Unit 3, Lesson 6

Rational Equations (Part 2)

Let's write and solve some more rational equations.

6.1 Two Directions

Noah likes to go for boat rides along a river with his family. In still water, the boat travels about 8 kilometers per hour. In the river, it takes them the same amount of time t to go upstream 5 kilometers as it does to travel downstream 10 kilometers.

6.2 A Rational River

Noah likes to go for boat rides along a river with his family. In still water, the boat travels about 8 kilometers per hour. In the river, it takes them the same amount of time t to go upstream 5 kilometers as it does to travel downstream 10 kilometers.

1. In still water, how long would it take to travel d kilometers in the boat?

2. If the speed of the river is r, write an expression for the time it takes to travel 5 kilometers upstream and an expression for the time it takes to travel 10 kilometers downstream.

3. Use your expressions to calculate the speed of the river. Explain or show your reasoning.

6.3 Rational Resistance

Circuits in parallel follow this law: The inverse of the total resistance is the sum of the inverses of each individual resistance. We can write this as: $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$, where there are n parallel circuits and R_T is the total resistance. Resistance is measured in ohms.

1. Two circuits are placed in parallel. The first circuit has a resistance of 40 ohms, and the second circuit has a resistance of 60 ohms. What is the total resistance of the two circuits?

2. Two circuits are placed in parallel. The second circuit has a resistance of 150 ohms more than the first. Write an equation for this situation showing the relationships between R_T and the resistance R of the first circuit.

3. For this circuit, Clare wants to use graphs to estimate the resistance of the first circuit R if R_T is 85 ohms. Describe how she could use a graph to determine the value of R, and then follow your instructions to find R.

Are you ready for more?

Two circuits with resistances of 40 ohms and 60 ohms have a combined resistance of 24 ohms when connected in parallel. If we had used two circuits that each had a resistance of 48 ohms, they would have had that same combined resistance. 48 is called the harmonic mean of 40 and 60.

A more familiar way to find the mean of two numbers is to add them up and divide by 2. This is the arithmetic mean. Here is how each kind of mean is calculated:

Harmonic mean of a and b:

$$\frac{2ab}{a+b}$$

Arithmetic mean of a and b:

$$\frac{a+b}{2}$$

The harmonic mean of 40 and 60 was 48, and their arithmetic mean is 50 because $\frac{40+60}{2} = 50$.

Experiment with other pairs of numbers. What can you conclude about the relationship between the harmonic mean and arithmetic mean?

Lesson 6 Summary

A boat travels about 6 kilometers per hour in still water. If the boat is on a river that flows at a constant speed of r kilometers per hour, it can travel at a speed of $6 + r$ kilometers per hour downstream and $6 - r$ kilometers per hour upstream. (And if the river current is the same speed as the boat, the boat wouldn't be able to travel upstream at all!)

On one particular river, the boat can travel 4 kilometers upstream in the same amount of time it takes to travel 12 kilometers downstream. Since time is equal to distance divided by speed, we can express the travel time as either $\frac{12}{6+r}$ hours or $\frac{4}{6-r}$ hours. If we don't know the travel time, we can make an equation using the fact that these two expressions are equal to one another and figure out the speed of the river.

$$\begin{aligned}
\frac{12}{6+r} &= \frac{4}{6-r} \\
\frac{12}{6+r} \cdot (6+r)(6-r) &= \frac{4}{6-r} \cdot (6+r)(6-r) \\
12(6-r) &= 4(6+r) \\
72 - 12r &= 24 + 4r \\
48 &= 16r \\
3 &= r
\end{aligned}$$

Substituting this value into the original expressions, we have $\frac{12}{6+3} = \frac{4}{3}$ and $\frac{4}{6-3} = \frac{4}{3}$, so these two expressions are equal when $r = 3$. This means that when the water flow in the river is about 3 kilometers per hour, it takes the boat 1 hour and 20 minutes to go 4 kilometers upstream and 1 hour and 20 minutes to go 12 kilometers downstream.

Even though we started out with a rational expression on each side of the equation, multiplying each side by the product of the denominators, $(6 + r)(6 - r)$, resulted in an equation similar to ones we have solved before. Multiplying to get an equation with no variables in denominators is sometimes called "clearing the denominators."