Unit 6, Lesson 1

Inputs and Outputs

Goals

- Describe (orally) how input-output diagrams represent rules.
- Identify a rule that describes the relationship between input-output pairs, and explain (orally) a strategy used for figuring out the rule.

Learning Targets

- I can write rules when I know input-output pairs.
- I know how an input-output diagram represents a rule.

Lesson Narrative

This lesson is the beginning of students' introduction to functions. While students will not use the word "function" until later, the purpose of this lesson is to focus students on the idea of input and output pairs connected by a rule.

In a partner activity, students take turns guessing each other's rules from input-output pairs. While some rules are straightforward, other rules require perseverance to identify a rule that depends on if the input is odd or even (MP1).

Next, students learn to make sense of rules represented by input-output diagrams to fill out a table with inputs and associated outputs. In each table, the first input-output pair is identical, illustrating that a single pair is insufficient for determining a rule. The last table returns to the topic of the *Warm-up* and introduces the idea that not all inputs are possible for a rule.

Teacher Notes for IM 6–8 Math Accelerated v.360
The *Lesson Synthesis* mentions how much of the work students do in grade 8 involves linear functions. This is also true of this course, and the idea that two input-output pairs is not always sufficient to determine a rule is still relevant.

Standards

Addressing	**8.F.1**
Building Toward	**8.F.1**

Instructional Routines

- MLR7: Compare and Connect

Required Materials

Materials to Copy

- Guess My Rule Cards (1 copy for every 4 students): Activity 2

Student Facing Learning Goals

Let's make some rules.

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1.1 Dividing by 0

Warm-up

5 min

Activity Narrative

The purpose of this activity is for students to see why an expression that contains the operation of dividing by zero can't be evaluated. They use their understanding of related multiplication and division equations to make sense of this.

Standards

Building Toward **8.F.1**

Launch

Arrange students in groups of 2. Tell students to consider the statements and try to find a value for x that makes the second statement true.

Give students 1 minute of quiet think time followed by 1 minute to share their thinking with their partner. Finish with a whole-class discussion.

Student Task Statement

Study the statements carefully.

- $12 \div 3 = 4$ because $12 = 4 \cdot 3$.
- $6 \div 0 = x$ because $6 = x \cdot 0$.

What value can be used in place of x to create true statements? Explain your reasoning.

Student Response

None. Sample explanation: There is no number that can be multiplied by zero to get something other than zero. Therefore, $x \cdot 0 = 6$ is never a true statement for any value of x.

Activity Synthesis

Select 2–3 groups to share their conclusions about x.

As a result of this discussion, we want students to understand that any expression where a number is divided by zero can't be evaluated. Therefore, we can state that there is no value for x that makes both equations true.

1.2 Guess My Rule

15 min

Activity Narrative

There is a digital version of this activity.

The purpose of this activity is to introduce the idea of input-output rules. Player 1 uses a rule written on a card that only

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they can see. Player 2 chooses inputs to tell Player 1 so that Player 1 can respond with the corresponding output. Player 2 guesses the rule on the card once they think they have enough input-output pairs to know what it is. Partners then reverse roles.

Monitor for students who use different strategies to determine the rule, such as:

- Choosing which numbers to give, such as always starting with 0 or 1, or choosing a sequence of consecutive whole numbers.
- Looking for a difference between the outputs that correspond to inputs that are even numbers and the outputs that correspond to inputs that are odd numbers.

Access for English Language Learners

This activity uses the *Compare and Connect* math language routine to advance representing and conversing as students use mathematically precise language in discussion.

Standards

Addressing **8.F.1**

Instructional Routines

- MLR7: Compare and Connect

Launch

Arrange students in groups of 2.

For students using the print version: Ask a student to act as your partner, and demonstrate the game using a simple rule that does not match one of the cards (like "divide by 2" or "subtract 4").

Ask groups to decide who will be Player 1 and who will be Player 2. Give each group the four rule cards, making sure that the simplest rules are at the top of the deck when face down. Tell students to be careful not to let their partner see what the rule is as they pick up the rule cards. If necessary, tell students that all numbers are allowed, including negative numbers.

Student Task Statement

Keep the rule cards face down. Decide who will go first.

1. Player 1 picks up a card and silently reads the rule without showing it to Player 2.
2. Player 2 chooses an integer and asks Player 1 for the result of applying the rule to that number.
3. Player 1 gives the result, without saying how they got it.
4. Keep going until Player 2 correctly guesses the rule.

After each round, the players switch roles.

Student Response

$x+7, 3x, 2x-5, \frac{x}{2}, 3x+1$

Building on Student Thinking

If students using the cards who difficulty with the rule on Card D, it may be because it involves conditional statements. Consider asking:

- “Which numbers have you tried so far as inputs? What were their outputs?”
- “What could you use to organize the inputs and outputs you have tried?”

