Unit 1, Lesson 17

Squares and Cubes

Goals

- Generalize a process for finding the volume of a cube, and justify (orally) why this can be abstracted as s^3.
- Include appropriate units (orally and in writing) when reporting lengths, areas, and volumes, e.g., cm, cm^2, cm^3.
- Interpret and write expressions with exponents ^{2} and ^{3} to represent the area of a square or the volume of a cube.

Learning Targets

- I can write and explain the formula for the volume of a cube, including the meaning of the exponent.
- When I know the edge length of a cube, I can find the volume and express it using appropriate units.

Lesson Narrative

In this lesson, students learn about perfect squares and perfect cubes. They see that these names come from the areas of squares and the volumes of cubes with whole-number side lengths. Students are introduced to **exponents** in this context.

Students learn to use the exponent ^{2} to express multiplication of two side lengths of a square and the exponent ^{3} to express multiplication of three edge lengths of a cube. They learn that the words **squared** and **cubed** can be used to describe expressions with exponents ^{2} and ^{3} and see the geometric motivation for this terminology. (The term "exponent" is deliberately not defined more generally at this time. Students will work with exponents in more depth in a later unit.)

Throughout the lesson, students attend to precision (MP6) as they think about the units for length, area, and volume. To write the formula for the volume of a cube, students also look for and express regularity in repeated reasoning (MP8).

Math Community
Today's activity is for students to individually reflect on the norms generated so far. During the *Cool-down*, students provide feedback on the norms, sharing those they agree with and those they feel need revision or removal. These suggestions will inform the next version of the classroom norms.

Standards

Building On	**4.MD.A.3, 5.MD.C.5.a**
Addressing	**6.EE.A, 6.EE.A.1**
Building Toward	**6.EE.A.1**

Instructional Routines

- 5 Practices
- MLR2: Collect and Display

Required Materials

Materials to Gather

- Math Community Chart: Activity 1, Cool-down
- Snap cubes: Activity 2

Required Preparation

Activity 2:

Prepare sets of 32 snap cubes for each group of 2 students.

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's investigate perfect squares and perfect cubes.

17.1 Perfect Squares

Warm-up

5 min

Activity Narrative

This activity introduces the concept of "perfect squares." It also includes opportunities to practice using units of measurement, which offers insights about students' knowledge from preceding lessons.

Some students may benefit from using physical tiles to reason about perfect squares. Provide access to square tiles, if available.

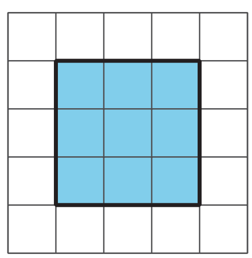
As students work, notice whether they use appropriate units for the questions about area.

Standards

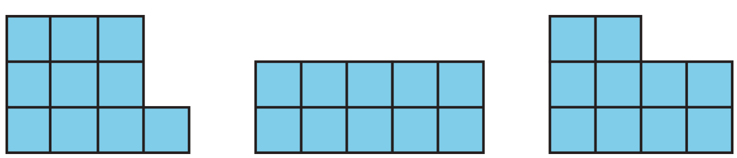
Building On **4.MD.A.3**

Launch

Tell students, "Some numbers are called 'perfect squares.' For example, 9 is a perfect square. Nine copies of a small square can be arranged into a large square." Display a square like this for all to see:



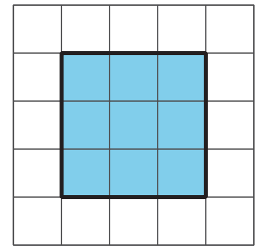
Explain that 10, however, is not a perfect square. Display images such as shown here, emphasizing that 10 small squares can not be arranged into a large square (the way 9 small squares can).



Tell students that in this warm-up they will find more numbers that are perfect squares. Give students 2 minutes of quiet think time to complete the activity.

Student Task Statement

1. The number 9 is a "perfect square." Find four numbers that are perfect squares and two numbers that are not perfect squares.
2. A square has side length 7 in. What is its area?
3. The area of a square is 64 sq cm. What is its side length?



Student Response

1. Sample response: Perfect squares: 9, 25, 4, 49, 100. Not perfect squares: $\frac{1}{2}$, 2, 3, 10.
2. 49 square inches
3. 8 centimeters

Building on Student Thinking

If students do not recall what the abbreviations km, cm, and sq stand for, provide that information.

