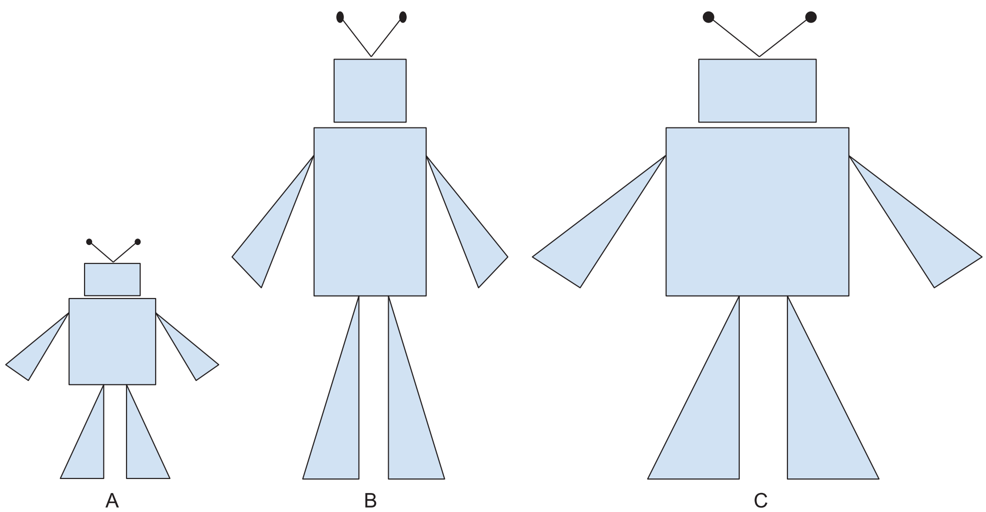
Unit 2 Family Support Materials

Similarity

In this unit, your student will be learning about similarity, sometimes called a scaled image. They study a variety of similar figures and write proofs about triangles using similarity, including a proof of the Pythagorean Theorem. Then they use the statements they've proven to solve new problems.

Students start with some comparisons. They look at different images to decide what stays the same and what changes with similar figures. In these images of robots, Images A and C are similar figures.

- How can you see that Image B is not similar to either Image A or Image C?
- What is the same about Images A and C? What is different?



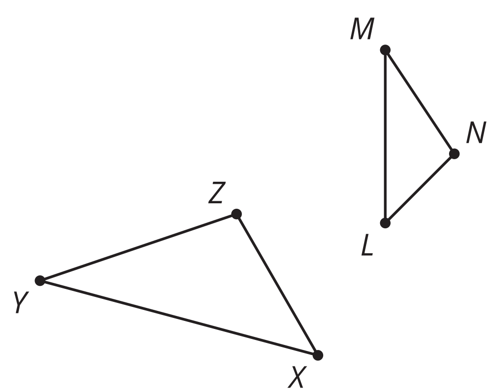
It looks like some parts of the shape stay the same no matter what. The rectangles stay rectangles in all 3 images. But in Image B, the sides of the rectangle for the head look almost the same. It might even be a square. That is how we can know that Image B is not similar to either Image A or C. The triangles for the legs in Image A are twice as tall as they are wide. This same ratio holds for Image C. The proportionality of corresponding sides is one of the characteristics of similar figures. Another characteristic of similar figures is that the corresponding angles stay the same. In Images A and C the antennas on top of the robot make an angle that looks almost like a right angle or maybe a little greater than 90°, but in Image B the angle between the antennas appears acute (less than 90°).

Recall that figures are called congruent if we can find rigid transformations (translation, rotation, reflection) that take one figure exactly onto the other figure so that every part lines up. Two figures are called similar if we can find any transformations (translation, rotation, reflection, dilation) that take one figure exactly onto the other figure so that every part lines up. The new transformation, dilation, stretches or shrinks the figure the same amount in every direction.

For the robots, Image C is a translation (slide to the side) and dilation (stretch or shrink) of Image A. To dilate an image we need to choose a scale factor to describe how much to stretch or shrink the figure. The scale factor to go from Image A to Image C is 2. Every segment will be twice as long after the dilation. Image B is a translation of Image A, but stretches only in the vertical (up and down) direction, so there is no dilation that lines up the figures because a dilation must stretch the same amount in every direction.

Maps are usually drawn so that they are similar to the layout of an area. A map that represents 1 kilometer using 2 centimeters (0.00002 km) on the map would have a scale factor of $\frac{1}{50,000}$ because every distance on the map is 50,000 times smaller than the actual distance. The image on the map is much smaller than the city it shows, but all of the angles between roads stay the same and the distances are all shrunk by the same amount, so the image is not distorted.

Here is a task to try with your student:

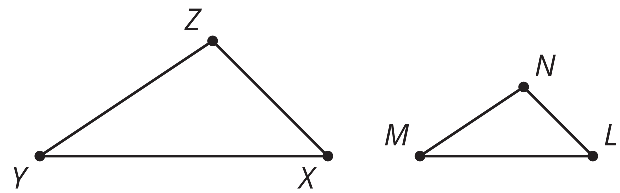


Triangles XYZ and LMN are similar triangles.

1. Redraw the triangles so the corresponding sides are easier to see. Name the corresponding sides and angles (the sides that would line up, after transforming them, to be on top of one another).
2. Angle X is 45 degrees, and angle N is 101 degrees. What are the measures of the other angles?
3. Side XY is 5 units long, and side LM is 3 units long.
    a. What is the scale factor of the dilation that takes triangle XYZ to triangle LMN?
    b. What is the scale factor of the dilation that takes triangle LMN to triangle XYZ?

CC BY NC 2024
Illustrative Mathematics®

Solution:



1. Angle X is corresponding to angle L.
   Angle Y is corresponding to angle M.
   Angle Z is corresponding to angle N.
   Side XY is corresponding to side LM.
   Side YZ is corresponding to side MN.
   Side ZX is corresponding to side NL.
2. Angle $L = 45^\circ$ because it corresponds to angle X, and corresponding angles in similar figures are congruent. Angle $Z = 101^\circ$ because it corresponds to angle N. Angle $M = Y = 34^\circ$ because the angles in a triangle add up to 180°, and $180 - 101 - 45 = 34$.
3.
   a. $\frac{3}{5} = 0.6$ because triangle XYZ needs to be shrunk to $\frac{3}{5}$ of its original size to be the same size as triangle LMN.

   b. $\frac{5}{3}$ because triangle LMN needs to be stretched to $\frac{5}{3}$ of its original size to be the same size as triangle XYZ.

CC BY NC 2024
Illustrative Mathematics®