Unit 6, Lesson 1

Rigid Transformations in a Plane

Goals

- Use coordinates to prove that given triangles are congruent.
- Use the structure of the coordinate plane to perform reflections, rotations, and translations.

Learning Targets

- I can reflect, rotate, and translate figures in the coordinate plane.
- I can use coordinates to prove that triangles are congruent.

Lesson Narrative

This lesson connects ideas from several previous units and extends them to the coordinate plane. In grade 8, students applied the Pythagorean Theorem to find the distance between two points in a coordinate system. Here, students calculate side lengths and angle measures, proving that given triangles are congruent. Students also draw and specify sequences of rigid transformations in the plane.

Each of these skills is a review, but the addition of the coordinate plane is novel. The goal is to prepare students to see transformations as functions using a new coordinate transformation notation that they will encounter in upcoming lessons. The notion of using the Pythagorean Theorem to calculate distances is a foundational idea that will reoccur in several lessons.

As students explore these ideas, they discover the structure provided by a coordinate grid (MP7). Students learn to use this structure, and to impose their own, by drawing auxiliary lines or right triangles to calculate lengths of segments.

Technology isn’t required for this lesson, but there are opportunities for students to choose to use appropriate technology to solve problems. We recommend making technology available.

Standards

Building On **8.G.B.8**

Addressing **HSG-CO.A.2, HSG-CO.A.5, HSG-CO.B**

Instructional Routines

- Draw It
- MLR5: Co-Craft Questions

Required Materials

Materials to Gather

- Scientific calculators: Activity 3

Required Preparation

Activity 2:

Dynamic geometry software can be used in this activity. If that is not available, provide access to the geometry toolkits for tracing paper and straight edges.

Lesson:

Dynamic geometry software can be used in& the *Cool-down* and the *Lesson Synthesis*. If that is not available, provide access to the geometry toolkits for tracing paper and straight edges.

Graph paper may be helpful to students in the *Lesson Synthesis*.

Student Facing Learning Goals

Let's try transformations with coordinates.

1.1 Traversing the Plane

Warm-up

5 min

Activity Narrative

In this activity, students make connections between transformations and the coordinate grid. When defining transformations, they will notice and make use of the structure created by the grid (MP7). This task also presents an opportunity to refresh students' memories of transformation language.

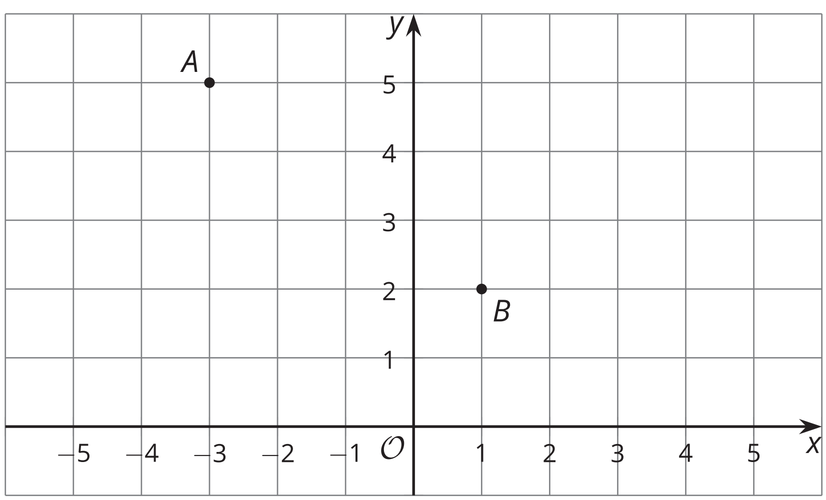
Monitor for students who draw a right triangle and use the Pythagorean Theorem to find the distance between points A and B. Throughout this unit, distance will be viewed as an application of the Pythagorean Theorem. Distance calculations using the Pythagorean Theorem will lead to the development of equations for circles and parabolas. There will be no need to introduce a separate distance formula.

Standards

Building On **8.G.B.8**

Addressing **HSG-CO.A.5**

Student Task Statement



1. How far is point A from point B?
2. What transformations will take point A to point B?

Student Response

1. 5 units
2. Sample response: Translate along the directed line segment from $(-3, 5)$ to $(1, 2)$.