Students may divide 64 by 2 for the third question. If students are having trouble with this, ask them to check by working backward i.e., by multiplying the side lengths to see if the product yields the given area measure.

Activity Synthesis

Invite students to share the examples and non-examples they found for perfect squares. Solicit some ideas on how they decided if a number is or is not a perfect square.

If a student asks about 0 being a perfect square, wait until the end of the lesson, when the exponent notation is introduced. 0 *is* a perfect square because $0^2 = 0$.

Briefly discuss students' responses to the last two questions, the last one in particular. If not already uncovered in discussion, highlight the reasoning for finding the side length of a square given its area. The area of a square is found by multiplying its two equal side lengths. So, if we know the area, we can find the side length by answering the question "What number times itself equals the area?"

Math Community
Display the Math Community Chart. Remind students that norms are agreements that everyone in the class shares responsibility for, so it is important that everyone understands the intent of each norm and can agree with it. Tell students that today's *Cool-down* includes a question asking for feedback on the drafted norms. This feedback will help identify which norms the class currently agrees with and which norms need revising or removing.

17.2 Building with 32 Cubes

15 min

Optional

Activity Narrative

There is a digital version of this activity.

In this activity, students revisit the meaning of volume and how to find it by building the largest cube possible from 32 snap cubes. Students also become familiar with two perfect cubes, 27 and 64, before the next activity introduces this term.

The activity prompts students to recall their work from grade 5—that the volume of a rectangular prism can be calculated in two different ways: by counting unit cubes that can be packed into the prism, and by multiplying the edge lengths of the prism.

Monitor the different approaches students take to find the volume of the built cube. Here are some likely ways, from less efficient to more efficient:

- Counting all of the snap cubes individually
- Counting the number of snap cubes per layer and then multiplying that by the number of layers
- Multiplying three edge lengths

This activity works best when each student has access to snap cubes. If physical cubes are not available, consider using the digital version of the activity. In the digital version, students use an applet that has 32 cubes with which to build a prism. For students who finish early, another applet with 64 cubes can be found in *Are You Ready for More?*

Standards

Building On **5.MD.C.5.a**

Instructional Routines

- 5 Practices

Launch

Arrange students in groups of 2. Give 32 snap cubes to each group. If centimeter cubes are available, have students work in centimeters instead of the generic units listed here. Give students 8–10 minutes to build the largest cube they can and answer the questions.

For groups who finish early, consider asking them to combine their cubes and build the largest single cube they can with 64 cubes. Then ask them to answer the same four questions as in the activity.

Select students who used each strategy described in the *Activity Narrative* to share later. Aim to elicit both key mathematical ideas and a variety of student approaches, especially from students who haven't shared recently.

Student Task Statement

Your teacher will give you 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?
2. What is the edge length of the cube you built?
3. What is the area of each face of the built cube? Be prepared to explain your reasoning.
4. What is the volume of the built cube? Be prepared to explain your reasoning.

Student Response

1. 27
2. 3 units
3. 9 square units
4. 27 cubic units

Building on Student Thinking

Students may neglect to write units for length or area and may need a reminder to do so.

When determining area, students may multiply a side by two instead of squaring it. When determining volume, they may multiply a side by three instead of cubing it. If this happens, ask them to count individual squares so that they can see that there is an error in their reasoning.

Are You Ready for More?

Combine your 32 snap cubes with another group's 32 snap cubes. Use them to build the largest single cube you can. Each small cube has an edge length of 1 unit.

1. How many snap cubes did you use?
2. What is the edge length of the cube you built?
3. What is the area of each face of the built cube? Show your reasoning.
4. What is the volume of the built cube? Show your reasoning.

Extension Student Response

1. 64 cubes
2. 4 units
3. 16 square units. Sample reasoning: $4 \cdot 4 = 16$
4. 64 cubic units. Sample reasoning: $4 \cdot 4 \cdot 4 = 16$

Activity Synthesis

Focus the whole-class discussion on the ways in which students found the volumes of the two cubes they built. Invite previously selected students to share their strategies. Sequence the discussion of the strategies in the order listed in the *Activity Narrative*, starting from the less efficient (counting individual cubes) to the most efficient (multiplying side lengths).

Connect the last strategy to the learning goals by emphasizing how the side length of a cube determines its volume. Specifically, highlight that the number 27 is $3 \cdot 3 \cdot 3$. If any group built a cube with 64 snap cubes, point out that the number 64 is $4 \cdot 4 \cdot 4$. These observations prepare students to think about perfect cubes in the next activity and about a general expression for the volume of a cube later in the lesson.

