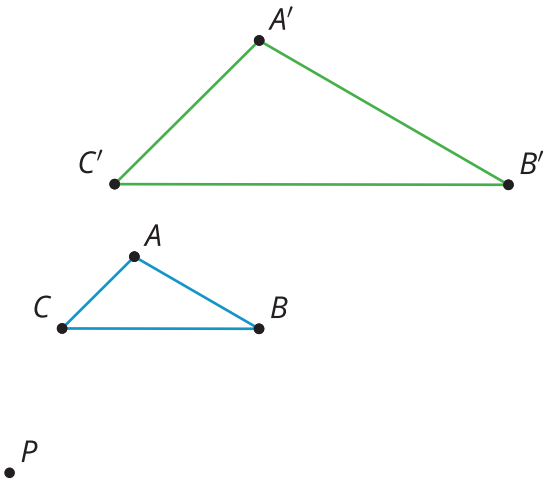
Unit 2, Lesson 4

Dilating Lines and Angles

Let's dilate lines and angles.

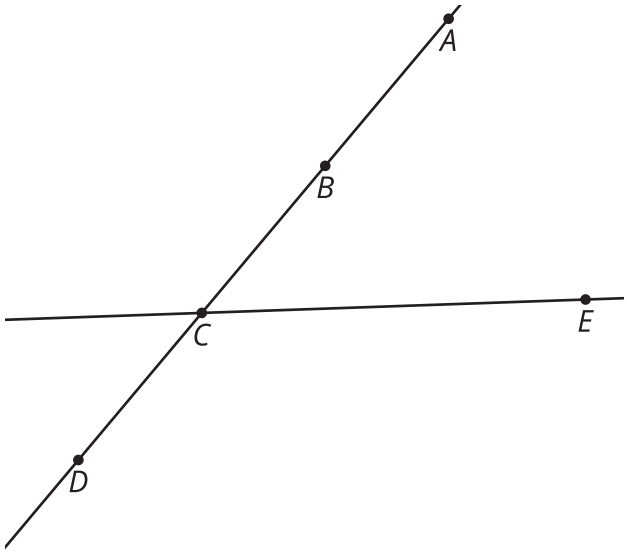
4.1 Angle Articulation



Triangle $A'B'C'$ is a dilation of triangle ABC using center P and a scale factor of 2.

1. What is the same about the two triangles? What is different? Make a conjecture about what stays the same after dilation.
2. Use the tools available to figure out if what you thought was true is definitely true for these triangles.
3. Do you think your conjecture will be true for any figure after dilation?

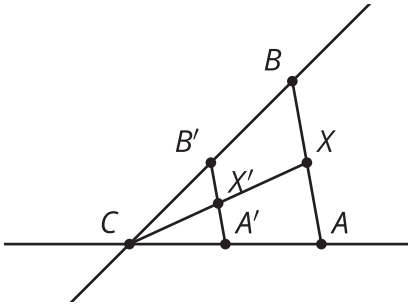
4.2 Dilating Lines



1. Dilate point A using center C and a scale factor of $\frac{3}{4}$.
2. Dilate point B using center C and a scale factor of $\frac{1}{3}$.
3. Dilate point D using center C and a scale factor of $\frac{3}{2}$.
4. Dilate line CE using center C and a scale factor of 2.
5. What happens when the center of dilation is on a line and then you dilate the line?

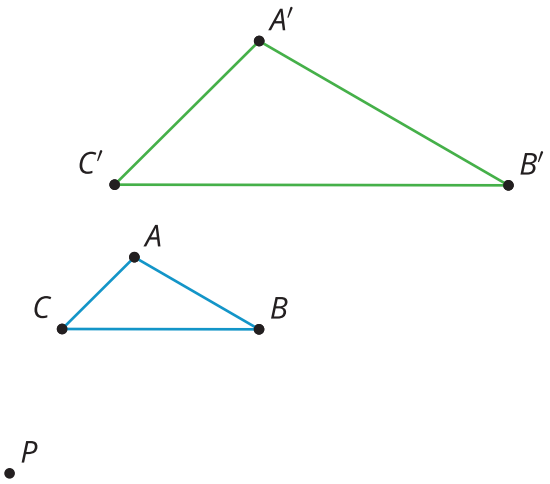
Are you ready for more?

- X is the midpoint of AB.
- B' is the image of B after being dilated by a scale factor of 0.5 using center C.
- A' is the image of A after being dilated by a scale factor of 0.5 using center C.



Call the intersection of CX and $A'B'$ point X'. Is point X' a dilation of point X? Explain or show your reasoning.

4.3 Proof in Parallel



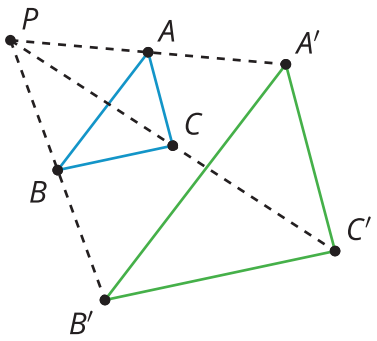
Jada dilates triangle ABC using center P and a scale factor of 2.

1. Prove the conjecture from your whole-class discussion.
2. In Jada's diagram the scale factor is greater than one. Would your proof have to change if the scale factor is less than one?

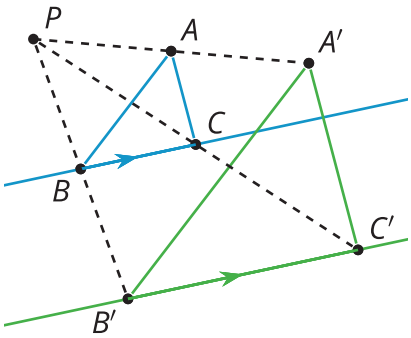
Lesson 4 Summary

When one figure is a dilation of the other, we know that corresponding side lengths of the original figure and the dilated image are in the same proportion, and are all related by the same scale factor, k. What is the relationship of corresponding angles in the original figure and the dilated image?

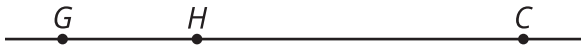
For example, if triangle ABC is dilated, using center P, with a scale factor of 2, we can verify experimentally that each angle in triangle ABC is congruent to its corresponding angle in triangle $A'B'C'$. $\angle A$ is congruent to $\angle A'$. $\angle B$ is congruent to $\angle B'$. $\angle C$ is congruent to $\angle C'$.



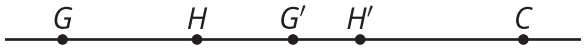
What is the image of a line not passing through the center of dilation? For example, what will be the image of line BC when it is dilated with center P and a scale factor of 2? We can use congruent corresponding angles to show that line BC is taken to parallel line $B'C'$.



What is the image of a line passing through the center of dilation?



For example, what will be the image of line GH when it is dilated with center C and a scale factor of $\frac{1}{2}$? When line GH is dilated with center C and a scale factor of $\frac{1}{2}$, line GH is unchanged, because dilations take points on a line through the center of a dilation to points on the same line, by definition.



So a dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.