Activity Synthesis

Invite a student who drew in a right triangle to share that method. If a student suggests the distance formula as an alternate method, ask the class how the formula connects to the Pythagorean Theorem. If no one uses the distance formula, there is no need to mention it.

Ask a few students to share their transformations. There are many possibilities. Transformations that take multiple steps are as valid as single-step transformations. If students use descriptions such as "Move 3 units down and 4 units right," connect this back to the language of translating and directed line segments. Remind students of this language by asking them to read the sentence frames for transformations from their reference chart:

- "Translate (object) along the directed line segment from (point) to (point)."
- "Rotate (object) (clockwise or counterclockwise) using (point) as the center by angle (measure)."
- "Reflect (object) across line (name/equation)."

Note that, during this unit, points could be named with letters (for example, point A) or with coordinates (for example, $(-3, 5)$). Similarly, lines could be named in various ways, such as "y-axis" or "$x = 0$."

1.2 Transforming with Coordinates

15 min

Activity Narrative

In this activity, students practice transforming a figure on the coordinate plane. Students may choose to use tracing paper and perform these transformations as if there were no grid. Other students may notice the structure of gridlines and look for patterns in the coordinates. During the *Activity Synthesis*, students are reminded that rigid transformations produce congruent figures. This helps prepare students for the next activity, in which they reason that given two congruent figures, there must be a sequence of transformations carrying one figure to the other.

Making dynamic geometry software and tracing paper available gives students an opportunity to choose appropriate tools strategically (MP5).

Standards

Addressing **HSG-CO.A.2, HSG-CO.A.5**

Instructional Routines

- Draw It

Launch

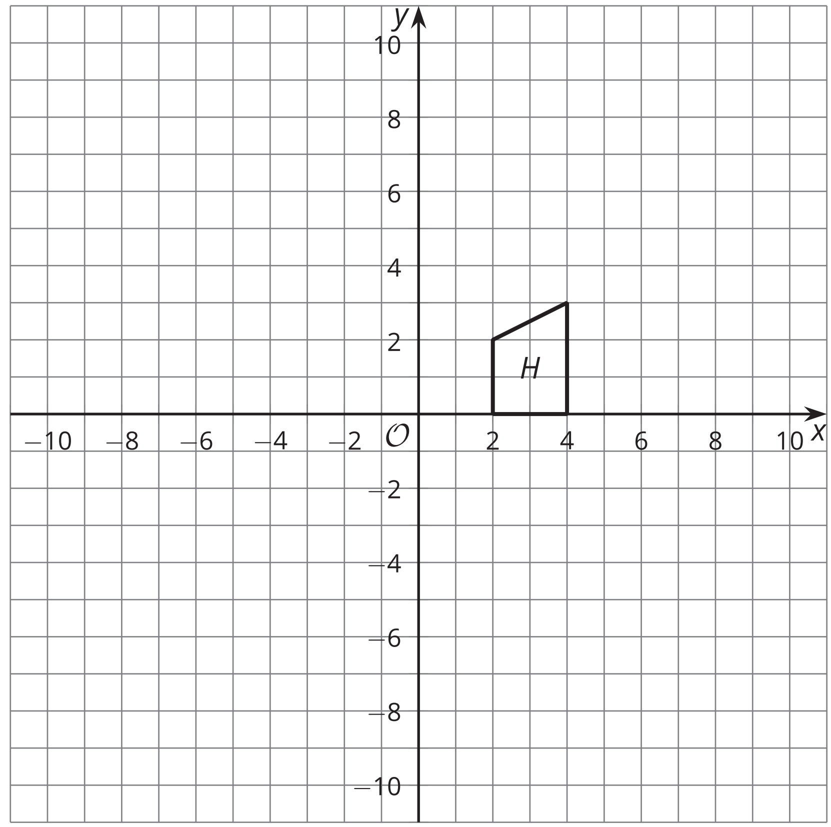
Access for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide access to Figure H at scale or to a large-scale Figure H and a large-scale grid to match.

