

Applying the Quadratic Formula (Part 2)

Let's use the quadratic formula and solve quadratic equations with care.

18.1 Bits and Pieces

Evaluate each expression for $a = 9$, $b = -5$, and $c = -2$

1. $-b$
2. b^2
3. $b^2 - 4ac$
4. $-b \pm \sqrt{a}$

18.2 Using the Formula with Care

Here are four equations, followed by attempts to solve them using the quadratic formula. Each attempt contains at least one error.

- Solve 1–2 equations by using the quadratic formula.
- Then, find and describe the error(s) in the worked solutions of the same equations as the ones you solved.

Equation 1: $2x^2 + 3 = 8x$

Equation 2: $x^2 + 3x = 10$

Equation 3: $9x^2 - 2x - 1 = 0$

Equation 4: $x^2 - 10x + 23 = 0$

Here are the worked solutions with errors:

Equation 1: $2x^2 + 3 = 8x$

$a = 2, b = -8, c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{64 - 24}}{4}$$

$$x = \frac{8 \pm \sqrt{40}}{4}$$

$$x = 2 \pm \sqrt{10}$$

Equation 2: $x^2 + 3x = 10$

$a = 1, b = 3, c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 40}}{2}$$

$$x = \frac{-3 \pm \sqrt{-31}}{2}$$

No solutions

Equation 3: $9x^2 - 2x - 1 = 0$

$a = 9, b = -2, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(9)(-1)}}{2}$$

$$x = \frac{2 \pm \sqrt{4 + 36}}{2}$$

$$x = \frac{2 \pm \sqrt{40}}{2}$$

Equation 4: $x^2 - 10x + 23 = 0$

$a = 1, b = -10, c = 23$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{(-10)^2 - 4(1)(23)}}{2}$$

$$x = \frac{-10 \pm \sqrt{-100 - 92}}{2}$$

$$x = \frac{-10 \pm \sqrt{-192}}{2}$$

No solutions

18.3 Sure about That?

1. The equation $h(t) = 2 + 30t - 5t^2$ represents the height, as a function of time, of a pumpkin that was catapulted up in the air. Height is measured in meters, and time is measured in seconds.
 - a. The pumpkin reached a maximum height of 47 meters. How many seconds after launch did that happen? Show your reasoning.
 - b. Suppose someone was unconvinced by your solution. Find another way (besides the steps you already took) to show your solution is correct.
2. The equation $r(p) = 80p - p^2$ models the revenue a band expects to collect as a function of the price of one concert ticket. Ticket prices and revenues are in dollars.

A band member says that a ticket price of either \$15.50 or \$74.50 would generate approximately \$1,000 in revenue. Do you agree? Show your reasoning.

 **Are you ready for more?**

Function g is defined by the equation $g(t) = 2 + 30t - 5t^2 - 47$. Its graph opens downward.

1. Find the zeros of function g without graphing. Show your reasoning.
2. Explain or show how the zeros you found can tell us the vertex of the graph of g .
3. Study the expressions that define functions g and h (which defined the height of the pumpkin). Explain how the maximum of function h , once we know it, can tell us the maximum of g .

Lesson 18 Summary

The quadratic formula has many parts in it. A small error in any one part can lead to incorrect solutions.

Suppose we are solving $2x^2 - 6 = 11x$. To use the formula, let's rewrite it in the form of $ax^2 + bx + c = 0$, which gives $2x^2 - 11x - 6 = 0$.

Here are some things to keep in mind:

- Use the correct values for a , b , and c in the formula.

$$x = \frac{-11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

Nope! b is -11 , so $-b$ is $-(-11)$, which is 11 , not -11 .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

That's better!

- Multiply 2 by a for the denominator in the formula.

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2}$$

Nope! The denominator is $2a$, which is $2(2)$, or 4 .

$$x = \frac{11 \pm \sqrt{(-11)^2 - 4(2)(-6)}}{2(2)}$$

That's better!

- Remember that squaring a negative number produces a positive number.

$$x = \frac{11 \pm \sqrt{-121 - 4(2)(-6)}}{4}$$

Nope! $(-11)^2$ is 121 , not -121 .

$$x = \frac{11 \pm \sqrt{121 - 4(2)(-6)}}{4}$$

That's better!

- Remember that a negative number times a positive number is a negative number.

$$x = \frac{11 \pm \sqrt{121 - 48}}{4}$$

Nope! $4(2)(-6) = -48$, and $121 - (-48)$ is $121 + 48$.

$$x = \frac{11 \pm \sqrt{121 + 48}}{4}$$

That's better!

- Follow the properties of algebra.

$$x = \frac{11 \pm \sqrt{169}}{4}$$

$$x = 11 \pm \sqrt{42.25}$$

$$x = \frac{11 \pm 13}{4}$$

That's better!

Nope! Both parts of the numerator, 11 and $\sqrt{169}$, get divided by 4. Also, $\frac{\sqrt{169}}{4}$ is not $\sqrt{42.25}$.

Let's finish by evaluating $\frac{11 \pm 13}{4}$ correctly:

$$x = \frac{11 + 13}{4}$$

$$\text{or } x = \frac{11 - 13}{4}$$

$$x = \frac{24}{4}$$

$$\text{or } x = -\frac{2}{4}$$

$$x = 6$$

$$\text{or } x = -\frac{1}{2}$$

To make sure our solutions are correct, we can substitute each solution back into the original equation and see whether it results in a true equation.

Checking 6 as a solution:

$$\begin{aligned} 2x^2 - 6 &= 11x \\ 2(6)^2 - 6 &= 11(6) \\ 2(36) - 6 &= 66 \\ 72 - 6 &= 66 \\ 66 &= 66 \end{aligned}$$

Checking $-\frac{1}{2}$ as a solution:

$$\begin{aligned} 2x^2 - 6 &= 11x \\ 2\left(-\frac{1}{2}\right)^2 - 6 &= 11\left(-\frac{1}{2}\right) \\ 2\left(\frac{1}{4}\right) - 6 &= -\frac{11}{2} \\ \frac{1}{2} - 6 &= -5\frac{1}{2} \\ -5\frac{1}{2} &= -5\frac{1}{2} \end{aligned}$$

We can also graph the equation $y = 2x^2 - 11x - 6$ and find its x -intercepts to see whether our solutions to $2x^2 - 11x - 6 = 0$ are accurate (or close to accurate).