Are You Ready for More?

If you have a rule, you can apply it several times in a row and look for patterns. For example, if your rule were “add 1” and you started with the number 5, then by applying that rule over and over again, you would get 6, then 7, then 8, etc., forming an obvious pattern.

Try this for the rules in this activity. That is, start with the number 5, and apply each of the rules a few times. Do you notice any patterns? What if you start with a different number?

Extension Student Response

1. 5, 12, 19, 26, 33, . . .
2. 5, 15, 45, 135, . . .
3. 5, 5, 5, 5, . . .
4. 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1

If the starting number is changed from a 5, the first two rules give the same patterns, adding 7 and tripling each time, but result in different numbers. The third pattern has a much harder pattern to recognize when 5 is not the starting value. The fourth pattern turns out to be incredibly difficult! Most mathematicians suspect that no matter *what* is the starting value, it will always eventually get back to 1 and starting cycling: 1, 4, 2, 1, 4, 2, 1, ... Computers have checked that this pattern appears for any starting number up to one billion billion, but no one knows for sure that it will *always* appear! For more information, research the Collatz Conjecture.

Activity Synthesis

The goal of this discussion is for students to understand what an input-output rule is and share strategies they used for figuring out the rule. Tell students we start with a number, called the *input*, and apply a rule to that number, which results in a number called the *output.* We say the output *corresponds* to that input.

To highlight these words, ask:

- “What is an example of an input from this activity?” (The input is the number that Player 2 chose, or it is the number placed in the ‘input’ box in the applet.)
- “Where in the activity was the rule applied?” (The rule was applied when Player 1 applied the rule to the input their partner told them, or it’s what happened in the black box in the applet.)
- “Where in the activity is the output?” (The output is the number that Player 1 said after applying the rule to the given input, or it is the number given in the black box in the applet.)

Invite previously identified students to briefly describe their strategy for figuring out one of the rules, recording their process for all to see. For example, for the rule “add 7,” they may describe the table they used to organize their input-output pairs and that their last row is x and $x+7$. Use *Compare and Connect* to help students compare, contrast, and connect the different strategies. Here are some questions for discussion:

- “What do the strategies have in common? How are they different?”
- “Did anyone have a similar strategy but would explain it differently?”
- “Are there any benefits or drawbacks to one strategy compared to another?”

Students might think the last rule isn't allowable because there were two “different” rules. Explain that a rule can be anything that always produces an output for a given input. Consider the rule “flip,” where the input is “coin.” The output may be heads or tails. We will consider several different types of rules in the following activities and lessons.

1.3 Making Tables

15 min

Activity Narrative

The purpose of this activity is for students to think of rules more broadly than simple arithmetic operations in preparation for the more abstract idea of a function, which is introduced in the next lesson.

Each problem begins with a diagram that represents a rule and is followed by a table for students to complete with input-output pairs that follow the rule. Monitor for students who notice that even though the rules are different, each one starts out with the same input-output pair: $\frac{3}{4}$ and 7. An important conclusion is that different rules can determine the same input-output pair.

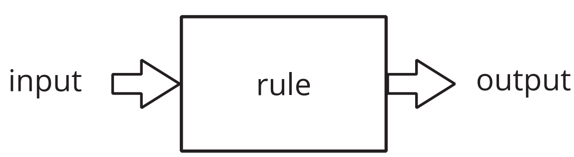
If using digital activities, there is a rule generator as an extension. Students give an input, and the generator gives an output. After a few inputs, students can guess a potential rule. The generator then gives feedback about the rule: “reasonable but not my rule,” “Correct! How did you know?” or “does not match data.”

Standards

Addressing **8.F.1**

Launch

Arrange students in groups of 2. Display the following diagram for all to see:



Tell students that this diagram is one way to think about input-output rules. For example, if the rule were “multiply by 2” and the input were $\frac{3}{2}$, then the output would be 3. Tell students they will use diagrams like this one during the activity to complete tables of input and output values.

Give students 3–5 minutes of quiet work time to complete the first three tables. Then give them time to share their responses with their partner and to resolve any differences.

Give partners 1–2 minutes of quiet work time for the final rule, and follow with a whole-class discussion. Depending on time, ask students to add only one additional input-output pair instead of two.

