

# Polynomial Division (Part 1)

Let's learn a way to divide polynomials.

## 12.1 Notice and Wonder: A Different Use for Diagrams

What do you notice? What do you wonder?

A.  $(x - 3)(x + 5) = x^2 + 2x - 15$

|      |       |       |
|------|-------|-------|
|      | $x$   | $5$   |
| $x$  | $x^2$ | $5x$  |
| $-3$ | $-3x$ | $-15$ |

B.  $(x - 1)(x^2 + 3x - 4) = x^3 + 2x^2 - 7x + 4$

|      |        |        |       |
|------|--------|--------|-------|
|      | $x^2$  | $3x$   | $-4$  |
| $x$  | $x^3$  | $3x^2$ | $-4x$ |
| $-1$ | $-x^2$ | $-3x$  | $+4$  |

C.  $(x - 2)(?) = (x^3 - x^2 - 4x + 4)$

|      |       |  |  |
|------|-------|--|--|
|      |       |  |  |
| $x$  | $x^3$ |  |  |
| $-2$ |       |  |  |

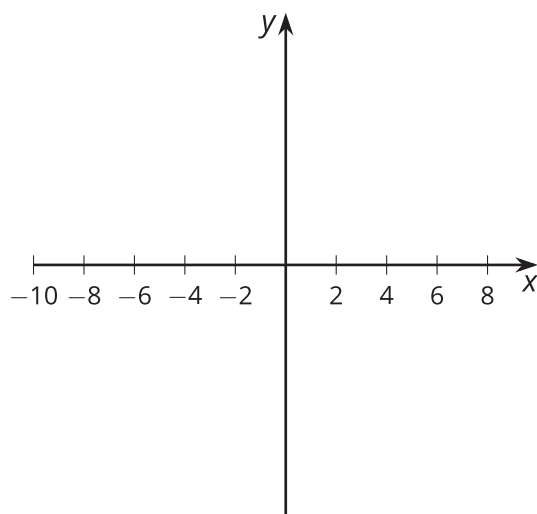
## 12.2 Factoring with Diagrams

Priya wants to sketch a graph of the polynomial  $f$  defined by  $f(x) = x^3 + 5x^2 + 2x - 8$ .

She knows  $f(1) = 0$ , so she suspects that  $(x - 1)$  could be a factor of  $x^3 + 5x^2 + 2x - 8$  and writes  $(x^3 + 5x^2 + 2x - 8) = (x - 1)(?x^2 + ?x + ?)$  and draws a diagram.

|      |       |  |  |
|------|-------|--|--|
|      |       |  |  |
| $x$  | $x^3$ |  |  |
| $-1$ |       |  |  |

1. Finish Priya's diagram.
2. Write  $f(x)$  as the product of  $(x - 1)$  and another factor.
3. Write  $f(x)$  as the product of three linear factors.
4. Draw a sketch of  $y = f(x)$ .



## 12.3

## More Factoring with Diagrams

Here are some polynomial functions with one or more known factors. Rewrite each polynomial as a product of linear factors.

Note: you may not need to use all the columns in each diagram. For some problems, you may need to make another diagram.

1.  $A(x) = x^3 - 7x^2 - 16x + 112, (x - 7)$

|     |         |   |  |  |  |
|-----|---------|---|--|--|--|
|     | $x^2$   |   |  |  |  |
| $x$ | $x^3$   | 0 |  |  |  |
| -7  | $-7x^2$ |   |  |  |  |

2.  $B(x) = 2x^3 - x^2 - 27x + 36, \left(x - \frac{3}{2}\right)$

|                |         |        |  |  |  |
|----------------|---------|--------|--|--|--|
|                | $2x^2$  |        |  |  |  |
| $x$            | $2x^3$  | $2x^2$ |  |  |  |
| $-\frac{3}{2}$ | $-3x^2$ |        |  |  |  |

3.  $C(x) = x^3 - 3x^2 - 13x + 15, (x + 3)$

|     |  |  |  |  |  |
|-----|--|--|--|--|--|
|     |  |  |  |  |  |
| $x$ |  |  |  |  |  |
| $3$ |  |  |  |  |  |

4.  $D(x) = x^4 - 13x^2 + 36, (x - 2), (x + 2)$

(Hint:  $x^4 - 13x^2 + 36 = x^4 + 0x^3 - 13x^2 + 0x + 36$ )

|  |  |  |  |  |  |
|--|--|--|--|--|--|
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

5.  $F(x) = 4x^4 - 15x^3 - 48x^2 + 109x + 30, (x - 5), (x - 2), (x + 3)$

|  |  |  |  |  |  |
|--|--|--|--|--|--|
|  |  |  |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

 **Are you ready for more?**

A diagram can also be used to divide polynomials even when a factor is not linear. Suppose we know  $(x^2 - 2x + 5)$  is a factor of  $x^4 + x^3 - 5x^2 + 23x - 20$ . We could write  $(x^4 + x^3 - 5x^2 + 23x - 20) = (x^2 - 2x + 5)(?x^2 + ?x + ?)$ . Make a diagram, and find the missing factor.

## Lesson 12 Summary

What are some things that could be true about the polynomial function defined by  $p(x) = x^3 - 5x^2 - 2x + 24$  if we know  $p(-2) = 0$ ?

- Thinking about the graph of the polynomial, the point  $(-2, 0)$  must be on the graph as a horizontal intercept.
- Thinking about the expression written in factored form,  $(x + 2)$  *could* be one of the factors, since  $x + 2 = 0$  when  $x = -2$ .

How can we figure out whether  $(x + 2)$  actually is a factor?

If we assume that  $(x + 2)$  is a factor, then there is some other polynomial  $q(x) = ax^2 + bx + c$  where  $a$ ,  $b$ , and  $c$  are real numbers and  $p(x) = (x + 2)q(x)$ . In the past we have expanded  $(x + 2)(ax^2 + bx + c)$  to find  $p(x) = (x + 2)q(x)$ . Instead, we can work out the values of  $a$ ,  $b$ , and  $c$  by thinking through the calculation backward.

One way to organize our thinking is to use a diagram. First, fill in  $(x + 2)$  and the leading term of  $p(x)$ ,  $x^3$ . From this we can see the leading term of  $q(x)$  must be  $x^2$ , meaning  $a = 1$ , since  $x \cdot x^2 = x^3$ .

|      |       |  |  |
|------|-------|--|--|
|      | $x^2$ |  |  |
| $x$  | $x^3$ |  |  |
| $+2$ |       |  |  |

We can fill in the rest of the diagram using similar thinking and paying close attention to the signs of each term. For example, we put in a  $2x^2$  in the bottom left cell because that's the product of 2 and  $x^2$ . But that means we need to have a  $-7x^2$  in the middle cell of the middle row, since that's the only other place we will get an  $x^2$  term, and we need to get  $-5x^2$  once all the terms are collected. Continuing in this way, we get the completed table:

|      |         |         |        |
|------|---------|---------|--------|
|      | $x^2$   | $-7x$   | $+12$  |
| $x$  | $x^3$   | $-7x^2$ | $+12x$ |
| $+2$ | $+2x^2$ | $-14x$  | $+24$  |

Collecting all the terms in the interior of the diagram, we see that  $x^3 - 5x^2 - 2x + 24 = (x + 2)(x^2 - 7x + 12)$ , so  $q(x) = x^2 - 7x + 12$ . Notice that the 24 in the bottom right was exactly what we needed, and it is how we know that  $(x + 2)$  is a factor of  $p(x)$ . With a bit more factoring, we can say that  $p(x) = (x + 2)(x - 3)(x - 4)$ .