17.3 Perfect Cubes

10 min

Activity Narrative

In this activity, students think about examples and non-examples of perfect cubes and find the volumes of cubes given their edge lengths. They see that the edge length of a cube determines its volume, notice the numerical expressions that can be written when calculating volumes, and write a general expression for finding the volume of a cube (MP8).

Some students may be unsure about writing the answer to the last question symbolically because it involves a variable.

Those students may prefer to write a verbal explanation. This is fine, because in an upcoming lesson they will learn to use exponential notation with numbers and variables.

Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Standards

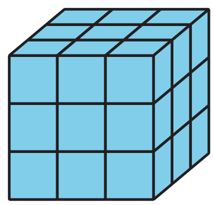
Building On	**5.MD.C.5.a**
Addressing	**6.EE.A**
Building Toward	**6.EE.A.1**

Instructional Routines

- MLR2: Collect and Display

Launch

Tell students, “Some numbers are called ‘perfect cubes.’ For example, 27 is a perfect cube.” Display a cube like this for all to see:



Arrange students in groups of 2. Give students a few minutes of quiet think time, and another minute to discuss their responses with their partner.

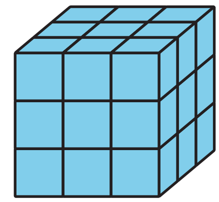
Use *Collect and Display* to create a shared reference that captures students’ developing mathematical language. Collect the language that students use to describe how they reason about the volume of a cube. Display phrases, such as “a number times itself and times itself again,” “multiply a number by itself twice,” “multiply three of the same number,” “multiply three edge lengths,” “a perfect square times the side length of the square,” “perfect cube,” and “not a perfect cube.” Also collect the language that students use to describe how they know if a number is a perfect cube. Display phrases, such as “I can’t make a cube that has a volume of _____” or “There is no number I can multiply by itself twice to get _____.”

Access for Students with Disabilities

Engagement: Provide Access by Recruiting Interest. Provide choice and autonomy. Provide access to snap cubes or blocks, or dot paper to help students analyze examples and non-examples of perfect cubes, paying close attention to the relationships between side lengths and volume.
Supports accessibility for: Visual-Spatial Processing, Organization

Student Task Statement

1. The number 27 is a "perfect cube." Find four other numbers that are perfect cubes and two numbers that are *not* perfect cubes.
2. A cube has a side length of 4 cm. What is its volume?
3. A cube has a side length of 10 inches. What is its volume?
4. A cube has a side length of s units. What is its volume?



Student Response

1. Sample responses: Cubes: 1, 8, 64, 125, 216, 1,000. Non-cubes: 2, 3, 4.
2. $4 \cdot 4 \cdot 4$ or 64 cubic cm
3. $10 \cdot 10 \cdot 10$ or 1,000 cubic inches
4. $s \cdot s \cdot s$ cubic units

Building on Student Thinking

Watch for students who use square units instead of cubic units. Remind them that volume is a measure of the space inside the cube and is measured in cubic units.

Students may multiply by 3 when finding the volume of a cube instead of multiplying three edge lengths (which happen to be the same number). Likewise, they may think a perfect cube is a number times 3. Suggest that they sketch or build a cube with that edge length and count the number of unit cubes. Or ask them to think about how to find the volume of a prism when the edge lengths are different (for instance, a prism that is 1 unit by 2 units by 3 units).

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display*. Ask students to share how they thought about the first question and decided if a number is or is not a perfect cube. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

Highlight the idea that multiplying three edge lengths allows us to determine volume efficiently, and that determining if a number is a perfect cube involves thinking about whether it is a product of three of the same number.

If a student asks about 0 being a perfect cube, wait until the end of the lesson, when exponent notation is introduced. 0 *is* a perfect cube because $0^3 = 0$.

Make sure that students see the answers to the last three questions written as expressions:

$$4 \cdot 4 \cdot 4$$

$$10 \cdot 10 \cdot 10$$

$$s \cdot s \cdot s$$

17.4 Introducing Exponents

15 min

Activity Narrative

This activity introduces students to the exponents "2" and "3" and the language that we use to talk about them. Students use and interpret this notation in the context of both geometric squares and their areas, and geometric cubes and their volumes. Students are likely to have seen exponent notation for 10^3 in their work on place values in grade 5. That experience would be helpful but is not necessary.