Supports accessibility for: Visual-Spatial Processing; Conceptual Processing

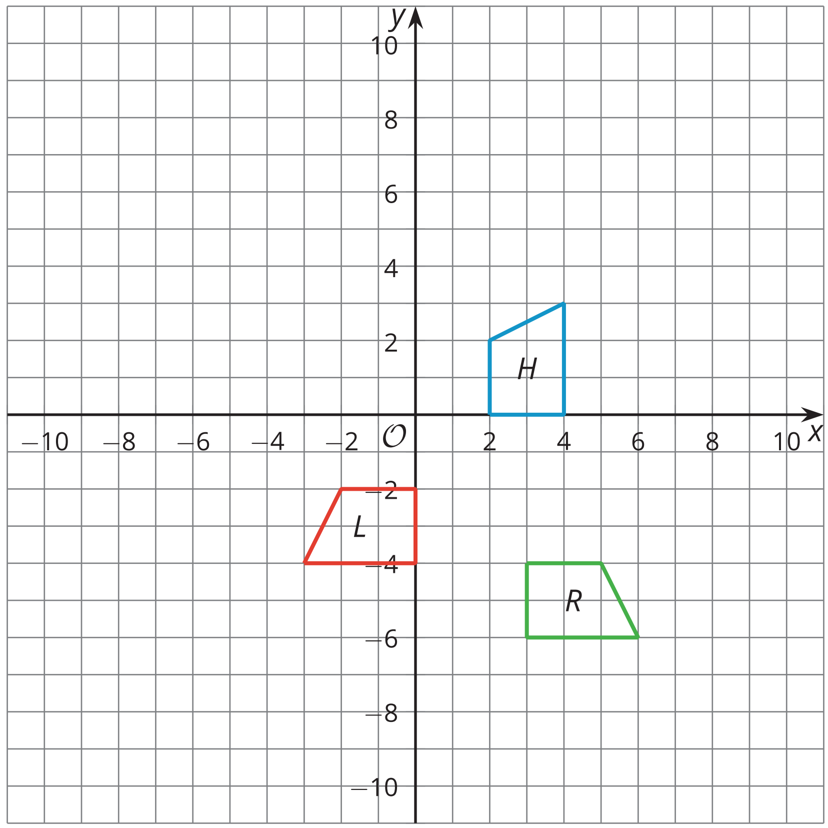
Student Task Statement

First, predict where each transformation will land. Next, carry out the transformation.



1. Rotate Figure H clockwise around center $(2, 0)$ by 90 degrees.
   Translate the image by the directed line segment from $(2, 0)$ to $(3, -4)$.
   Label the result R.
2. Reflect Figure H across the y-axis.
   Rotate the image counterclockwise around center $(0, 0)$ by 90 degrees.
   Label the result L.

Student Response



Activity Synthesis

Invite students to share strategies such as "Reflecting across the y-axis makes the x-values negative and keeps the y-values the same." If students do not notice patterns like this one, there is no need to mention them. In a subsequent lesson, students will investigate the effect of transformations on coordinates.

Ask students what they notice about the three figures. (The figures are trapezoids. The figures have three right angles. All three figures are congruent.) Ask students how they know the figures are congruent. (They are congruent by definition of rigid transformations.)

1.3 Congruent by Coordinates

15 min

Activity Narrative

In this activity students calculate side lengths and angle measures of triangles on the coordinate plane. In the process they demonstrate the two triangles are congruent. During the synthesis they discuss the minimum requirements for a proof of triangle congruence, since calculating *all* side lengths and angle measures goes above and beyond what is necessary. Finally, students specify a sequence of rigid transformations taking one triangle to the other.

