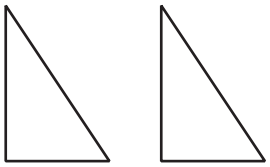
Similarity

Let's explore similar figures.

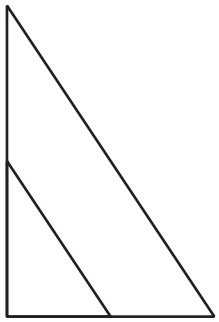
12.1 Which Three Go Together: Triangles

Which 3 go together? Why do they go together?

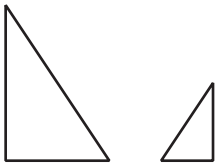
A



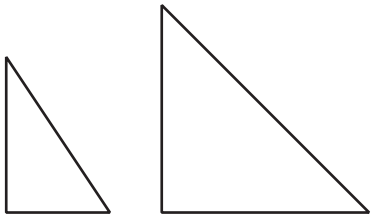
B



C

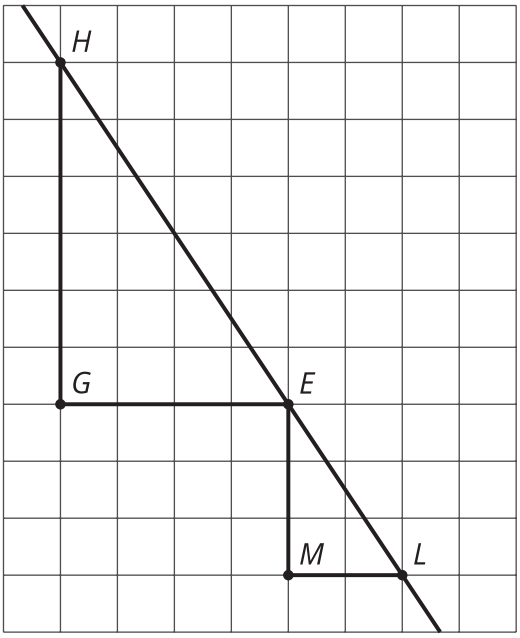


D

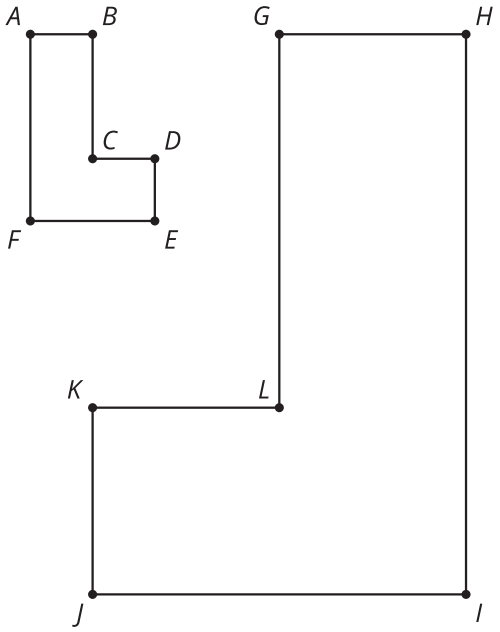


12.2 Similarity Transformations

1. Triangle EGH and triangle LME are **similar**. Find a sequence of translations, rotations, reflections, and dilations that shows this.