Note that the term "exponent" is deliberately *not* defined more generally at this time. Students will work with exponents in more depth in a later unit.

As students work, observe how they approach the last two questions. Identify a couple of students who approach the fourth question differently so they can share later. Also notice whether students include appropriate units, written using exponents, in their answers.

Access for English Language Learners

This activity uses the *Collect and Display* math language routine to advance conversing and reading as students clarify, build on, or make connections to mathematical language.

Standards

Addressing **6.EE.A.1**

Instructional Routines

- MLR2: Collect and Display

Launch

Ask students if they have seen an expression such as 10^3 before. Tell students that in this expression, the 3 is called an **exponent**. Explain the use of the exponents "2" and "3":

- "When we multiply two of the same number together, such as $5 \cdot 5$, we say that we are 'squaring' the number and can write the expression as 5^2. The raised 2 in 5^2 is called an 'exponent.'"
- "Because 5^2 is 25, we can write $5^2 = 25$ and say, '5 **squared** is 25. We can also say that 25 is a 'perfect square.'"
- "When we multiply three of the same number together, such as $4 \cdot 4 \cdot 4$, we say that we are 'cubing' the number. We can write it as 4^3. The raised 3 in 4^3 is called an 'exponent.'"
- "Because 4^3 is 64, we can write $4^3 = 64$ and say, '4 **cubed** is 64.' We also say that 64 is a 'perfect cube.'"

Explain that we can also use exponents as a shorthand for the units used for area and volume:

- A square with side length 5 inches has an area of 25 square inches, which we can write as 25 in^2.
- A cube with an edge length 4 centimeters has a volume of 64 cubic centimeters, which we can write as 64 cm^3.

Ask students to read a few areas and volumes in different units (for instance, 100 ft^2 is read "100 square feet" and 125 yd^3 is read "125 cubic yards").

Keep students in groups of 2. Give students 3–4 minutes of quiet time to complete the activity and a minute to discuss their response with their partner. Ask partners to note any disagreements so they can be discussed.

Use *Collect and Display* to direct attention to words collected and displayed from an earlier activity. Collect the language students use when discussing their responses to the questions about volume. Display words and phrases, such as "perfect square," "squaring," "squared," "an exponent of 2," "perfect cube," "cubing _____," "_____ cubed is ____," and "an exponent of 3."

Student Task Statement

Make sure to include correct units of measure as part of each answer.

1. A square has side length 10 cm. Use an **exponent** to express its area.
2. The area of a square is 7^2 sq in. What is its side length?
3. The area of a square is 81 m^2. Use an exponent to express this area.
4. A cube has edge length 5 in. Use an exponent to express its volume.
5. The volume of a cube is 6^3 cm^3. What is its edge length?
6. A cube has edge length s units. Use an exponent to write an expression for its volume.

Student Response

1. 10^2 cm^2
2. 7 inches
3. 9^2 m^2
4. 5^3 in^3
5. 6 cm
6. s^3 units^3 or s^3 cubic units

Building on Student Thinking

Upon seeing the expression 6^3, some students may neglect to interpret the question, automatically calculate, and conclude that the edge length is 216 cm. Ask them to check their answer by finding the volume of a cube with edge length 216 cm.

Are You Ready for More?

The number 15,625 is both a perfect square and a perfect cube. It is a perfect square because it equals 125^2. It is also a perfect cube because it equals 25^3. Find another number that is both a perfect square and a perfect cube. How many of these can you find?

Extension Student Response

The smallest examples are 0, 1, 64, 729, and 4,096.

Activity Synthesis

Direct students' attention to the reference created using *Collect and Display.* Ask partners to share disagreements in their responses, if any. Then focus the whole-class discussion on the last two questions. Select a couple of previously identified students to share their interpretations of the question about a cube with an edge length of 5 inches. Invite students to borrow language from the display as needed and update the reference to include additional phrases as they respond.

Highlight that a cube with a volume of 6^3 cubic units has an edge length of 6 units, because we know there are $6 \cdot 6 \cdot 6$ unit cubes in a cube with that edge length.

In other words, we can express the volume of a cube using a number (216), a product of three numbers ($6 \cdot 6 \cdot 6$), or an expression that uses exponent (6^3). This idea can be extended to all cubes. The volume of a cube with edge length s is:

$$s \cdot s \cdot s$$

$$s^3$$

Students will have more opportunities to generalize the expressions for the volume of a cube in the next lesson.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Maintain a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of exponents in the context of geometry. Terms may include: "squared," "cubed," "exponent," and "base."
Supports accessibility for: Conceptual Processing, Language

Lesson Synthesis

Review the language and notation for squaring and cubing a number. Remind students that we use this notation for square units and cubic units, too.