Standards

Addressing **HSG-CO.A.5, HSG-CO.B**

Instructional Routines

- MLR5: Co-Craft Questions

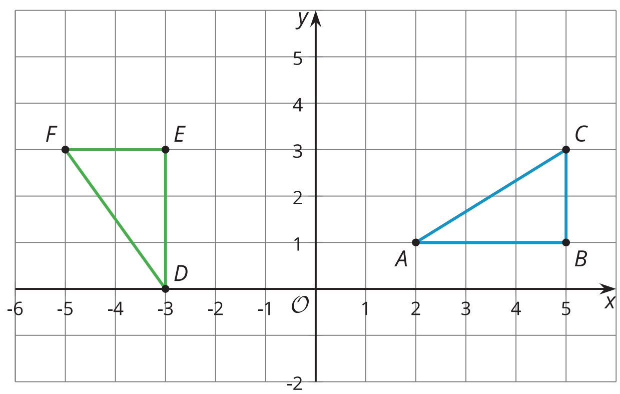
Launch

Tell students they can either leave answers as exact values or round sides to the nearest tenth and angles to the nearest degree.

Access for English Language Learners

MLR5 Co-Craft Questions. Keep books or devices closed. Display only the image, without revealing the questions, and ask students to record possible mathematical questions that could be asked about the situation. Invite students to compare their questions before revealing the task. Ask, "What do these questions have in common? How are they different?" Reveal the intended questions for this task and invite additional connections.
Advances: Reading, Writing

Student Task Statement



1. Calculate the length of each side in triangles ABC and DEF.
2. Calculate the measure of each angle in triangles ABC and DEF.
3. The triangles are congruent. How do you know this is true?
4. Because the triangles are congruent, there must be a sequence of rigid motions that takes one to the other. Find a sequence of rigid motions that takes triangle ABC to triangle DEF.

Student Response

1. $AB = DE = 3, BC = EF = 2, AC = DF = \sqrt{13}$ (or equivalent)
2. $m\angle A = m\angle D = 34^\circ, m\angle B = m\angle E = 90^\circ, m\angle C = m\angle F = 56^\circ$
3. Sample response: All corresponding parts are congruent, so the triangles are congruent.
4. Sample response: Rotate triangle ABC 90 degrees counterclockwise around center $(0, 0)$. Translate the image by the directed line segment from $(-1, 2)$ to $(-3, 0)$.

Building on Student Thinking

If students are stuck on finding the measures of the angles, suggest they look at their reference chart for concepts from a prior unit that can help.

Are You Ready for More?

What single transformation would take triangle ABC to triangle DEF?

Extension Student Response

Sample response: Rotate triangle ABC 90 degrees counterclockwise around center $(0, -2)$.

Activity Synthesis

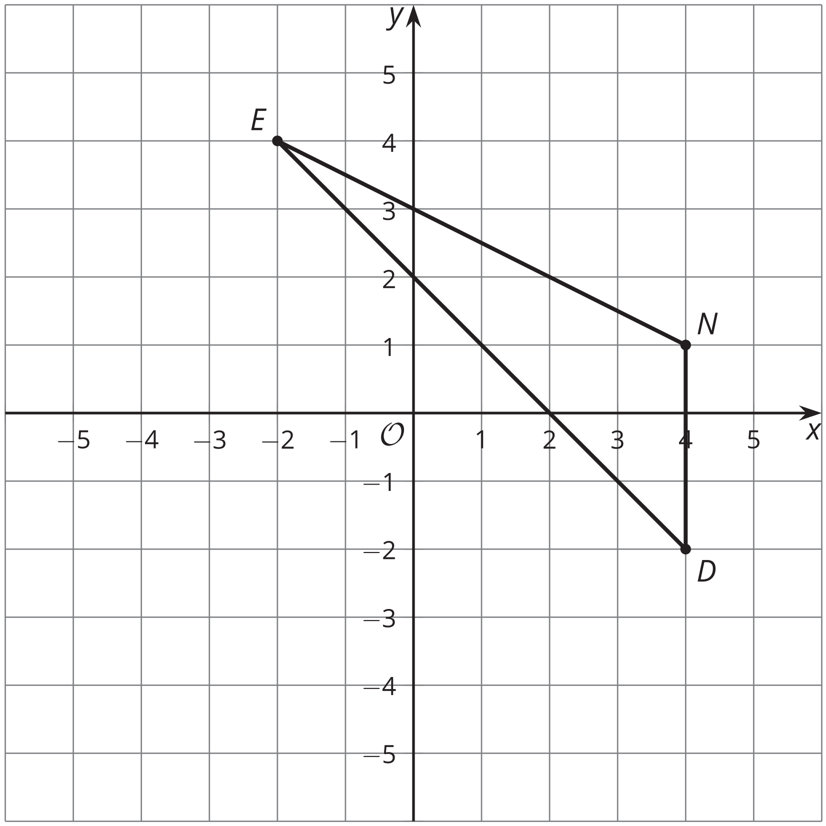
Invite a student to share their solution for calculating the measure of angle A. Ask the class why they can use trigonometric ratios to find angle measures. (Triangle ABC is a right triangle.)

