



Using Graphs to Find Average Rate of Change

Let's measure how quickly the output of a function changes.

7.1 Temperature Drop

Here are the recorded temperatures at three different times on a winter evening.

time	4 p.m.	6 p.m.	10 p.m.
temperature	25°F	17°F	8°F

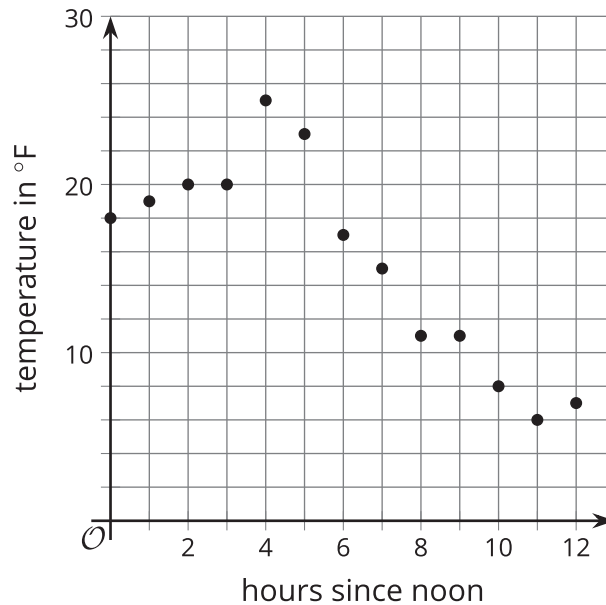
- Tyler says the temperature dropped faster between 4 p.m. and 6 p.m.
- Mai says the temperature dropped faster between 6 p.m. and 10 p.m.

Who do you agree with? Explain your reasoning.

7.2 Drop Some More

The table and graph show a more complete picture of the temperature changes on the same winter day. The function T gives the temperature in degrees Fahrenheit, h hours since noon.

h	$T(h)$
0	18
1	19
2	20
3	20
4	25
5	23
6	17
7	15
8	11
9	11
10	8
11	6
12	7



- Find the **average rate of change** for the following intervals. Explain or show your reasoning.
 - between noon and 1 p.m.
 - between noon and 4 p.m.
 - between noon and midnight
- Remember Mai and Tyler's disagreement? Use average rate of change to show which time period—4 p.m. to 6 p.m. or 6 p.m. to 10 p.m.—experienced a faster temperature drop.



- ### 7.3 Populations of Two States

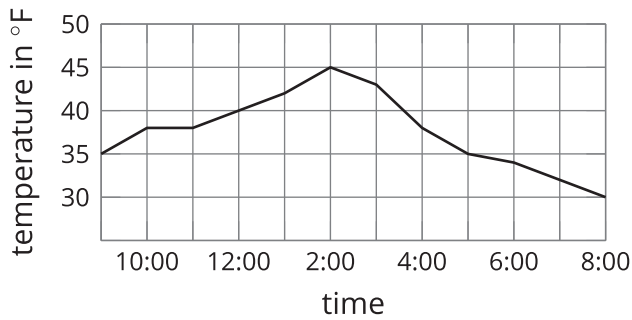
The graph displays the population growth of California and Texas over a century. California's population, represented by a solid blue line, shows a steady increase, starting at 2 million in 1910 and reaching approximately 38 million by 2010. Texas's population, represented by a dashed red line, also shows a steady increase, starting at 3 million in 1910 and reaching approximately 25 million by 2010. The two lines intersect around 1935, after which California's population grows at a faster rate than Texas's.

Year	California (millions)	Texas (millions)
1910	2	3
1920	3	4
1930	5	6
1940	7	7
1950	11	8
1960	16	10
1970	21	11
1980	24	14
1990	30	17
2000	34	21
2010	38	25

1.
 - a. Estimate the average rate of change in the population in each state between 1970 and 2010. Show your reasoning.
 - b. In this situation, what does each rate of change mean?
2. Which state's population grew more quickly between 1900 and 2000? Show your reasoning.

Lesson 7 Summary

Here is a graph of one day's temperature as a function of time.



The temperature was 35°F at 9 a.m. and 45°F at 2 p.m., an increase of 10°F over those 5 hours.

The increase wasn't constant, however. The temperature rose from 9 a.m. and 10 a.m., stayed steady for an hour, then rose again.

- On average, how fast was the temperature rising between 9 a.m. and 2 p.m.?

Let's calculate the **average rate of change** and measure the temperature change per hour. We do that by finding the difference in the temperature between 9 a.m. and 2 p.m. and dividing it by the number of hours in that interval.

$$\text{average rate of change} = \frac{45 - 35}{5} = \frac{10}{5} = 2$$

On average, the temperature between 9 a.m. and 2 p.m. increased 2°F per hour.

- How quickly was the temperature falling between 2 p.m. and 8 p.m.?

$$\text{average rate of change} = \frac{30 - 45}{6} = \frac{-15}{6} = -2.5$$

On average, the temperature between 2 p.m. and 8 p.m. dropped by 2.5°F per hour.

In general, we can calculate the average rate of change of a function f between input values a and b by dividing the difference in the outputs by the difference in the inputs.

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a}$$

If the two points on the graph of the function are $(a, f(a))$ and $(b, f(b))$, the average rate of change is the slope of the line that connects the two points.