- When we multiply two of the same number, such as $10 \cdot 10$, we say that we are "squaring" the number. We write, for example, $10^2 = 100$, and say, "Ten squared is one hundred."
- When we multiply three of the same number, such as $10 \cdot 10 \cdot 10$, we say that we are "cubing" the number. We write, for example, $10^3 = 1{,}000$, and say, "Ten cubed is one thousand."
- Exponents are used to write square units and cubic units. The area of a square with a side length of 7 km is 49 km^2. The volume of a cube with an edge length of 2 millimeters is 8 mm^3.

A note about materials for an upcoming unit:

For the first lesson in the unit on ratios, students will need to bring in a personal collection of 10–50 small objects. Examples include rocks, seashells, trading cards, or coins. Inform or remind students about this.

17.5 Exponent Expressions

Cool-down

5 min

Standards

Addressing **6.EE.A.1**

Launch

Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and these questions:

- What norm(s) should stay the way they are?
- What norm(s) do you think should be made more clear? How?
- What norms are missing that you would add?
- What norm(s) should be removed?

Ask students to respond to one or more of the questions after completing the *Cool-down* on the same sheet. Make sure students know they can make suggestions for both student and teacher norms.

After collecting the *Cool-downs*, identify themes from the norms questions. There will be many opportunities throughout the year to revise the classroom norms, so focus on revision suggestions that multiple students made to share in the next exercise. One option is to list one addition, one revision, and one removal that the class has the most agreement about.

Student Task Statement

1. Which is larger, 5^2 or 3^3?
2. A cube has an edge length of 21 cm. Use an exponent to express its volume.

Student Response

1. $3^3 = 27$ and $5^2 = 25$, so 3^3 is larger than 5^2.
2. 21^3 cm^3 or 21^3 cubic centimeters

Responding to Student Thinking

More Chances

Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 17 Summary

When we multiply two of the same numbers together, such as $5 \cdot 5$, we say that we are *squaring* the number. We can write it like this:

$$5^2$$

The raised 2 in 5^2 is called an **exponent**.

Because $5 \cdot 5 = 25$, we write $5^2 = 25$ and we say, "5 **squared** is 25."

When we multiply three of the same numbers together, such as $4 \cdot 4 \cdot 4$, we say that we are *cubing* the number. We can write it like this:

$$4^3$$

Because $4 \cdot 4 \cdot 4 = 64$, we write $4^3 = 64$ and we say, "4 **cubed** is 64."

We also use an exponent for square units and cubic units.

- A square with a side length of 5 inches has an area of 25 in^2.
- A cube with an edge length of 4 cm has a volume of 64 cm^3.

To read 25 in^2, we say “25 square inches,” just like before.

The area of a square with a side length of 7 kilometers is 7^2 km^2. The volume of a cube with an edge length of 2 millimeters is 2^3 mm^3.

In general, the area of a square with a side length of s is s^2, and the volume of a cube with an edge length of s is s^3.

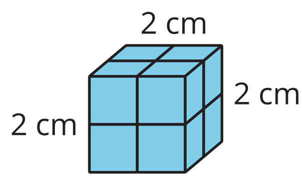
Glossary

- cubed
- exponent
- squared

Lesson 17 Practice Problems

1 Student Task Statement

What is the volume of this cube?



Solution

8 cu cm ($2 \cdot 2 \cdot 2 = 8$)

2 Student Task Statement

a. Decide if each number on the list is a perfect square.

16	125
20	144
25	225
100	10,000

b. Write a sentence that explains your reasoning.

Solution

a. All of these numbers, except 20 and 125, are perfect squares.

b. Sample response: Perfect squares can be found by multiplying a whole number by itself.

3 Student Task Statement

a. Decide if each number on the list is a perfect cube.

1	27
3	64
8	100
9	125

b. Explain what a perfect cube is.

Solution

a. All of the numbers except 3, 9, and 100 are perfect cubes.

b. Sample response: Perfect cubes can be found by using a whole number as a factor three times.

4

Student Task Statement

a. A square has a side length of 4 cm. What is its area?

b. The area of a square is 49 m^2. What is its side length?

c. A cube has an edge length of 3 in. What is its volume?