Invite students to share how they determined that the triangles were congruent. Here are some questions for the discussion:

- "Did we have to show that every pair of corresponding side lengths and angles are congruent in order to determine that the triangles are congruent?" (No. It is possible to use other strategies, like congruence theorems or rigid transformations.)
- "How did calculating the side lengths help show that the triangles are congruent?" (Three side lengths means that we can use the Side-Angle-Side Triangle Congruence Theorem.)
- "Is there a different Triangle Congruence Theorem you could have used with this figure?" (Since the triangles are right triangles, we could have use the Side-Angle-Side Triangle Congruence Theorem.)
- "How does finding a sequence of rigid motions help us determine whether shapes are congruent?" (If we didn't have side lengths or angle measures, we could still use a sequence of rigid motions to show that two figures are congruent.

Lesson Synthesis

Arrange students in groups of 2. Display triangle END shown here. Instruct students to work with their partners to find a set of coordinates that forms a triangle congruent to END. Then the students should explain how they determined that the triangles are congruent.



Sample responses:

- We found the coordinates $E'(-3,1)$, $N'(3,-2)$, and $D'(3,-5)$. Translate triangle END by the directed line segment from $(-2,4)$ to $(-3,1)$. Each vertex of triangle END will coincide with the corresponding vertices of triangle $E'N'D'$, so the triangles are congruent.
- We found the coordinates $E'(2,4)$, $N'(-4,1)$, and $D'(-4,-2)$. Then we calculated the lengths of the segments. The lengths of EN and $E'N'$ are each $\sqrt{45}$ units. The lengths of DE and $D'E'$ are each $\sqrt{72}$ units. The lengths of ND and $N'D'$ are each 3 units. So, the triangles are congruent by the Side-Side-Side Triangle Congruence Theorem.

Invite a few pairs of students to present their triangles and explanations. If possible, select at least one pair who used transformations to determine congruence and at least one pair who used calculations of side length or angle measure.

1.4

A Transformed Triangle

Cool-down

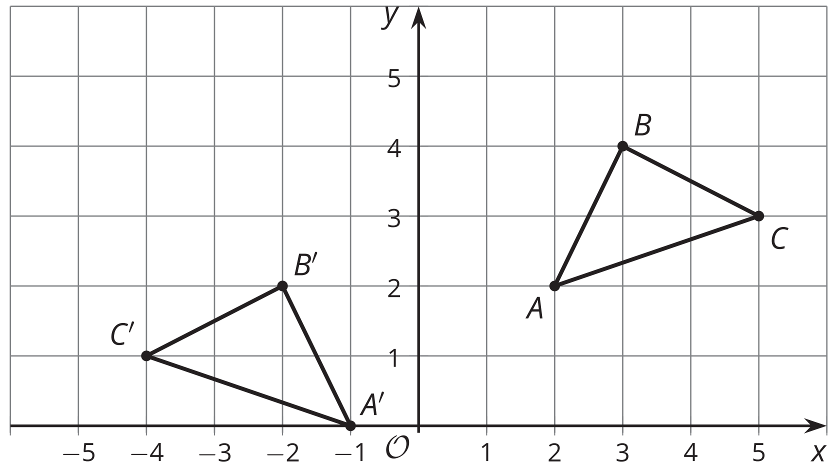
5 min

Standards

Addressing **HSG-CO.A.5**

Student Task Statement

Triangle $A'B'C'$ is the image of triangle ABC after a sequence of rigid motions.



Find a sequence of transformations that takes triangle ABC to triangle $A'B'C'$.