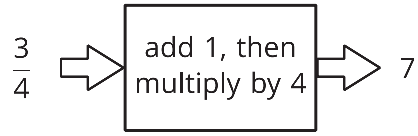
Access for Students with Disabilities

Representation: Internalize Comprehension. Begin by asking, “Does this problem/situation remind anyone of something we have done before?”
Supports accessibility for: Memory, Attention

Student Task Statement

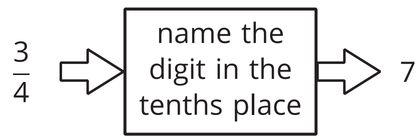
For each input-output rule, fill in the table with the outputs that go with the given inputs. Add two more input-output pairs to the table.

1.



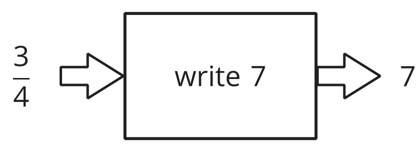
input	output
$\frac{3}{4}$	7
2.35	
42	

2.



input	output
$\frac{3}{4}$	7
2.35	
42	

3.



input	output
$\frac{3}{4}$	7
2.35	
42	

Pause here until your teacher directs you to the last rule.

4.

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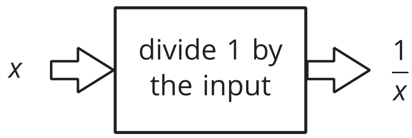

input	output
$\frac{3}{7}$	$\frac{7}{3}$
1	
0	

Student Response

Answers vary for the last two rows in each table.

1.

input	output
$\frac{3}{4}$	7
2.35	13.4
42	172
9	40
1	8

2.

input	output
$\frac{3}{4}$	7
2.35	3
42	0
7.31	3
4.95	9

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3.

input	output
$\frac{3}{4}$	7
2.35	7
42	7
511	7
91	7

4.

input	output
$\frac{3}{7}$	$\frac{7}{3}$
1	1
0	no output
2	$\frac{1}{2}$

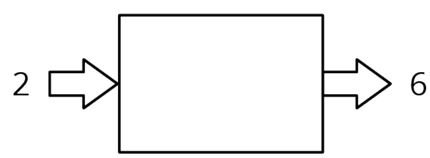
Building on Student Thinking

Students may have trouble thinking of “write 7” as a rule. Emphasize that a rule can be anything that produces a well-defined output, even if it ignores the value of the input. Students who know about infinite decimal expansions might wonder about the second rule because, for example $0.999... = 1.0$, so the same number could have two outputs. If this comes up, discuss how the rule might be refined in this case.

Activity Synthesis

The purpose of this discussion is for students to see rules as more than arithmetic operations on numbers and to consider how sometimes not all inputs are possible.

Display a rule diagram with input 2, output 6, and a blank space for the rule for all to see.



Select 2–3 previously identified students, and ask what the rule for the input-output pair might be. Display these possibilities next to the diagram for all to see. For example, students may suggest the rules such as “Add 4,” “Multiply by 3,” or “Add 1, then multiply by 2.”

The last rule, “1 divided by the input,” calls back to the *Warm-up*. Explain to students that not all inputs are possible for a rule. To highlight this idea, ask:

- “Why was 0 not a valid input for the last rule?” (1 divided by 0 does not exist.)
- “What are some other situations when a rule might not have a valid input?” (Any time an operation requires us to divide by 0, or when the input must be non-negative, such as a side length of a square.)

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- "How does using a variable x to denote the input and $\frac{1}{x}$ to denote the output help us understand the function rule?" (We can clearly see the relationship between the input and output.)

Lesson Synthesis

The main focus of this lesson are the parallel ideas of using sets of inputs and outputs to identify the rule describing the relationship between them and using a rule to create a set of inputs and outputs. When we have an input-output table that represents only some of the input-output pairs, we can guess a rule but we won't know the rule for sure until we see it. For some rules, there are some numbers that are not allowed as inputs because the rule does not make sense.

To highlight some things to remember about input-output rules, ask:

- "Which rule would you rather make an input-output table for: 'Divide 10 by the input' or 'Write the current year'?" (I would rather make a table for the second rule since the outputs would all just be the current year. The table for the first rule will have fractions, and we can't input 0 since 0 divided by 10 does not exist.)
- "What input can you not use with the rule 'Divide 10 by the input added to 3'?" (We cannot use -3 since $-3+3=0$ and 0 divided by 10 does not exist.)

To conclude the discussion, poll the class to find out how many input-output pairs students think they would need to figure out a rule, and record their answers for all to see. While much of the work students do in grade 8 involves linear functions, and two is a sufficient number of pairs, if students think two pairs are always enough, point out that both the first and second rule in this activity share the pair $\frac{3}{4}$ and 7 and the pair -1 and 0. Depending on the context, two input-output pairs is not always sufficient to determine the rule.