Solution

a. 16 cm^2

b. 7 m

c. 27 in^3

5

from Unit 1, Lesson 16

Student Task Statement

Prism A and Prism B are rectangular prisms.

- Prism A is 3 inches by 2 inches by 1 inch.
- Prism B is 1 inch by 1 inch by 6 inches.

Select **all** statements that are true about the two prisms.

A. They have the same volume.

B. They have the same number of faces.

C. More inch cubes can be packed into Prism A than into Prism B.

D. The two prisms have the same surface area.

E. The surface area of Prism B is greater than that of Prism A.

Solution

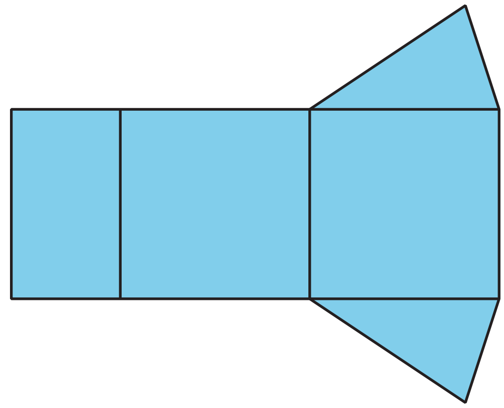
A, B, E

6 from Unit 1, Lesson 14

Student Task Statement

a. What polyhedron can be assembled from this net?

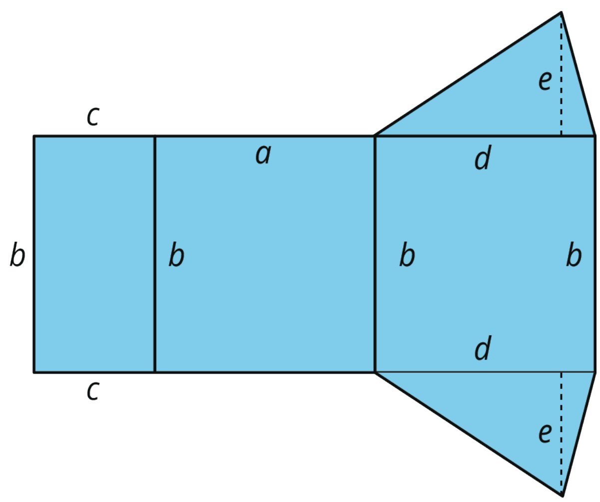
b. What information would you need to find its surface area? Be specific, and label the diagram as needed.



Solution

a. Triangular prism

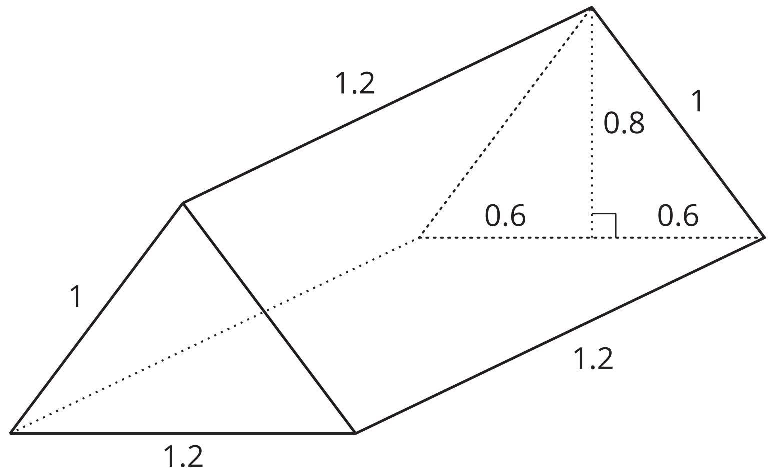
b. Length and width of each rectangular face (as shown in the diagram), as well as the height of the triangular faces



7 from Unit 1, Lesson 15

Student Task Statement

Find the surface area of this triangular prism. All measurements are in meters.



Solution

4.8 square meters. Sample reasoning:

- There are two triangular faces with an area of 0.48 square meters each. $\frac{1}{2} \cdot (1.2) \cdot (0.8) = 0.48$
- There are two rectangular faces with area of 1.2 square meters each. $1 \cdot (1.2) = 1.2$
- There is one rectangular face with an area of $(1.2) \cdot (1.2) = 1.44$ square meters.
- $2 \cdot (0.48) + 2 \cdot (1.2) + (1.44) = 4.8$, or 4.8 square meters.