Student Response

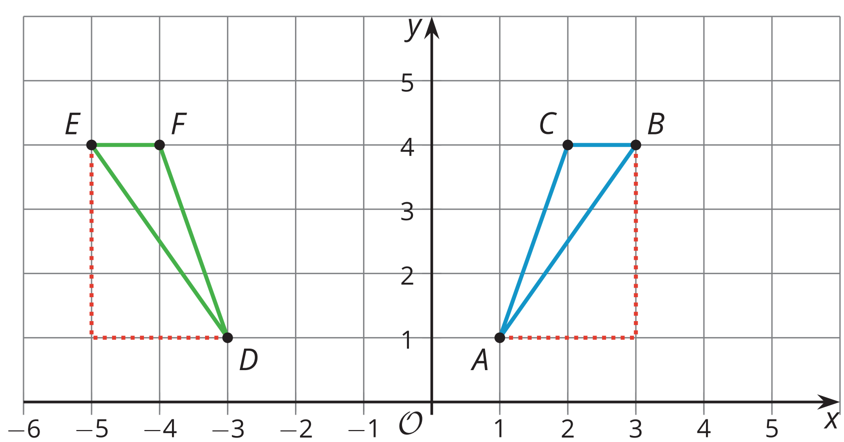
Sample response: Reflect triangle ABC across the y-axis. Then translate by the directed line segment from $(-2, 2)$ to $(-1, 0)$.

Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

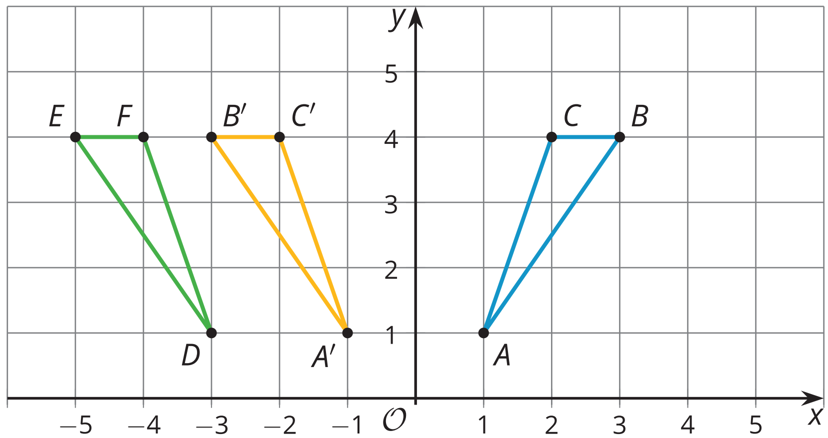
Lesson 1 Summary

The triangles shown here look like they might be congruent. Since we know the coordinates of all the vertices, we can compare side lengths using the Pythagorean Theorem—if we draw line segments (see the red dotted lines) that create two right triangles that have segments AB and DE as their respective hypotenuses. The length of segment AB is $\sqrt{13}$ units because this segment is the hypotenuse of a right triangle with vertical side length of 3 units and horizontal side length of 2 units. The length of segment DE is $\sqrt{13}$ units as well, because this segment is also the hypotenuse of a right triangle with leg lengths of 3 and 2 units.



The other sides of the triangles are congruent as well: The lengths of segments BC and FE are 1 unit each, and the lengths of segments AC and DF are each $\sqrt{10}$ units, because they are both hypotenuses of right triangles with leg lengths 1 and 3 units (those lines are not shown, but could be drawn). So triangle ABC is congruent to triangle DEF by the Side-Side-Side Triangle Congruence Theorem.

Since triangle ABC is congruent to triangle DEF, there is a sequence of rigid motions that takes triangle ABC to triangle DEF. Here is one possible sequence: First, reflect triangle ABC across the y-axis. Then translate the image by the directed line segment from $(-1, 1)$ to $(-3, 1)$.



