Using Radians

Let's see how radians can help us calculate sector areas and arc lengths.

17.1 What Fraction?

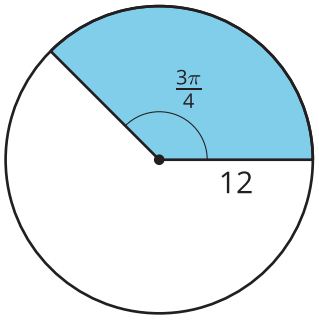
A circle with radius 24 inches has a sector with central angle $\frac{\pi}{3}$ radians.

1. What fraction of the whole circle is represented by this sector?

2. Find the area of the sector.

17.2 A Sector Area Shortcut

Lin and Elena are trying to find the area of the shaded sector in the image.



Lin says, "We've found sector areas when the central angles are given in degrees, but here the central angle is in radians. Should I start by finding the area of the full circle?"

Elena says, "I saw someone using the formula $\frac{1}{2}r^2\theta$, where θ is the measure of the angle in radians, and r is the radius. But I don't know where that came from."

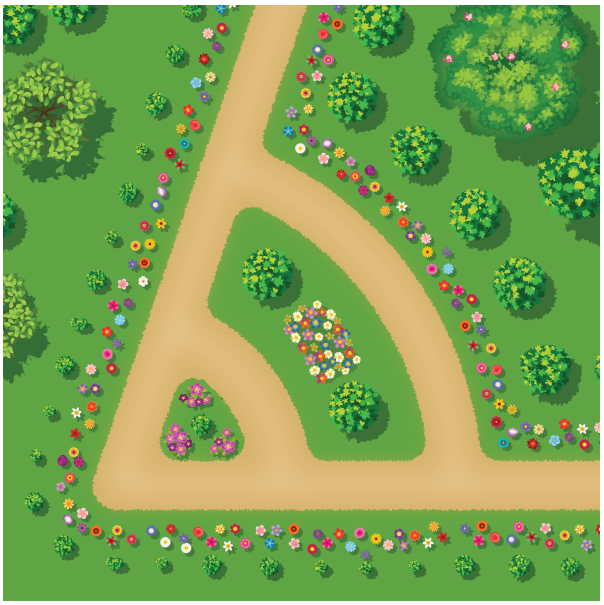
1. Compare and contrast finding sector areas for central angles measured in degrees and those measured in radians.

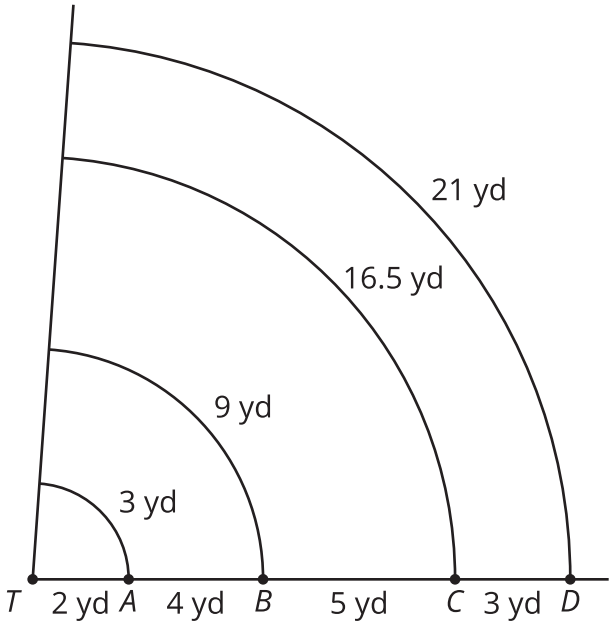
2. Explain why the formula that Elena saw works.

3. Find the area of the sector.

17.3 An Arc Length Shortcut

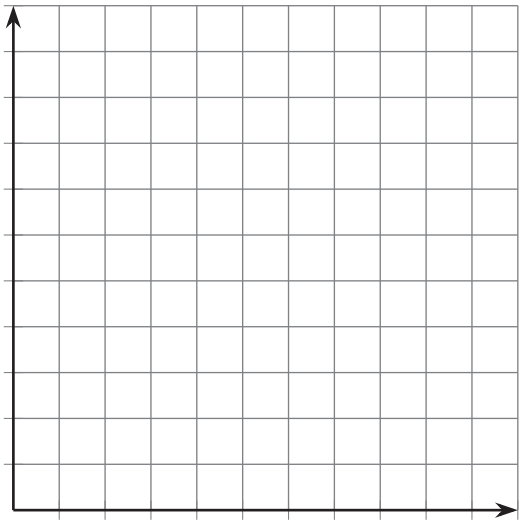
A city is designing a new park. One feature of the park is a garden designed for walking meditation. The space they have chosen is 14 yards long on one edge. There will be alternating circular arcs of plants and paved paths.





1. Create a table that shows the arc length ℓ of the curved paths as a function of the radius r of the straight connecting paths.

2. Plot the points from your table on the coordinate grid, and connect them.



3. The points should form a line. Write an equation for this line, using the variables ℓ and r.

4. What does the slope of the line mean in the context of the curved and connecting paths?

5. The city is considering building another curved path starting at point E that will be an additional 6 yards past point D. How far would the walk be from one straight connector path to the other?

Are you ready for more?

Suppose a Ferris wheel with a 50 foot radius takes 2 minutes to complete 1 rotation.

1. After 20 seconds, through how many radians would a rider on the Ferris wheel have traveled?

2. How long would it take to travel through 3 radians?

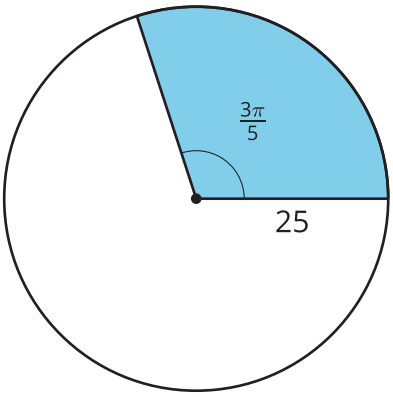
3. What is the rotational speed of the rider on the Ferris wheel in radians per second?

4. A decorative light is attached to the Ferris wheel 25 feet from the center of the wheel. What is the rotational speed of the light in radians per second?

Lesson 17 Summary

Suppose we want to find the area of a sector of a circle whose central angle is θ radians and whose radius is r units. First we find the area of the whole circle, πr^2 square units. Then we find the fraction of the circle represented by the sector. The complete circle measures 2π radians. So the fraction is $\frac{\theta}{2\pi}$. Multiply the fraction by the area to get $\frac{\theta}{2\pi} \cdot \pi r^2$. This can be rewritten as $\frac{1}{2} r^2 \theta$.

For example, let the radius be 25 units and the radian measure of the sector's central angle be $\frac{3\pi}{5}$ radians. Substitute these values into the formula we just created to get $\frac{1}{2}(25)^2 \cdot \frac{3\pi}{5}$. This can be rewritten as $\frac{375\pi}{2}$, which is about 589 square units. Using the formula was a bit quicker than going through the process of finding the specific fraction of the circle represented by this sector and multiplying by the circle's area.



Additionally, we can write a formula for arc length based on the definition of radian measure: $\theta = \frac{\ell}{r}$, where θ is the central angle measure in radians, ℓ is the arc length, and r is the radius. Rewrite the definition to solve for arc length. We get $\ell = r\theta$.