



# Coordinate Proof

Let's use coordinates to prove theorems and to compute perimeter and area.

## 14.1

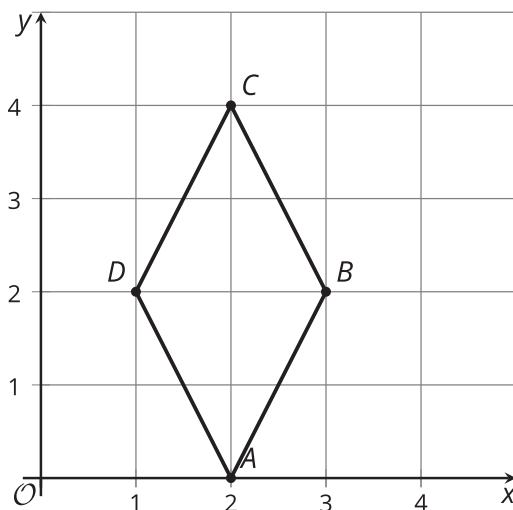
## Which Three Go Together: Coordinate Quadrilaterals

Which three go together? Why do they go together?

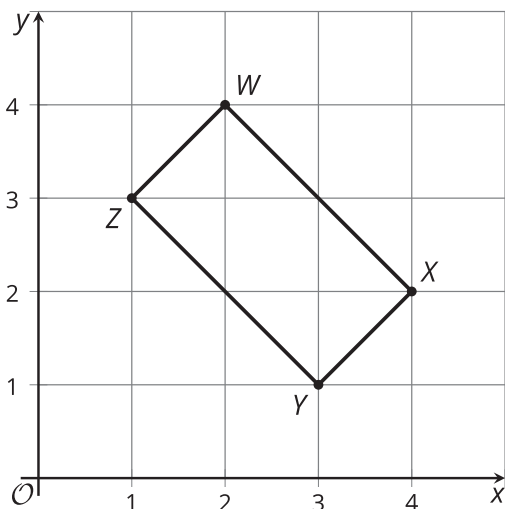
**A**



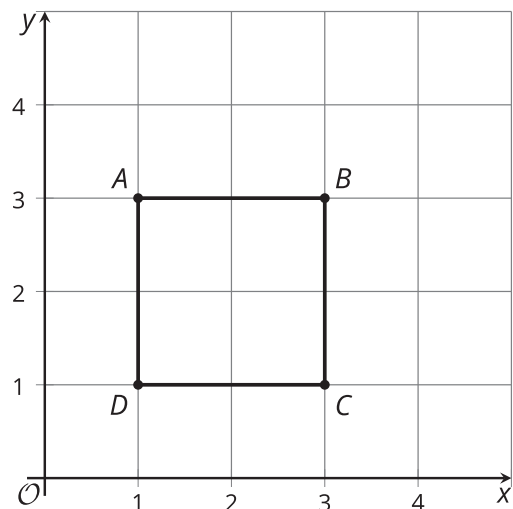
**B**



**C**



**D**



## 14.2

## Name This Quadrilateral

A quadrilateral has vertices  $(0, 0)$ ,  $(4, 3)$ ,  $(13, -9)$ , and  $(9, -12)$ .

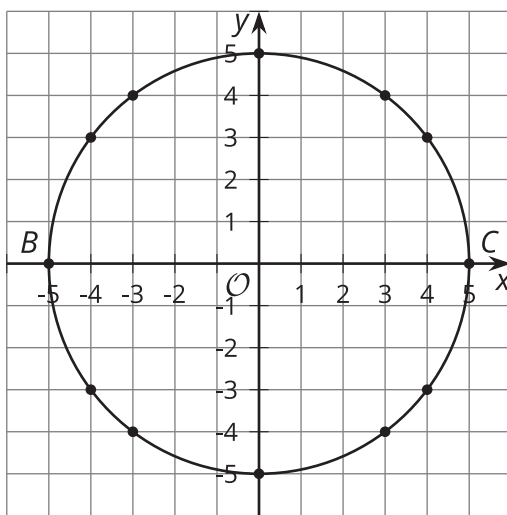
1. What type of quadrilateral is it? Explain or show your reasoning.
2. Find the perimeter of this quadrilateral.
3. Find the area of this quadrilateral.

**Are you ready for more?**

1. A parallelogram has vertices  $(0, 0)$ ,  $(5, 1)$ ,  $(-2, 10)$ , and  $(3, 10)$ . Find the area of this parallelogram.
2. Consider a general parallelogram with vertices  $(0, 0)$ ,  $(a, b)$ ,  $(kb, ka)$ , and  $(a - kb, b + ka)$ , where  $a$  and  $b$  are positive, and a scale factor of  $k$ . Show that the parallelogram is a rectangle, then write an expression for its area in terms of  $a$ ,  $b$ , and  $k$ .

## 14.3 Circular Logic

The image shows a circle with several points plotted on the circle.



1. How does segment  $BC$  relate to the circle?
2. Choose one of the plotted points on the circle and call it  $D$ . Each student in the group should choose a different point. Draw segments  $BD$  and  $DC$ . What does the measure of angle  $BDC$  appear to be?
3. Calculate the slopes of segments  $BD$  and  $DC$ . What do your results tell you?
4. Compare your results to those of others in your group. What did they find?
5. Using your group's results, write a conjecture that captures what you are seeing.

## Lesson 14 Summary



What kind of shape is quadrilateral  $ABCD$ ? It looks like it might be a rhombus. To check, we can calculate the length of each side. Using the Pythagorean Theorem, we find that the lengths of segments  $AB$  and  $CD$  are  $\sqrt{45}$  units, and the lengths of segments  $BC$  and  $DA$  are  $\sqrt{37}$  units. All side lengths are between 6 and 7 units long, but they are not exactly the same. So our calculations show that  $ABCD$  is not really a rhombus, even though at first glance we might think it is.

We did just show that two pairs of opposite sides of  $ABCD$  are congruent. This means that  $ABCD$  must be a parallelogram. Checking slopes confirms this. Sides  $AB$  and  $CD$  both have a slope of  $\frac{1}{2}$ . Sides  $BC$  and  $DA$  both have a slope of 6.

Can we find the area of triangle  $EFG$ ? That seems tricky, because we don't know the height of the triangle using  $EG$  as the base. However, angle  $EFG$  seems like it could be a right angle. In that case, we could use sides  $EF$  and  $FG$  as the base and height.

To see if  $EFG$  is a right angle, we can calculate slopes. The slope of  $EF$  is  $\frac{8}{6}$  or  $\frac{4}{3}$ , and the slope of  $FG$  is  $-\frac{3}{4}$ . Since the slopes are opposite reciprocals, the segments are perpendicular and angle  $EFG$  is indeed a right angle. This means that we can think of  $EF$  as the base and  $FG$  as the height. The length of  $EF$  is 10 units and the length of  $FG$  is 5 units. So the area of triangle  $EFG$  is 25 square units because  $\frac{1}{2} \cdot 10 \cdot 5 = 25$ .