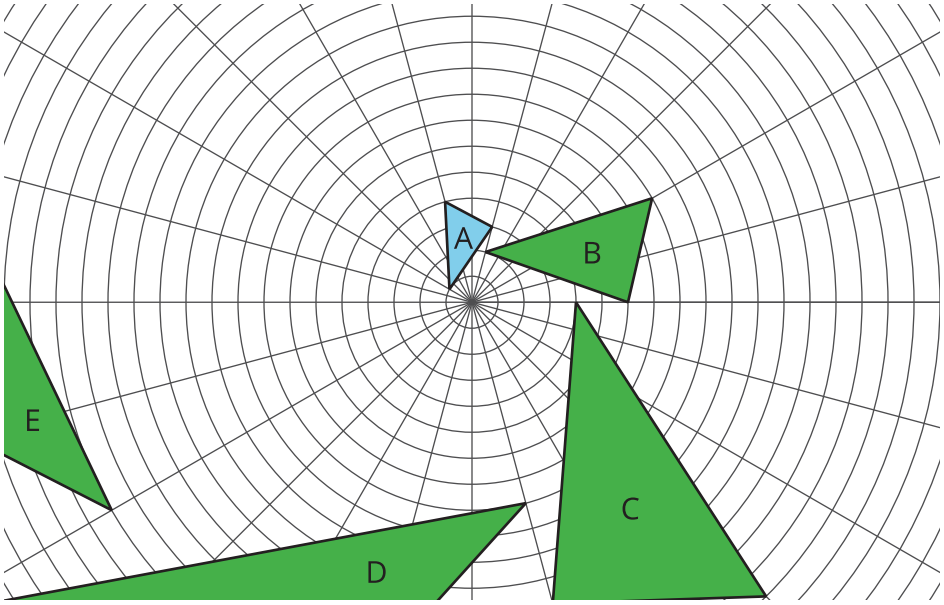


2. Hexagon $ABCDEF$ and hexagon $HGLKJI$ are similar. Find a sequence of translations, rotations, reflections, and dilations that shows this.



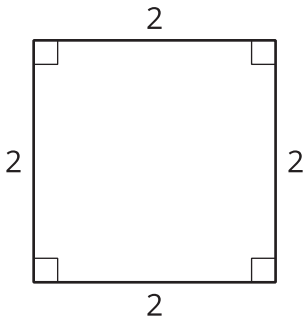
Are you ready for more?

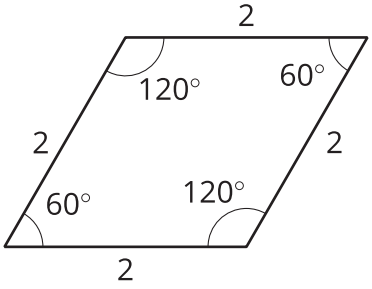
The same sequence of transformations that takes Triangle A to Triangle B, also takes Triangle B to Triangle C, and so on. Describe a possible sequence of transformations.



12.3 Are They Similar?

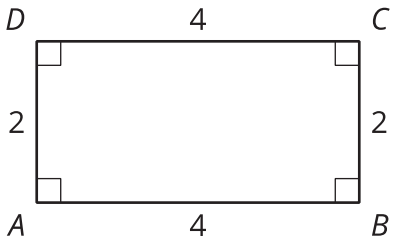
1. Let's look at a square and a rhombus.

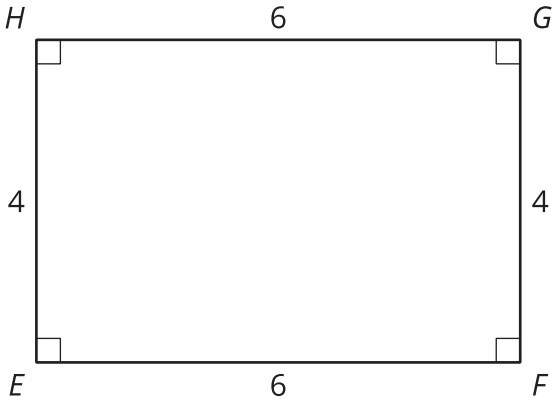




Priya says, "These polygons are similar because their side lengths are all the same." Clare says, "These polygons are not similar because the angles are different." Do you agree with either Priya or Clare? Explain your reasoning.

2. Now, let's look at rectangles $ABCD$ and $EFGH$.

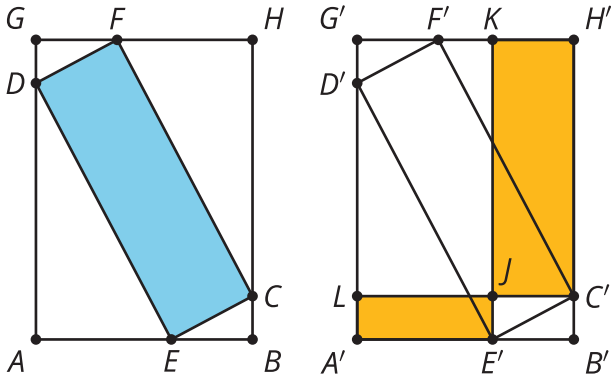




Jada says, "These rectangles are similar because all of the side lengths differ by 2." Lin says, "These rectangles are similar. I can dilate AD and BC using a scale factor of 2 and AB and CD using a scale factor of 1.5 to make the rectangles congruent. Then I can use a translation to line up the rectangles." Do you agree with either Jada or Lin? Explain your reasoning.

Are you ready for more?

