Let’s Make a Box

Let’s investigate volumes of different boxes.

1.1 Which Three Go Together: Boxes

Which three go together? Why do they go together?

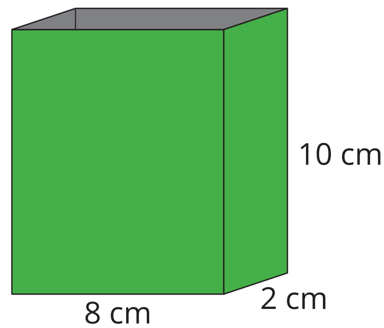
A

length: 4cm

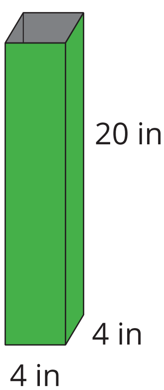
width: 8cm

height: 10cm

B

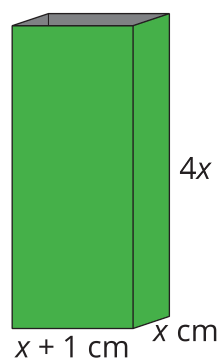


C



D

$V = 320 \text{ cm}^3$



1.2 Building Boxes

Your teacher will give you some supplies to construct an open-top box.

1. Cut out a square from each corner of a sheet of paper, and then fold up the sides.
2. Calculate the volume of your box, and complete the table with your information.

side length of square cutout (in)	length (in)	width (in)	height (in)	volume of box (in^3)
1				

1.3 Building the Biggest Box

1. The volume $V(x)$ in cubic inches of the open-top box is a function of the side length x in inches of the square cutouts. Make a plan to figure out how to construct the box with the largest volume.

   Pause here so your teacher can review your plan.

2. Write an expression for $V(x)$.

3. Use graphing technology to create a graph representing $V(x)$. Approximate the value of x that would allow you to construct an open-top box with the largest volume possible from one piece of paper.

Are you ready for more?

The surface area $A(x)$, in square inches, of the open-top box is also a function of the side length x, in inches, of the square cutouts.

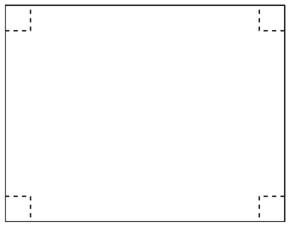
1. Find one expression for $A(x)$ by adding the area of the five faces of our open-top box.

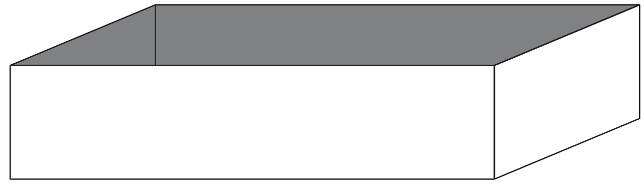
2. Find another expression for $A(x)$ by subtracting the area of the cutouts from the area of the paper.

3. Show algebraically that these two expressions are equivalent.

Lesson 1 Summary

A box can be created by removing squares from each corner of a rectangle of paper.





Let $V(x)$ be the volume of the box in cubic inches, where x is the side length, in inches, of each square removed from the four corners.

To define V using an expression, we can use the fact that the volume of a cube is $(length)(width)(height)$. If the piece of paper we start with is 3 inches by 8 inches, then:

$$V(x) = (3 - 2x)(8 - 2x)(x)$$

What are some reasonable values for x? Cutting out squares with side lengths less than 0 inches doesn't make sense, and similarly, we can't cut out squares larger than 1.5 inches, since the short side of the paper is only 3 inches (since $3 - 1.5 \cdot 2 = 0$). You may remember that the name for the set of all the input values that make sense to use with a function is the domain. Here, a reasonable domain is somewhere larger than 0 inches but less than 1.5 inches, depending on how well we can cut and fold!

By graphing this function, it is possible to find the maximum value within a specific domain. Here is a graph of $y = V(x)$ for x values between 0 and 1.5. It looks like the largest volume we can get for a box made this way from a 3-inch by 8-inch piece of paper is about 7.4 in^3.