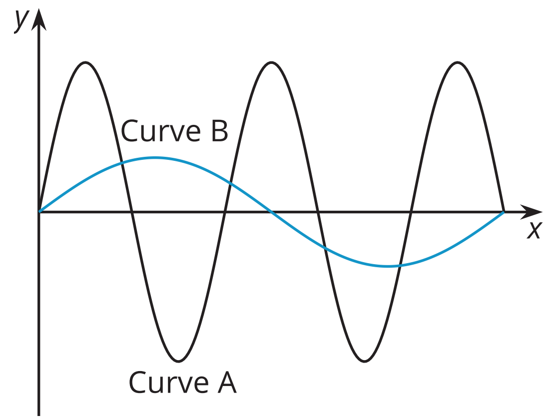
Features of Trigonometric Graphs (Part 1)

Let’s compare graphs and equations of trigonometric functions.

16.1 Musical Notes

Here are pictures of sound waves for two different musical notes:



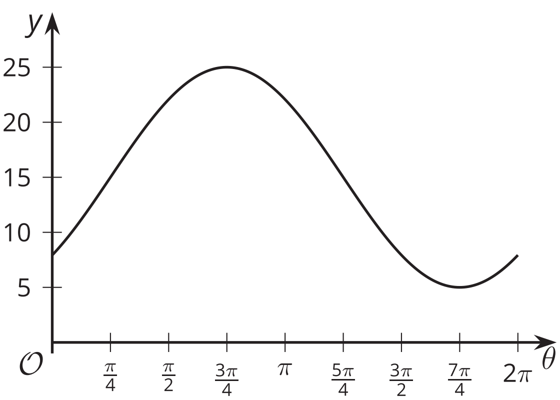
16.2 Card Sort: Equations and Graphs

Your teacher will give you a set of cards. Take turns with your partner to match each graph with the equation and description that it represents. More than 1 equation or description can match the same graph.

1. For each match that you find, explain to your partner how you know it’s a match.
2. For each match that your partner finds, listen carefully to your partner's explanation. If you disagree, discuss your thinking and work to reach an agreement.

Are you ready for more?

1. Use the sine function to find an equation for this graph.



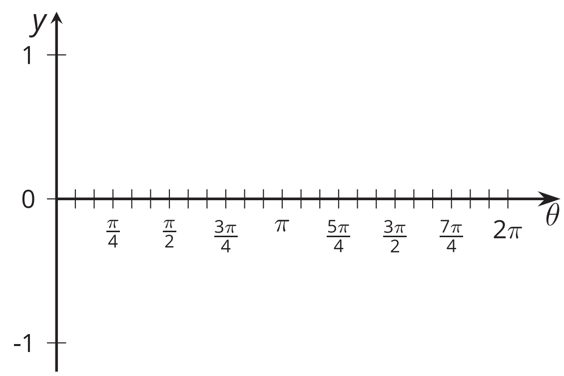
2. Use the cosine function to find another equation for the same graph.

16.3 Double the Sine

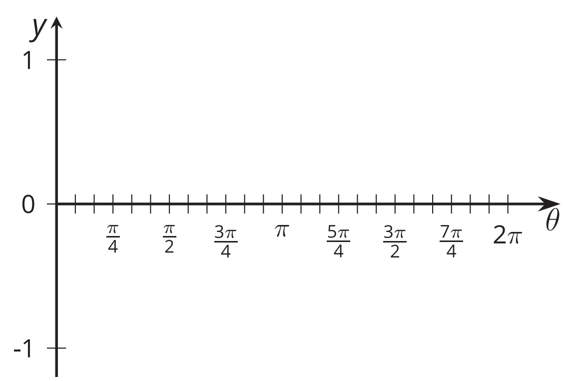
1. Complete the table of values for the expression $\sin(2\theta)$

θ	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin(2\theta)$											

2. Plot the values and sketch a graph of the equation $y = \sin(2\theta)$. How does the graph of $y = \sin(2\theta)$ compare to the graph of $y = \sin(\theta)$?

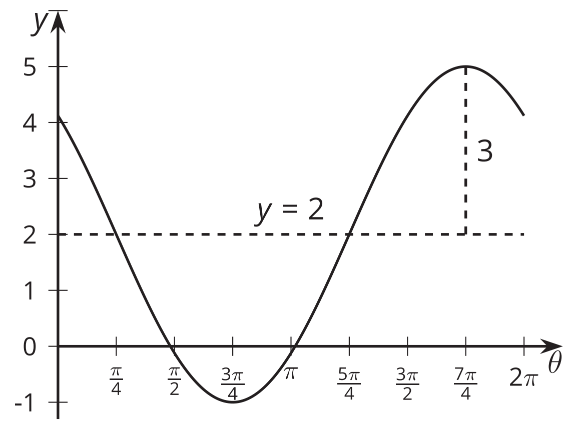


3. Predict what the graph of $y = \cos(4\theta)$ will look like and make a sketch. Explain your reasoning.

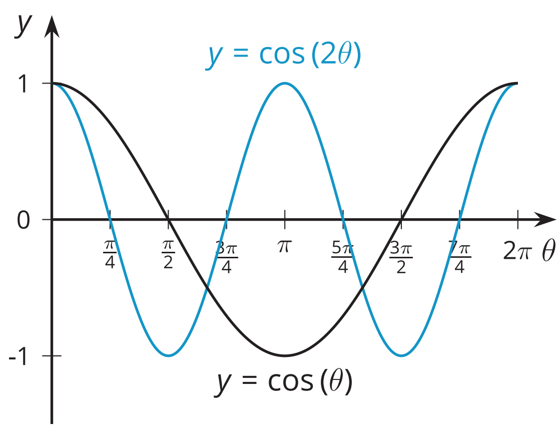


Lesson 16 Summary

We can find the amplitude and midline of a trigonometric function using the graph or from an equation. For example, let’s look at the function given by the equation $y = 3\cos\left(\theta + \frac{\pi}{4}\right) + 2$. We can see that the midline of this function is 2 because of the vertical translation up by 2. This means the horizontal line $y = 2$ goes through the middle of the graph. The amplitude of the function is 3. This means that the maximum value it takes is 5, 3 more than the midline value, and the minimum value it takes is -1, 3 less than the midline value. The horizontal translation is $\frac{\pi}{4}$ to the left, so instead of having, for example, a minimum at π, the minimum is at $\frac{3\pi}{4}$. Here is what the graph looks like:



Another type of transformation is one that affects the period, and that is when a horizontal scale factor is used. For example, let's look at the equation $y = \cos(2\theta)$, where the variable, θ, is multiplied by a number. Here, 2 is the scale factor affecting θ. When $\theta = 0$, we have $2\theta = 0$ so the graph of this cosine equation starts at $(0, 1)$, just like the graph of $y = \cos(\theta)$. When $x = \pi$, we have $2\theta = 2\pi$, so the graph of $y = \cos(2\theta)$ goes through two full periods in the same horizontal span that it takes $y = \cos(\theta)$ to complete one full period, as shown in their graphs.



Notice that the graph of $y = \cos(2\theta)$ has the same general shape as the graph of $y = \cos(\theta)$ (same midline and amplitude) but the waves are compressed together. And what if we wanted to give the graph of cosine a stretched appearance? Then we could use a horizontal scale factor between 0 and 1. For example, the graph of $y = \cos(\frac{\theta}{6})$ has a period of 12π.