Points A through H are translated to the right to create points A' through H'. All of the following are rectangles: $GHBA$, $FCED$, $KH'C'J$, and $LJE'A'$. Which is greater, the area of blue rectangle $DFCE$ or the total area of yellow rectangles $KH'C'J$ and $LJE'A'$?

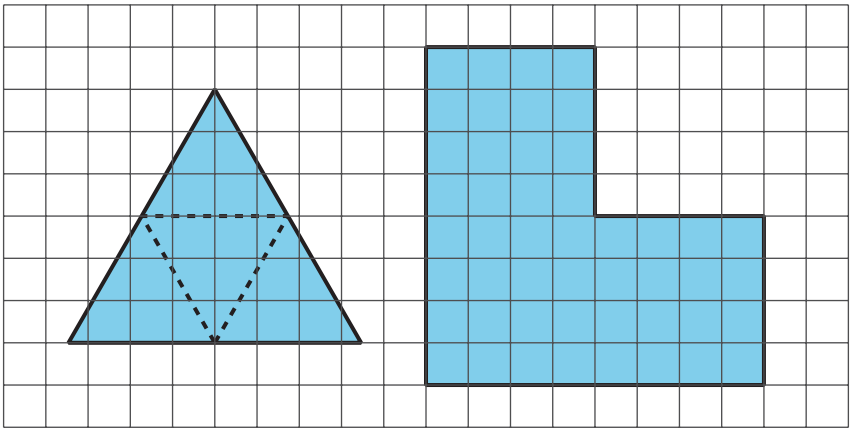


12.4 Find Someone Similar

Your teacher will give you a card. Find someone else in the room who has a card with a polygon that is similar but not congruent to yours. When you have found your partner, work with them to explain how you know that the two polygons are similar.

Are you ready for more?

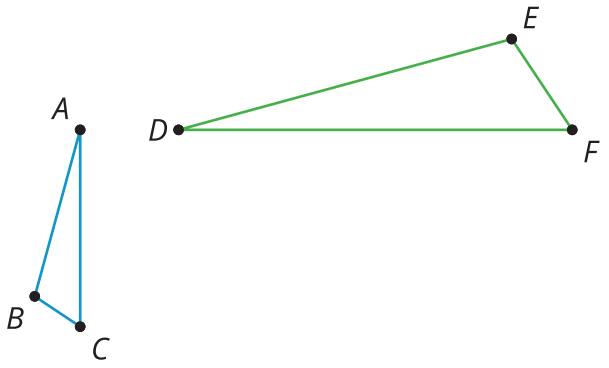
On the left is an equilateral triangle where dashed lines have been added, showing how an equilateral triangle can be partitioned into smaller similar triangles.



Find a way to do this for the figure on the right, partitioning it into smaller figures which are each similar to that original shape. What's the fewest number of pieces you can use? The most?

Lesson 12 Summary

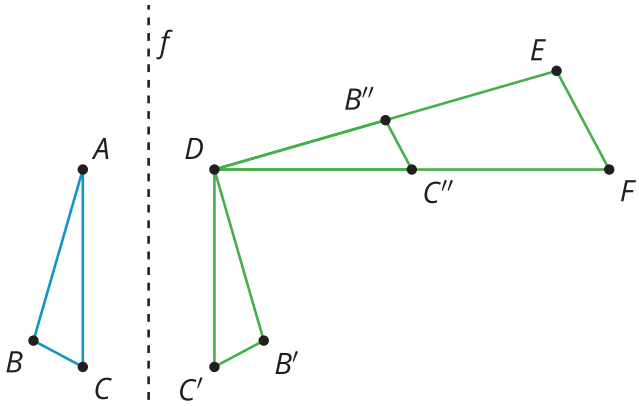
Let's show that triangle ABC is similar to triangle DEF:



Two figures are **similar** if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

One way to get from triangle ABC to triangle DEF follows these steps:

- Reflect triangle ABC across line f
- Rotate 90° counterclockwise around D
- Dilate with center D and scale factor 2



Another way to show that triangle ABC is similar to triangle DEF would be to dilate triangle DEF by a scale factor of $\frac{1}{2}$ with center of dilation at D, then translate D to A, then rotate it 90° clockwise around D, and finally reflect it across the vertical line containing DF so it matches up with triangle ABC.

When two polygons are similar:

- Every angle and side in one polygon has a corresponding angle and side in the other polygon.
- All pairs of corresponding angles have the same measure.
- Each side length in one figure is multiplied by the same scale factor to get the corresponding side length in the other figure.