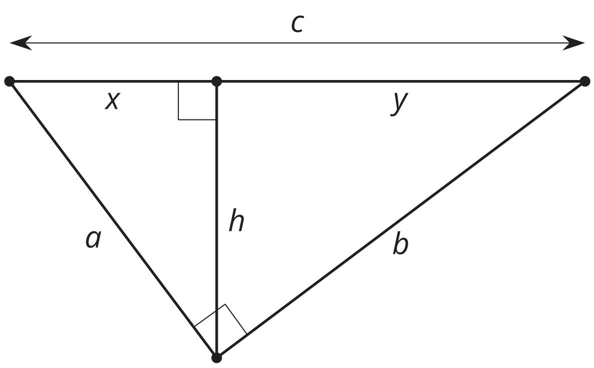
Lesson 14: Proving the Pythagorean Theorem

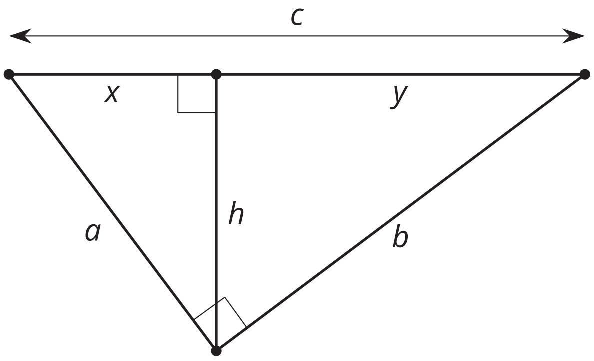
- Let's prove the Pythagorean Theorem.

14.1: Notice and Wonder: Variable Version



What do you notice? What do you wonder?

14.2: Prove Pythagoras Right



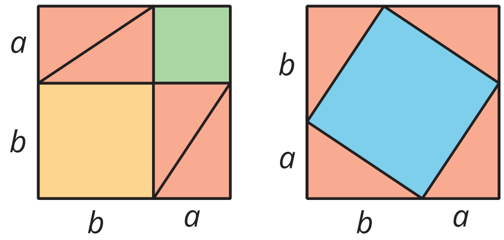
Elena is playing with the equivalent ratios she wrote in the warm-up. She rewrites $\frac{a}{x} = \frac{c}{a}$ as $a^2 = xc$. Diego notices and comments, “I got $b^2 = yc$. The a^2 and b^2 remind me of the Pythagorean Theorem.” Elena says, “The Pythagorean Theorem says that $a^2 + b^2 = c^2$. I bet we could figure out how to show that.”

1. How did Elena get from $\frac{a}{x} = \frac{c}{a}$ to $a^2 = xc$?

2. What equivalent ratios of side lengths did Diego use to get $b^2 = yc$?

3. Prove $a^2 + b^2 = c^2$ in a right triangle with legs length a and b and hypotenuse length c.

14.3: An Alternate Approach



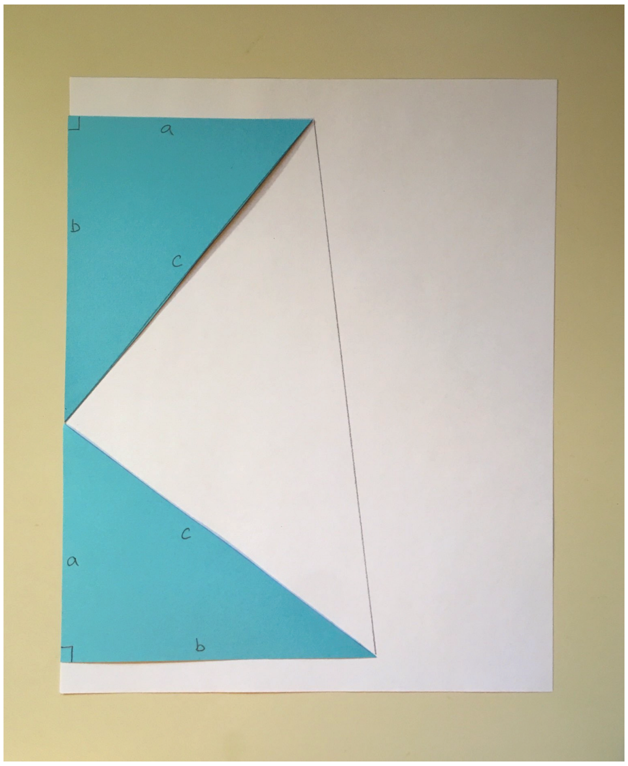
When Pythagoras proved his theorem he used the 2 images shown here. Can you figure out how he used these diagrams to prove $a^2 + b^2 = c^2$ in a right triangle with hypotenuse length c?

Are you ready for more?

James Garfield, the 20th president, proved the Pythagorean Theorem in a different way.

- Cut out 2 congruent right triangles
- Label the long sides b, the short sides a and the hypotenuses c.
- Align the triangles on a piece of paper, with one long side and one short side in a line. Draw the line connecting the other acute angles.

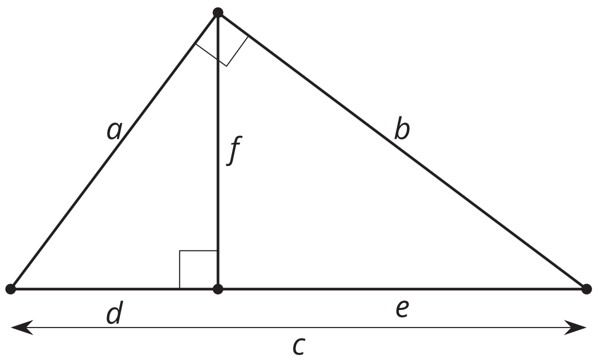
How does this diagram prove the Pythagorean Theorem?



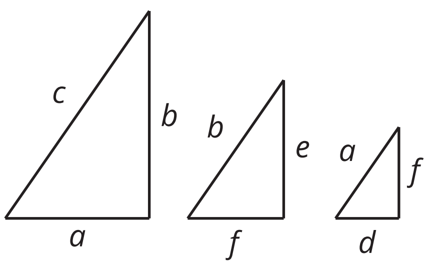
Lesson 14 Summary

In any right triangle with legs a and b and hypotenuse c, we know that $a^2 + b^2 = c^2$. We call this the Pythagorean Theorem. But why does it work?

We can use an altitude drawn to the hypotenuse of a right triangle to prove the Pythagorean Theorem.



We can use the Angle-Angle Triangle Similarity Theorem to show that all 3 triangles are similar. Because the triangles are similar, corresponding side lengths are in the same proportion.



Because the largest triangle is similar to the smaller triangle, $\frac{c}{a} = \frac{a}{d}$. Because the largest triangle is similar to the middle triangle, $\frac{c}{b} = \frac{b}{e}$. We can rewrite these equations as $a^2 = cd$ and $b^2 = ce$.

We can add the 2 equations to get that $a^2 + b^2 = cd + ce$ or $a^2 + b^2 = c(d + e)$. From the original diagram we can see that $d + e = c$, so $a^2 + b^2 = c(c)$ or $a^2 + b^2 = c^2$.

Using the Pythagorean Theorem we can describe a triangle's angles without ever drawing it. For example, a triangle with side lengths 8, 15, and 17 is right because $17^2 = 8^2 + 15^2$. A triangle with side lengths 8, 15, and 18 is obtuse because $18^2 > 8^2 + 15^2$. A triangle with side lengths 8, 15, and 16 is acute because $16^2 < 8^2 + 15^2$.