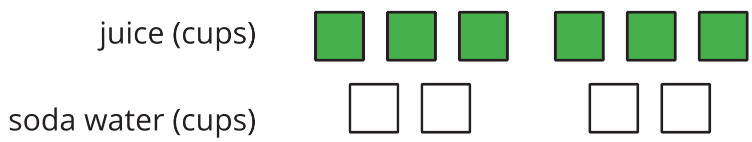
Unit 2 Family Support Materials

Introducing Ratios

Section A: What Are Ratios?

A **ratio** is an association between two or more quantities. For example, the cups of juice and the cups of soda water in a drink recipe form a ratio. Ratios can be represented with diagrams. Here is one diagram for a drink recipe:



Here are some correct ways to describe this diagram:

- The ratio of cups of juice to cups of soda water is $6 : 4$.
- The ratio of cups of soda water to cups of juice is 4 to 6.
- There are 6 cups of juice for every 4 cups of soda water.

We can also use other numbers to describe the quantities in this situation. For instance, we can say that there are 3 cups of juice for every 2 cups of soda water. The ratios $6 : 4$ and $3 : 2$ are **equivalent** because mixing juice and soda water in these amounts would make drinks that taste the same.

Two situations that can be described with **equivalent ratios** are the same in some important way. For example, mixing 1 ml of black paint and 10 ml of white paint would create the same shade of gray as mixing 3 ml of black paint and 30 ml of white paint, so these ratios of black paint to white paint are equivalent.

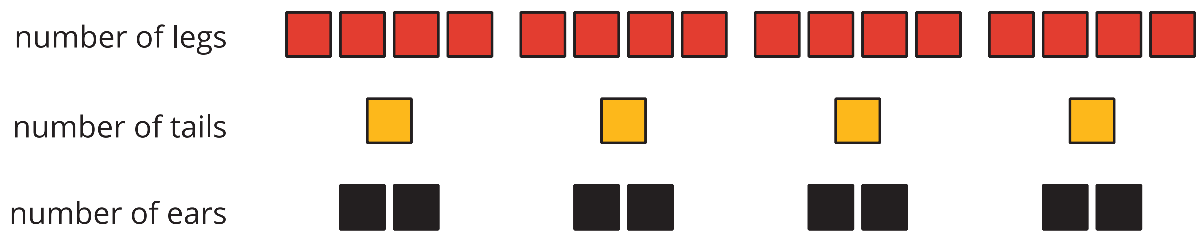
Here is a task to try with your student:

There are 4 horses in a stall. Each horse has 4 legs, 1 tail, and 2 ears.

1. Draw a diagram that shows the ratio of legs, tails, and ears in the stall.
2. Complete each statement.
    - The ratio of _______ to _______ to _______ is _______ : _______ : _______.
    - There are _______ ears for every tail. There are _______ legs for every ear.

Solution:

1. Sample response:

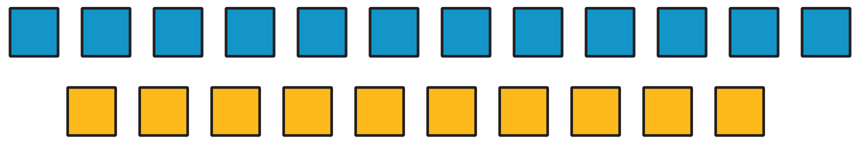


2. Sample response: The ratio of legs to tails to ears is $16 : 4 : 8$. There are 2 ears for every tail. There are 2 legs for every ear.

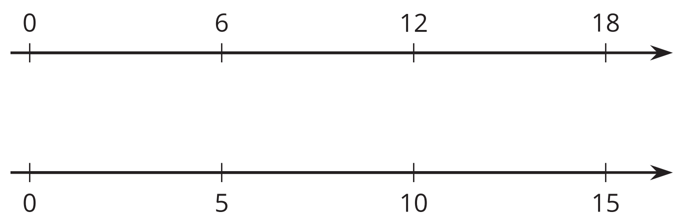
Section C: Representing Equivalent Ratios

There are different ways to represent equivalent ratios.