1.4 What's the Rule?

Cool-down

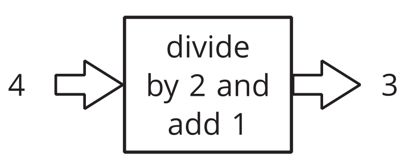
5 min

Standards

Addressing **8.F.1**

Student Task Statement

Fill in the table for this input-output rule:



input	output
0	
2	
-8	
100	

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Student Response

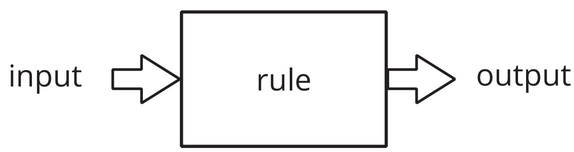
In each row, the output should be one more than half of the input.

input	output
0	1
2	2
-8	-3
100	51

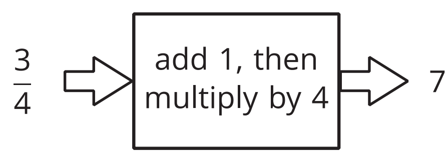
Responding to Student Thinking

More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

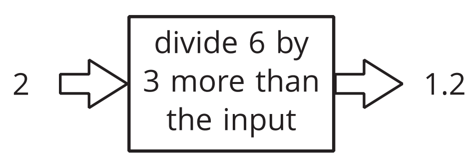
Lesson 1 Summary



An *input-output rule* is a rule that takes an allowable input and uses it to determine an output. For example, the following diagram represents the rule that takes any number as an input, then adds 1, multiplies by 4, and gives the resulting number as an output.



In some cases, not all inputs are allowable, and the rule must specify which inputs will work. For example, this rule is fine when the input is 2:



But if the input were -3, we would need to evaluate $6 \div 0$ to get the output.

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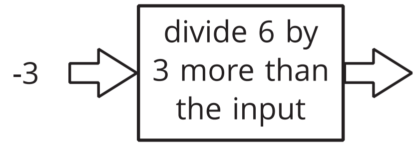

So, when we say that the rule is “Divide 6 by 3 more than the input,” we also have to say that -3 is not allowed as an input.

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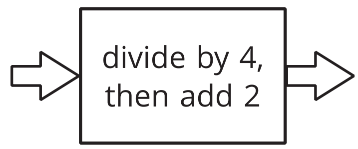
Lesson 1 Practice Problems

1 Student Task Statement

Given the rule:



Complete the table for the function rule for the following input values:

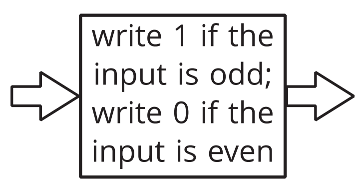
input	0	2	4	6	8	10
output						

Solution

2, 2.5, 3, 3.5, 4, 4.5

2 Student Task Statement

Here is an input-output rule:



Complete the table for the input-output rule:

input	-3	-2	-1	0	1	2	3
output							

Solution

1, 0, 1, 0, 1, 0, 1

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3 from Unit 5, Lesson 17

Student Task Statement

Andre's school orders some new supplies for the chemistry lab. The online store shows a pack of 10 test tubes that costs \$4 less than a set of nested beakers. In order to fully equip the lab, the school orders 12 sets of beakers and 8 packs of test tubes.

a. Write an equation that shows the cost of a pack of 10 test tubes, t, in terms of the cost of a set of nested beakers, b.

b. The school office receives a bill for the supplies in the amount of \$348. Write an equation with t and b that describes this situation.

c. Since t is in terms of b in the first equation, this expression can be substituted into the second equation where t appears. Write an equation that shows this substitution.

d. Solve the equation for b.

e. How much did the school pay for a set of beakers? For a pack of test tubes?

Solution

a. $t = b - 4$ (or equivalent)
b. $8t + 12b = 348$ (or equivalent)
c. $8(b - 4) + 12b = 348$ (or equivalent)
d. $b = 19$
e. \$19 for a set of beakers and \$15 for a pack of test tubes

4 from Unit 5, Lesson 16

Student Task Statement

Solve: $\begin{cases} y = x - 4 \\ y = 6x - 10 \end{cases}$

Solution

$\left(\frac{6}{5}, \frac{-14}{5}\right)$. Sample reasoning: Substituting $x - 4$ for y into the second equation, we get $x - 4 = 6x - 10$. Solving this equation gives $x = \frac{6}{5}$. Substituting $x = \frac{6}{5}$ into $y = x - 4$, we get $y = \frac{-14}{5}$.

5 from Unit 4, Lesson 11

Student Task Statement

For what value of x do the expressions $2x + 3$ and $3x - 6$ have the same value?

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Solution

$x = 9$

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