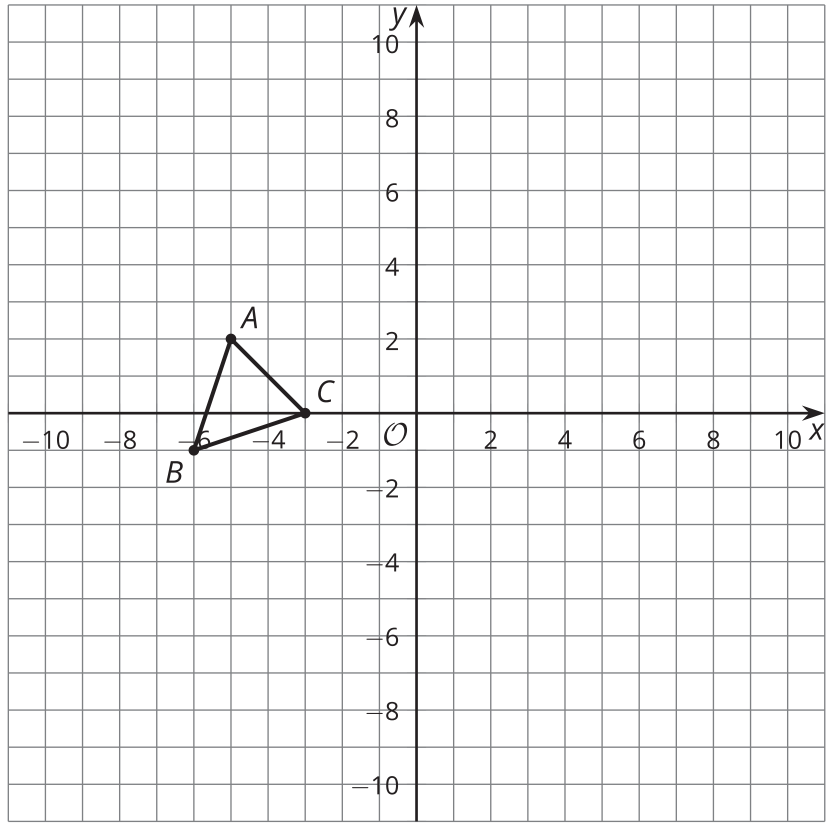
Lesson 1 Practice Problems

1 Student Task Statement

Reflect triangle ABC over the line $x = -3$.

Translate the image by the directed line segment from $(0, 0)$ to $(4, 1)$.

What are the coordinates of the vertices in the final image?



Solution

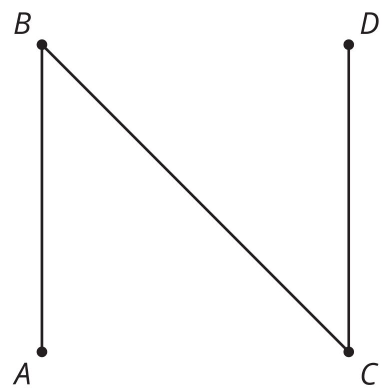
$A''(3, 3)$, $B''(4, 0)$, and $C''(1, 1)$

2

from Unit 1, Lesson 14

Student Task Statement

Three line segments form the letter N. Rotate the letter N counterclockwise around the midpoint of segment BC by 180 degrees. Describe the result.



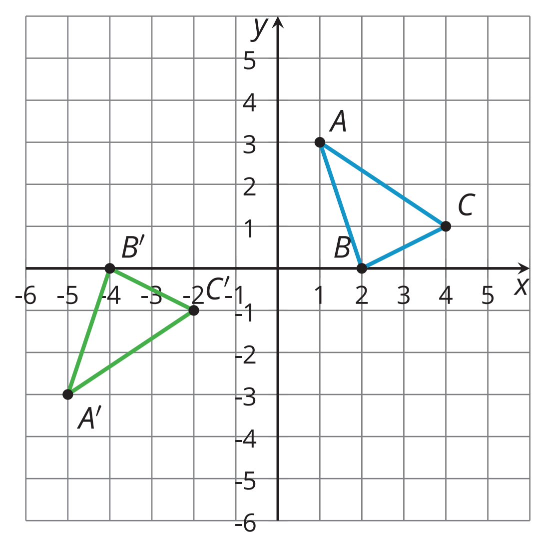
Solution

The image is the same letter N.