Let's say that the sixth grade class is selling raffle tickets at a price of \$6 for 5 tickets. At that rate, 10 tickets cost \$12. Some students may use diagrams with shapes to represent the situation. For example, here is a diagram representing 10 tickets for \$12.



Drawing so many shapes becomes impractical. **Double number line diagrams** can be a quicker way to show equivalent ratios. Here is a diagram that represents the price in dollars for different numbers of raffle tickets all sold *at the same rate* of \$6 for 5 tickets.



The diagram can be partitioned, extended, and marked up to find the prices for other numbers of tickets—including the price for 1 ticket, which is the **unit price**.

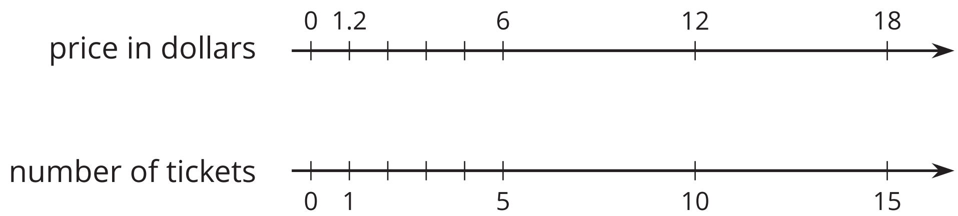
Here is a task to try with your student:

Raffle tickets cost \$6 for 5 tickets.

1. How many tickets can you get for \$90?
2. What is the price of 1 ticket?

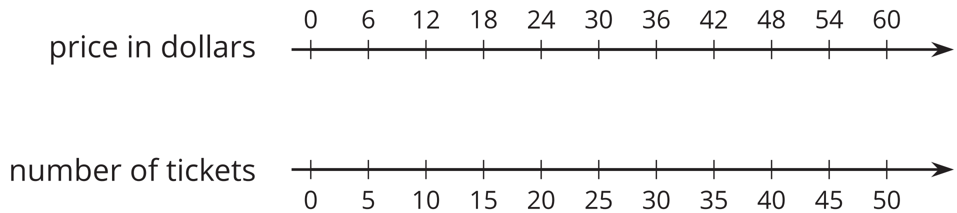
Solution:

1. 75 tickets. Sample reasoning:
    - Extend the double number line shown and observe that \$90 is lined up with 75 tickets.
    - If \$6 buys 5 tickets, then \$60 buys 50 tickets and \$30 buys 25 tickets. So, \$90, which is $60 + 30$, buys $50 + 25$, or 75 tickets.
2. \$1.20. Sample reasoning: Divide the number line into 5 equal intervals, as shown. Reason that the price in dollars of 1 ticket must be $6 \div 5$.


price in dollars
0
1.2
6
12
18
number of tickets
0
1
5
10
15

Section D: Solving Ratio and Rate Problems

Let's think about an example that we saw before: The sixth grade class is selling raffle tickets at a price of \$6 for 5 tickets. If we tried to find the price of 300 tickets by extending the double number line diagram here, it would take 5 times more paper!



Double number line diagrams are hard to use in problems with large amounts. A *table* is a better choice to represent this situation. Tables of equivalent ratios are useful because you can arrange the rows in any order. For example, a student may find the price for 300 raffle tickets by making the table shown.

	price in dollars	**number of tickets**	
	6	5	
÷ 5	1.20	1	÷ 5
• 300	360	300	• 300

Although students can choose any representation that helps them solve a problem, it is important that they get comfortable with tables because they are used for a variety of purposes throughout future mathematics courses.

Here is a task to try with your student:

At a constant speed, a train travels 45 miles in 60 minutes. At this rate, how far does the train travel in 12 minutes? If you get stuck, consider creating a table.

Solution:

9 miles. Sample reasoning:

time in minutes	distance in miles
60	45
1	0.75
12	9