Unit 3, Lesson 10

Using Trigonometric Ratios to Find Angles

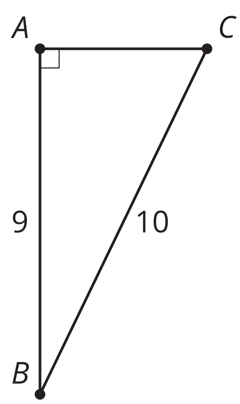
Let’s work backwards to find angles in right triangles.

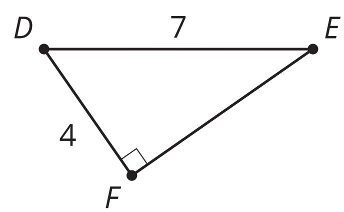
10.1 Once More with the Table

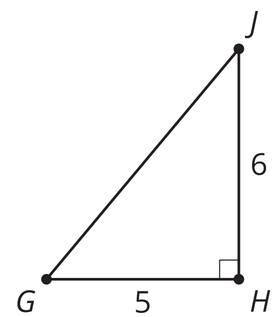
A triangle with side lengths 3, 4, and 5 is a right triangle by the converse of the Pythagorean Theorem. What are the measures of the acute angles?

10.2 From Ratios to Angles

Find all unknown side lengths and angle measures.







10.3 Leaning Ladders

A good rule of thumb for a safe angle to use when leaning a ladder is the angle formed by your body when you stand on the ground and hold your arms out parallel to the ground.

1. What are the angles in the triangle formed by your body and the ladder?
2. What are the angles in the triangle formed by the ladder, the ground, and the railing of the house? Explain or show your reasoning.
3. You have a ladder that is 13 feet long and need to climb to a roof that is 12 feet tall.
   a. If you put the top of the ladder at the top of the wall, what angle is formed between the ladder and the ground?
   b. Is it possible to adjust the ladder to a safe angle? If so, give someone instructions to do so. If not, explain why not.

Are you ready for more?

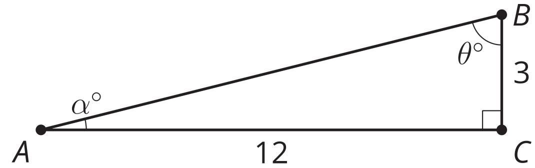
People have various proportions to their body. Suppose that someone's height to arm length ratio is $5 : 1$.

1. What are the angles in the triangle formed by their body and the ladder?
2. How far off are the angles in this triangle from the angles in the safe ratio, $4 : 1$?
3. What could this person do to make the ladder have an angle closer to the angle for a safe ladder?

Lesson 10 Summary

Using trigonometric ratios and a calculator, the unknown side lengths and angle measures of right triangles can be found.

Using the Right Triangle Table, we can estimate angle measures as in previous lessons. However, with a calculator, we can find angles more precisely.



The side opposite angle A is 3 units long, and the side adjacent to A is 12 units long. So to find angle A, we write an equation using tangent: $\tan(\alpha) = \frac{3}{12}$. To find the measure of angle A, we ask the calculator, "What angle has a tangent of $\frac{3}{12}$?" To ask that, we use *arctangent* by writing $\arctan\left(\frac{3}{12}\right)$. The **arctangent** of a positive number is the measure of an acute angle whose tangent is that number. If we know the cosine of an angle, we use *arccosine* to look up the angle measure (the **arccosine** of a number between 0 and 1 is the measure of an acute angle whose cosine is that number). And if we know the sine of an angle, we use *arcsine* (the **arcsine** of a number between 0 and 1 is the measure of an acute angle whose sine is that number). So $\alpha = \arctan\left(\frac{3}{12}\right)$, which means angle A measures about 14 degrees.

Angle B can be calculated using another trigonometric equation or the Triangle Angle Sum Theorem. Let's use arctangent again. We know $\tan(\theta) = \frac{12}{3}$, so $\theta = \arctan\left(\frac{12}{3}\right)$, which is about 76 degrees. This matches the answer we get with the Triangle Angle Sum Theorem: $180 - 90 - 14 = 76$.