3 ## Student Task Statement

Triangle ABC has coordinates $A(1, 3)$, $B(2, 0)$, and $C(4, 1)$. The image of this triangle after a sequence of transformations is triangle $A'B'C'$ where $A'(-5, -3)$, $B'(-4, 0)$, and $C'(-2, -1)$.

Write a sequence of transformations that takes triangle ABC to triangle $A'B'C'$.



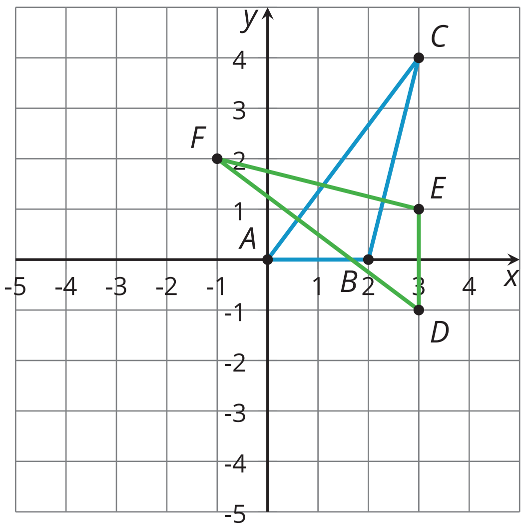
Solution

Sample response: Translate triangle ABC by the directed line segment from $(2, 0)$ to $(-4, 0)$. Then reflect the translated image across the x-axis.

4

Student Task Statement

Show using transformations or segment lengths that triangle ABC is congruent to triangle DEF.



Solution

Sample responses:

- Both triangles have corresponding sides of lengths 2, $\sqrt{17}$, and 5 so they are congruent by the Side-Side-Side Congruence Theorem.
- We can rotate triangle ABC 90 degrees counterclockwise around $(0, 0)$ and then translate by the directed line segment from $(0, 0)$ to $(3, -1)$. The final image coincides with triangle DEF, so the triangles are congruent.

5 from Unit 5, Lesson 17

Student Task Statement

The density of water is 1 gram per cm^3. An object floats in water if its density is less than water's density, and it sinks if its density is greater than water's. Will a 1.17 gram diamond in the shape of a pyramid whose base has area 2 cm^2 and whose height is 0.5 centimeters sink or float? Explain your reasoning.

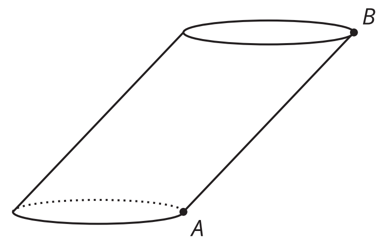
Solution

Sample response: It will sink. Its density is about 3.51 grams per cm^3 which is more than 1 gram per cm^3.

6 from Unit 5, Lesson 11

Student Task Statement

Technology required. An oblique cylinder with a base of radius 2 units is shown. The top of the cylinder can be obtained by translating the base by the directed line segment AB which has length 16 units. The segment AB forms a 30° angle with the plane of the base. What is the volume of the cylinder?



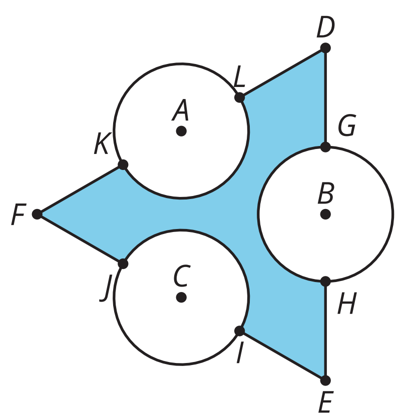
Solution

32π cubic units

7 from Unit 1, Lesson 22

Student Task Statement

This design began from the construction of an equilateral triangle. Record at least 3 rigid transformations (rotation, reflection, translation) that take parts of the diagram to congruent parts of the diagram.



Solution

Sample response: Rotate the figure clockwise by 120 degrees around the center of the triangle. This rigid transformations takes the entire figure to itself. Translation by directed line segment AB takes the circle centered at A to the circle centered at B. Reflection across the perpendicular bisector of segment DE takes the entire figure to itself.