. If we know the hypotenuse and the leg of a right triangle, we can be confident they are congruent.



Lesson 11 Practice Problems

1 Student Task Statement

Which of the following criteria *always* proves triangles congruent? Select **all** that apply.

A. 3 congruent angles

B. 3 congruent sides

C. Corresponding congruent Side-Angle-Side

D. Corresponding congruent Side-Side-Angle

E. Corresponding congruent Angle-Side-Angle

Solution

B, C, E

2 Student Task Statement

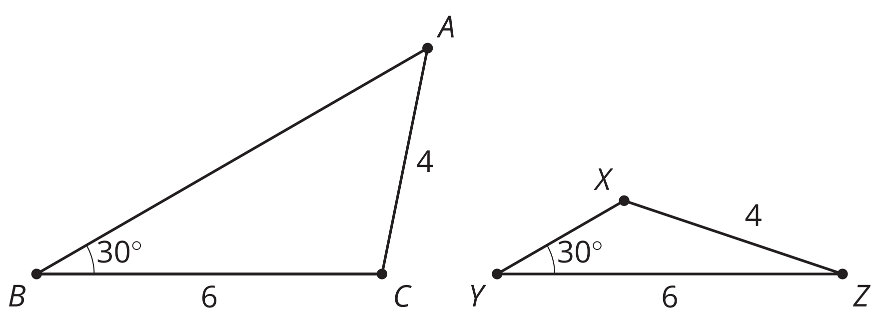
Here are some measurements for triangle ABC and triangle XYZ:

- Angle ABC and angle XYZ are both 30°.
- BC and YZ both measure 6 units.
- CA and ZX both measure 4 units.

Lin thinks these triangles must be congruent. Priya says she knows they might not be congruent. Construct 2 triangles with the given measurements that aren't congruent. Explain why triangles with 3 congruent parts aren't necessarily congruent.

Solution

Sample response:



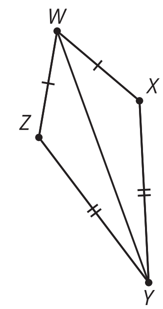
This is the ambiguous case. We know 2 pairs of corresponding sides are congruent, but the corresponding congruent angles are *not* between the congruent sides, and the shorter side is opposite the given angle. So this is not enough to prove the triangles congruent.

3 from Unit 2, Lesson 9

Student Task Statement

Jada states that diagonal WY bisects angles ZWX and ZYX. Is she correct? Explain your reasoning.

$\overline{WZ} \cong \overline{WX}, \overline{ZY} \cong \overline{XY}$



Solution

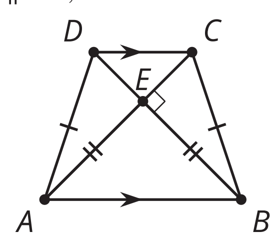
Yes. Sample reasoning: Triangle WYZ is congruent to triangle WYX using the Side-Side-Side Triangle Congruence Theorem. Since corresponding parts of congruent triangles are congruent, angle ZWY is congruent to angle XWY, and angle ZYW is congruent to angle XYW. Since the angles are congruent, WY must be the angle bisector of both angles.

4 from Unit 2, Lesson 10

Student Task Statement

Select **all** true statements based on the diagram.

$\overline{DA} \cong \overline{CB}, \overline{AE} \cong \overline{BE},$
$\overline{DC} \parallel \overline{AB}, \overline{CA} \perp \overline{DB}$



A. Angle CBE is congruent to angle DAE.

B. Angle CEB is congruent to angle DEA.

C. Segment DA is congruent to segment CB.

D. Segment DC is congruent to segment AB.

E. Line DC is parallel to line AB.

F. Line DA is parallel to line CB.

Solution

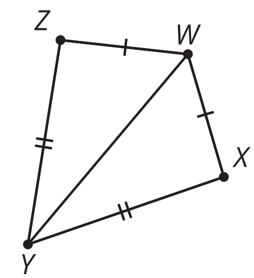
A, B, C, E

5 from Unit 2, Lesson 9

Student Task Statement

$WXYZ$ is a kite. Angle WXY has a measure of 94 degrees, and angle ZWX has a measure of 112 degrees. Find the measure of angle ZYW.

$\overline{WX} \cong \overline{WZ}, \overline{XY} \cong \overline{ZY}$



Solution

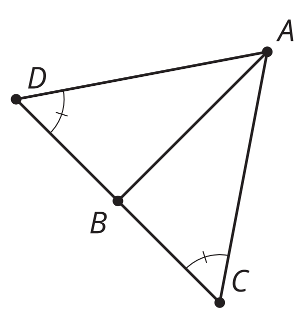
Angle ZYW has a measure of 30 degrees.

6 from Unit 2, Lesson 8

Student Task Statement

Andre is thinking through a proof using a reflection to show that a triangle is isosceles given that its base angles are congruent. Complete the missing information for his proof.

$\angle D \cong \angle C$



Construct AB such that AB is the perpendicular bisector of segment CD. We know angle ADB is congruent to ______1______. DB is congruent to ______2______ since AB is the perpendicular bisector of CD. Angle ______3______ is congruent to angle ______4______ because they are both right angles. Triangle ABC is congruent to triangle ______5______ because of the ______6______ Triangle Congruence Theorem. AD is congruent to ______7______ because they are corresponding parts of congruent triangles. Therefore, triangle ADC is an isosceles triangle.

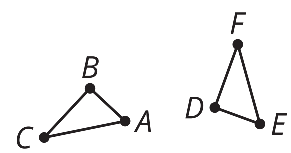
Solution

1. ACB
2. CB
3. ABD or ABC
4. ABC or ABD (whichever was not used in number 3)
5. ABD
6. Angle-Side-Angle
7. AC

7 from Unit 2, Lesson 3

Student Task Statement

The triangles are congruent. Which sequence of rigid motions takes triangle DEF onto triangle BAC?



A. Translate DEF using directed line segment EA. Rotate $D'E'F'$ using A as the center so that D' coincides with C. Reflect $D''E''F''$ across line AC.

B. Translate DEF using directed line segment EA. Rotate $D'E'F'$ using A as the center so that D' coincides with C. Reflect $D''E''F''$ across line AB.

C. Translate DEF using directed line segment EA. Rotate $D'E'F'$ using A as the center so that D' coincides with B. Reflect $D''E''F''$ across line AC.

D. Translate DEF using directed line segment EA. Rotate $D'E'F'$ using A as the center so that D' coincides with B. Reflect $D''E''F''$ across line AB.

Solution

D